A new hybrid method for demoldability analysis of discrete geometries

Jorge Manuel Mercado-Colmenero [a], M. A. R. Paramio [b], Jesus Maria Perez-Garcia [c], Cristina Martin-Doñate [d*]

[a, b, d] Department of Engineering Graphics Design and Projects. University of Jaen. Spain

[c] Department of Mechanical Engineering. Politechnical University of Madrid. Spain

*Corresponding Author[d]

Campus Las Lagunillas, s/n. Building A3-210

23071 Jaen (Spain)

Phone: +34 953212821, Fax: +34 953212334

E-mail: jmercado@ujaen.es [a], marubio@ujaen.es [b], jesusmaria.perez@upm.es [c], cdonate@ujaen.es [d]

Abstract

In this paper, a new method for demoldability automatic analysis of parts to be manufactured in plastic injection is presented. The algorithm analysis is based on the geometry of the plastic part, which is discretized by a triangular mesh, posing a hybrid discrete demoldability analysis of both the mesh nodes and facets. A first preprocessing phase classifies mesh nodes according to their vertical dimension, assigning each node a plane perpendicular to the given parting direction. By selective projection of facets, closed contours which serve as the basis for calculating the demoldability of the nodes are created. The facets are then cataloged according to demoldability nodes that comprise demoldable, non-demoldable and semi-demoldable facets. Those facets listed as semi-demoldable are fragmented into demoldable and non-demoldable polygonal regions, causing a redefinition of the original mesh as a new virtual geometry. Finally, non-demoldable areas are studied by redirecting the mesh in the direction of the sliding side, and again applying the processing algorithm and cataloging nodes and facets. Resoluble areas of the piece through mobile devices in the mold are obtained. The hybrid analysis model (nodes and facets) takes advantage of working with a discrete model of the plastic part (nodes), supplemented by creating a new virtual geometry (new nodes and facets) that complements the original mesh, providing the designer not only with information about the geometry of the plastic piece but also information on their manufacture, exactly like a CAE tool. The geometry of the part is stored in arrays with information about their manufacture for use in downstream applications.

Keywords: Demoldability analysis, geometric analysis, injection molding, computer aided manufacturing

1. Introduction

Injection molding is a manufacturing process of plastic parts which can make parts with complex geometries and freeform surfaces. It is suitable for high production volumes because of its reduced cycle time. Essentially, an injection mold consists of two parts called the upper and lower cavity, within which the plastic is injected, leading to the desired body. The main parameters that largely determine the structure of a mold are: the presence of undercuts, the chosen parting direction, and the parting line generated between the various mold components. Demoldability geometric analysis aims to determine these parameters automatically based on the definition of the geometry of the part to be molded. Selecting the parting direction and the parting surface is of great significance so that the number of side cores is reduced to a strict minimum. The number and volume of the side cores increases the mold complexity and the costs of manufacture and the time spent in the manufacturing cycle of the pieces to be processed and, ultimately, its production cost.

During the conceptual phase of the design process, customers often impose a large number of changes in the geometry of the part. It is necessary to validate demoldability quickly, detecting and differentiating automated small undercuts and areas that cannot be manufactured. Moreover, in the preparation of price studies for the manufacturing of plastic parts, a quick and accurate review of part geometry in their manufacture can be a determining factor in achieving a deal. The commercial CAD systems used today by companies for the resolution of analysis demoldability still involve a great deal of manual interaction on the part of the designer. The knowledge of the CAD tool is not sufficient and great experience is required in the manufacture of plastic parts.

Reducing design and manufacturing time, establishing good precision and quality in the finished piece and being able to make design changes quickly are the main concerns of the designers of new products in the industry of injection molding. Solving these problems requires a complete automation of the analysis process of the plastic part for its manufacture. Numerous methods at the research level have addressed the analysis of geometric demoldability, but they have many disadvantages in that they are either linked to the modeler to analyzing the piece internally, or need additional computing devices such as GPU's, or are not valid for analyzing all kinds of plastic parts.

In order to solve the problems described above, a new method for analyzing the geometric demoldability and calculation of the parting line is presented. This method is based on the discrete model of the plastic part consisting of a triangular mesh. This methodology develops a hybrid analysis of the part mesh, analyzing the demoldability of both mesh nodes and their facets. A first preprocessing phase classifies discrete mesh nodes according to their manufacturability. Based on this information, a second algorithm processes the mesh facets and color them based on their demoldability. The geometry of the plastic part is stored in separate arrays of nodes and facets with information on their manufacture for use in downstream applications. The new analysis algorithm developed results in a new virtual mesh of the plastic part that complements the initial nodes and adds new semi-facets to provide information not only on the part geometry but also on part manufacture. The resulting mesh represents the demoldability map of the plastic part. Nodes and facets are colored just as with a CAE tool. This algorithm improves the methods developed so far, since is valid for all kinds of plastic parts, for any modeler, and requires no additional hardware, allowing changes to be implemented quickly during the early stages of design and studies of price.

2. Background and related work

Demoldability analysis and calculation of the parting line are considered extremely important tasks in the design of the plastic part due to their influence on the quality and the costs of the manufacturing process. Both academic and industrial researchers have been seeking new methodologies to address the recognition of the surface of the plastic part in its application to the analysis of demoldability in order to achieve high levels of quality while reducing design time and production.

Many methodologies for determining the demoldability of the part along the parting direction are based on visibility analysis. The visibility map of a surface is the set of all directions from which the surface is completely visible. In terms of geometrical analysis of the surface of the plastic part, a parallelism between the concepts of visibility and demoldability can be established. The complexity of calculating the visibility of a surface obviously depends on the complexity of the surface itself which, in the extreme case of freeform surfaces, would be determined by calculating the visibility of each of its points. One of the first methods used for analyzing parts demoldability applying visibility maps was proposed by Chen et al. [1], who introduced the concept of the pocket for the calculation of non-demoldable areas. Chen et al. [2] created an algorithm for obtaining the optimal parting direction, by dividing objects into pockets. In these pockets visibility and moldability play the same role. Chen et al. [3] set both global and local visibility levels in order to extend their work to internal undercuts features and to decompose them into separable portions (based on complete visibility) and undercuts (on partial visibility). Weinstein and Manoochehri [4] obtained optimal parting directions by means of the location of common visibility areas of concave surfaces, the partition line being obtained by analyzing the surfaces belonging to the convex area.

Much research is based on features recognition in casting and plastic parts. A feature is a geometric region of the piece which has additional information about its manufacture. Currently, the solid modeling of the piece does not store this information explicitly. The methods of extracting features allow the development of methodologies that enable us to extract the required information and enter it as an input in a structured algorithm. On the plastic parts, undercut features encompass geometric regions of the piece, which are not accessible along the parting direction. Identifying undercut features directly affects the determination of the partition line and the design of upper and lower cavity, as well as the design of the mobile devices of the mold. Researchers have used different techniques for determining undercut features. Fu et al. developed a complete solution for the computer-aided design of plastic injection molds including the definition, classification and recognition of undercut features [5], determining the parting direction [6], parting line and surfaces [7], upper and lower cavities [8], and design of side cores [9]. Ran and Fu [10] proposed a methodology for obtaining the automatic design of internal pins in injection mold CAD via the automatic recognition of undercut features. Wuerger and Gadh [11], [12] used the concavity features to locate the parting direction, using the convex hull of the part and then the Boolean difference with the piece. Kurt and Gadh [13] extended this research to the concavity features created by extrusion or rotation of a flat section. Lu and Lee [14] performed an analysis of the undercut features method by means of the elements of interference. Yin et al. [15] provided an algorithm to recognize undercut features for near net shapes. Ye et al. [16] proposed an undercut features recognition hybrid method that takes advantage of graph recognition methods. With this method the plastic part is configured by ‘extended attributed face edge graphs’, while undercut features recognition is based on searching for the ‘cut sets’ of undercut sub-graphs. In [17] Ye extended their work to side core design. Based on the curvature properties of entities in the B-Rep model Zhang et al. [18] describe an approach to recognizing DP features. Bassi et al. [19] proposed an automatic mold feature recognition system to recognize protusion and depression as well as intersecting depression features. Other methods combine features recognition algorithms with visibility and accessibility analysis. Surti et al. [20] proposed a projection based methodology to analyze the visibility of a part from a given parting direction without discretizing the part. Singh et al. [21] describe an automated identification, classification, division and determination of the parting direction of complex undercut features of die-cast parts. Then the undercut features are classified using a rule -based algorithm. The 3D solid object in the format B-Rep, boundary representation, means the geometric and topological representation of the object may not be unique, since it depends on the procedures used for the CAD model generation, or the internal kernels used by the CAD program. Furthermore, the recognition of features for a piece created in a CAD system is linked to the modeler with which it was created. Another important issue in the automated recognition of features is the treatment of the interacting features. An interacting feature is the result of the intersection of multiple features. The difficulty in recognizing the interacting features is given mainly because, in the interaction between features, a destruction of the adjacency relationships between features that were in contact occurs. Furthermore, the decomposition from a complex feature to several individual features creates problems of multiple interpretations, generated mainly by the sequences in which the features are recognized

Nee et al. [22,23] addressed the problem of demoldability by classifying plastic part surfaces according to their orientation with respect to the parting direction and their connection to each other. This method uses the dot product between the normal surface vector and parting direction in order to calculate the parting line. Hui and Tan [24] presented a method for obtaining the optimal parting direction using several parameters such as blocking factor and preference value. This set of values is calculated for all proposed parting directions. Ravi and Srinivasan [25] proposed a set of nine criteria for obtaining partition lines of plastic injection parts and casting parts, and applied an automatic process in order to achieve the optimal solution according to this set of criteria. All of the above-mentioned methods face difficulties in recognizing undercut features from solid objects with free form surfaces. Rappaport and Rosenbloom [26] introduced the concept of moldability in simple polygons, linking it to that of monotonicity. Ahn et al. [27] established the conditions that must be accomplished for a polyhedral part to be moldable in two directions and in [28] they extended this work to arbitrary objects. Nevertheless, the complex implementation of their method lengthens considerably the running time of the complete algorithm. Chen and McMains [29] introduced an algorithm for calculating all the Undercut free parting directions for extrusions. This algorithm is based on a previous study on 2D profile analysis addressed in [30,31].

Mold design methods based on fuzzy logic algorithms are involved with the fact that several influence parameters such as parting direction, parting line, undercut features etc. interact among themselves. It is necessary to develop a methodology that studies these parameters in an integrated way. Yin et al. [32] presented a hybrid approach to obtaining an optimal combination of these parameters. Chen et al. [33] developed a method for finding and locating the minimum bounding box volume of tridimensional CAD parts and proposed a weighting scheme based on fuzzy logic for the automatic selection of parting directions in plastic parts injection.

Other authors have conducted their research into the geometric resolution of multipiece molds based on the study of accessibility. The technology of multipiece molds largely overcomes the restrictions imposed by traditional molds. The multipart mold has several parting directions, with each of its parts incorporating a different parting line. The algorithms encompassed by this group, mainly developed by Huang et al. [34] and Priyadarshi et al. [37], are based on accessibility studies conducted by Dhaliwal. Dhaliwal et al. [35, 36] presented an algorithm for calculating global accessibility cones for each side of a polyhedral object. A point belonging to a geometric entity is accessible from a given direction if a semi-infinite ray can be drawn from it towards the given direction without intersecting inside the geometric entity. Based on this concept a parallel between accessibility and demoldability is established. The inaccessible region in every facet due to the presence of another facet is calculated. Priyadarshi et al. [37] developed an algorithm for automating the design of multi-part permanent molds. First, a set of parting directions d are located based on the geometry of the part. Then an analysis of accessibility is performed at each facet along the chosen parting direction, checking oclusion of each facet with the remaining facets. However these molds are of limited application, and are used more in the field of prototype tooling. Multipiece mold design problems are computationally very challenging. Existing techniques work well if the number of mold pieces in the final solution is relatively small (Bourne et al [38]). Chen and Rosen [39] presented a region-based approach to automated mold design for simple two-piece molds as well as molds with many additional moving sections. With this approach part faces are partitioned into regions, each of which can be formed by a single mold piece. Later they presented an approach [40] based on a reverse glue operation for generating parting faces. A problem definition of parting face generation for a region is provided. Correspondingly, three face generating criteria are identified.

Other researchers perform a demoldability analysis of the plastic part by means of techniques based on making cuts to the model. The result is a variable set of sections, which are then traversed by a bundle of lines to obtain a set of border points. The demoldability and the parting line analysis are deduced by studying the points obtained [41], [42], [43], [44]. Ganter et al. [41] analyzed sections made to the plastic part and established a set of nine rules based on the experience as well as a vast bibliography. Wong et al. [42] improves the algorithm extending the scope to pieces with partition lines with various contours and increasing the accuracy of the algorithm. Rubio et al. [43] use a technique of discretization of 3D solids to solve the problem of demoldability, performing a process of analysis by planes. More recently Martin et al. [44],[45] propose a method of discretization of the solid, which transforms the solid body into a grid of points, reallocating points in the mesh based on their demoldability. While the main advantage of these methods is that they analyze the part geometry externally, they do not achieve the accuracy of methods based on recognition of features.

Some authors use tessellated models as input to their algorithms. Tessellations of the plastic part provide a great advantage when the piece incorporates complicated and freeform surfaces. Some recognition methods locate non-demoldable areas in the plastic part based on the use of GPU'S. The first applications of programmable GPUs were described by Khardekar et al. [46]. Khardekar et al. [46], [47] use the GPU as a means of recognizing parting directions which have no undercuts. They propose a method to detect undercuts and thus enlighten as well as to locate minimal and insufficient demolding angles. Other authors also use tessellated models to obtain the parting direction and the parting line. Chakraborty et al [48] identifies non-convex regions, determining the set of possible parting directions as well as the accessibility along each parting direction. Finally the best parting direction is selected, and a parting surface corresponding to the best parting direction is generated. The best parting direction is the one that achieves the best value for a factor which is the sum of several individual factors, including visibility, flatness of the parting line, and demolding depth. Singh et al. [49] use a tesselated model and discuss the classification of the die cast part surfaces, identifying undercuts and protusions, and determining the parting line. By employing Euler operations, Shin and Lee [50] presented a procedure for locating side cores and upper and lower cavities by means of identifying interference surfaces (primary and secondary) of the mold. Banerjee and Gupta [51] described algorithms for generating shapes of side actions. Given a set of undercut facets on a polyhedral part and the main mold opening directions, candidate retraction space for every undercut facet is computed. This space represents the set of candidate translation vectors that can be used by the side action. Li et al[52] develop a method which computes a smooth parting line which runs through a band of triangles whose normals are approximately perpendicular to the parting direction. The methods presented here use a faceted model as a basis for the implementation of undercut recognition algorithms. However, many of these assume that the part to be produced is moldable, without internal side cores or undercut features that interact. Moreover these methods work with the normal vector of the facets, making the calculations complex and usually requiring additional hardware

This paper proposes a new method for performing the analysis of demoldability and the calculation parting line for a given parting direction. The algorithm works with an initial file representing discrete surface geometry, and obtains as a result a new virtual discrete geometry which incorporates information of the molding manufacture of the piece. It works with the coordinates of the points and is valid for all kinds of parts and CAD systems independently of the modeler used. The algorithm detects not only lateral undercuts resolvable through side cores, but non-demoldable areas as well, allowing the designer to make changes in the early phases of design.

3. Methodology

Let B be the 3D part to be manufactured. The algorithm proposed in this article must be valid for use in any part to be produced with the injection molding technique.

$$\forall \mathcal{B} \in \mathbb{R}^3$$

First the solid $\mathcal{B}$ is transformed into a discrete mesh $\mathcal{B}'$ consisting of a set of triangular facets and nodes. Thus the discrete mesh $\mathcal{B}'$ is defined as a vector of dimensions 1x2, formed firstly by a rank nx3 matrix $\mathcal{B}'_f$ grouping the entire set of triangular facets (n being the total number of mesh facets), and secondly by the rank 3nx3 matrix $\mathcal{B}'_n$ grouping the whole set of nodes of the mesh. This allows us to define the geometry of the part externally Fig 1 (a) and 1 (b) and thus avoid the problems of dependence of the CAD model.

$$\forall \mathcal{B} \rightarrow \mathcal{B}' \in \mathcal{M}_{1x2}(\mathbb{R}) \rightarrow \mathcal{B}' = \{\mathcal{B}'_f \quad \mathcal{B}'_n\}$$

Where first term $\mathcal{B}'_{11}$ of matrix $\mathcal{B}'$, represents the set of facets of the mesh.

$$\mathcal{B}'_{11} = \mathcal{B}'_f \rightarrow \mathcal{B}'_f \in \mathcal{M}_{\mathrm{n}x\mathrm{m}}\ (\mathbb{R})$$

$$\mathcal{B}'_f \coloneqq \mathscr{b}'_f(\mathrm{ij})_{\mathrm{n}x\mathrm{m}}$$

Futhermore matrix $\mathcal{B}'_f$ could be expressed by nodes that compose it. For instance given a facet F_i, it can be defined by its position in matrix $\mathcal{B}'_f$.

$$\mathscr{b}'^{\,i}_f = \mathrm{F_i}$$

Or it can be defined by nodes $\{\mathrm{P_{i1}}, \dots, \mathrm{P_{ij}}, \dots, \mathrm{P_{im}}\}$ that compose it.

$$\mathrm{F_i} = \{\mathrm{P_{i1}}, \dots, \mathrm{P_{ij}}, \dots, \mathrm{P_{im}}\} \rightarrow \mathscr{b}'_f(\mathrm{ij}) = \mathrm{P_{ij}}$$

$$1 \le \mathrm{i} \le \mathrm{n}\ ;\ 1 \le \mathrm{j} \le \mathrm{m}$$

Where n represents the total number of mesh facets and m represents the number of nodes per facet. By working only with triangular facets, then m = 3.

$$\mathcal{B}'_{12} = \mathcal{B}'_n \rightarrow \mathcal{B}'_n \in \mathcal{M}_{\mathrm{n}\cdot 3x3}\ (\mathbb{R})$$

Where first term $\mathcal{B}'_{12}$ of matrix $\mathcal{B}'$, represents the set of nodes of the mesh.

$$\mathcal{B}'_n \coloneqq \mathscr{b}'_n(\mathrm{ij})_{\mathrm{n}\cdot 3x3}$$

Given a node P_{ij}, it can be defined by its position in matrix $\mathcal{B}'_n$.

$$\mathscr{b}'^{\,i}_n = \mathrm{P_{ij}}$$

Or it can be defined by its coordinates x, y, z.

$$\mathrm{P_{ij}} = \{\mathrm{x_{ij}}, \mathrm{y_{ij}}, \mathrm{z_{ij}}\}$$

$$1 \le \mathrm{i} \le \mathrm{n} \cdot 3\ ;\ 1 \le \mathrm{j} \le 3$$

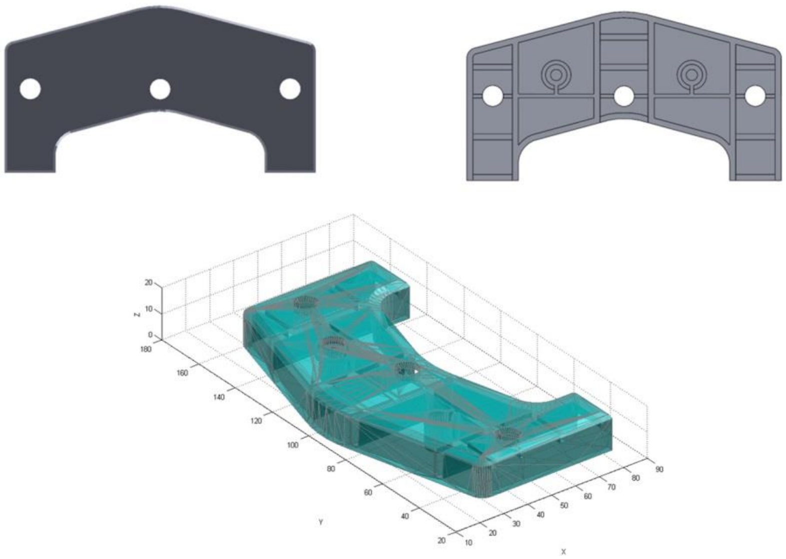
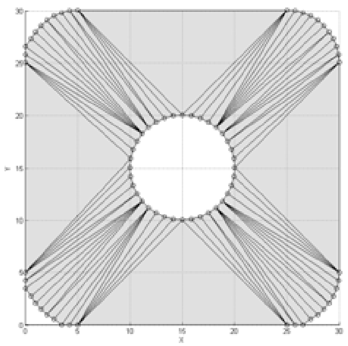
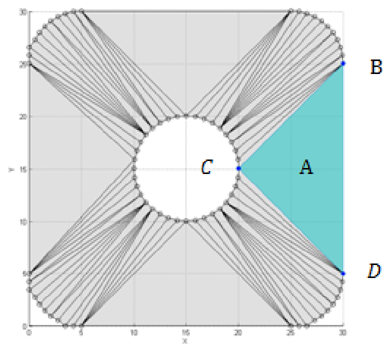


Fig1a.*- Converting a CAD model to a virtual faceted model.*



$$\text{Nodes}: \mathcal{B}'_n = \begin{Bmatrix} P_{11} \\ P_{12} \\ P_{13} \\ \vdots \\ P_{n\cdot m1} \\ P_{n\cdot m2} \\ P_{n\cdot m3} \end{Bmatrix} = \begin{bmatrix} x_{11} & y_{11} & z_{11} \\ x_{12} & y_{12} & z_{12} \\ x_{13} & y_{13} & z_{13} \\ \vdots & \ddots & \vdots \\ x_{n\cdot m1} & y_{n\cdot m1} & z_{n\cdot m1} \\ x_{n\cdot m2} & y_{n\cdot m2} & z_{n\cdot m2} \\ x_{n\cdot m3} & y_{n\cdot m3} & z_{n\cdot m3} \end{bmatrix}$$

$$\text{Facets}: \mathcal{B}'_f = \begin{Bmatrix} F_1 \\ \vdots \\ F_n \end{Bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & P_{n3} \end{bmatrix}$$



Elements	Definition
A	$F_i = \{P_{i1}, P_{i2}, P_{i3}\}$
B	$P_{i1} = \{x_{i1}, y_{i1}, z_{i1}\}$
C	$P_{i2} = \{x_{i2}, y_{i2}, z_{i2}\}$
D	$P_{i3} = \{x_{i3}, y_{i3}, z_{i3}\}$

Fig1b.*- Definition of facets and nodes, triangular shell elements.*

Each node $P_{ij}(x_{ij}, y_{ij}, z_{ij})$ belonging both to a facet F_i and to the mesh $\mathcal{B}'_n$, must be unique for that facet F_i.

$$\begin{Bmatrix} \forall\, F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in \mathcal{B}'_f \\ 1 \le i \le n \\ \forall\, P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\} \in \mathcal{B}'_n \\ 1 \le i \le n\cdot 3\ ;\ 1 \le j \le 3 \end{Bmatrix} \rightarrow \{x_{i1}, y_{i1}, z_{i1}\} \neq \{x_{i2}, y_{i2}, z_{i2}\} \neq \{x_{i3}, y_{i3}, z_{i3}\}$$

3.1. Classification and recognition of the mesh elements. Pre-processing

During the preprocessing phase the P_{ij} elements of the geometric mesh $\mathcal{B}'_n$ will be classified according to their z_{ij} coordinate, along the direction vector $(\vec{V_z})$ parallel to the parting direction and in the positive direction, Fig. 2. The main parting direction is determined by the user as a starting data so that the choice can be changed if convenient, once the analysis is finished.

Let $V_l \in \mathbb{R}^2$ be a one-dimensional vector, which contains the set of z_{ij} coordinates of the points $P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\}$ belonging to $\mathcal{B}'_n$, sorted in descending order of $(\vec{V_z})$. A set of planes of analysis Pl_k parallel to the plane XOY and perpendicular to $(\vec{V_z})$ will be assigned for each value of the vector V_l.

Then the facets $F_i \in \mathcal{B}'_f$, will be classified in levels (sub-arrays Pl_k) in a multidimensional array $\Pi_k \in \mathbb{R}^n$. Each dimension k of it will correspond to a plane of analysis Pl_k, containing all facets $F_i \in \mathcal{B}'_f$, so that at least one of its vertices $\{P_{i1}, P_{i2}, P_{i3}\}$ belongs to the plane Pl_k. Fig.2

$$V_l \in \mathcal{M}_{1xq}(\mathbb{R}) \rightarrow V_l = \{z_1, z_2, \ldots z_k, \ldots, z_q\}$$

$$\text{If} \left\{ \begin{array}{c} Pl_k : z = Cons \;;\; z = z_k = Cons \\ (z_{i1} = z_k) \wedge (z_{i2} = z_k) \wedge (z_{i3} = z_k) \\ 1 \leq i \leq n \cdot 3 \\ 1 \leq k \leq q \\ \Pi_k \in \mathbb{R}^n \end{array} \right\} \rightarrow F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in \Pi_k$$

Where n represents the total number of mesh facets and q the number of z coordinates nodes unequal.

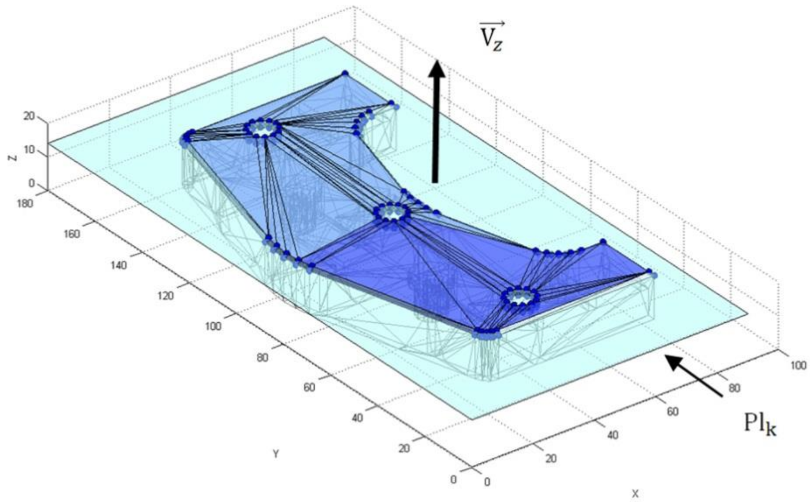


Fig 2- *Defining the set of nodes / facets belonging to Pl_k.*

3.2. Algorithm of analysis and classification of mesh nodes, Processing.

After the grouping of the different facets $F_i \in \mathcal{B}'_f$, according to the value z_{ij} of their respective nodes $\{P_{i1}, P_{i2}, P_{i3}\}$, along the parting direction $(\vec{V_z})$, mesh $\mathcal{B}'$ is processed in order to study its demoldability, analyzing geometrically the facets $F_i \in \mathcal{B}'_f$ and the nodes $P_{ij} \in \mathcal{B}'_n$ according to demoldability criteria. First, the $\mathcal{B}'$ geometry is evaluated, according to the parting direction $(\vec{V_z})$ in the positive direction, in order to estimate the geometric regions of the part to be manufactured by the upper cavity. Subsequently, a reorientation of the part in the opposite direction to the one analyzed previously allows the regions that belong to the lower cavity to be located.

Initially the different facets F_i and points P_{ij} contained in each of the levels (Pl_k sub-arrays) of the multidimensional array $\Pi_k \in \mathbb{R}^n$ are selected. Thus for a sequential Pl_k plane, a closed polygonal analysis contour $C_k \in \mathbb{R}^2$ will be defined from the projection $Proj[\sum_1^{k-1} F_{i,k}]$ of all facets $\sum_1^{k-1} F_{i,k}$ contained in superior levels [1,k-1] of the array $\Pi_k \in \mathbb{R}^n$, Fig.3.

$$\forall\, F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in \Pi_k :\ z_k = Cons\ \rightarrow C_k = \left[Proj[F_{i,k}]\right] \in \mathbb{R}^2$$

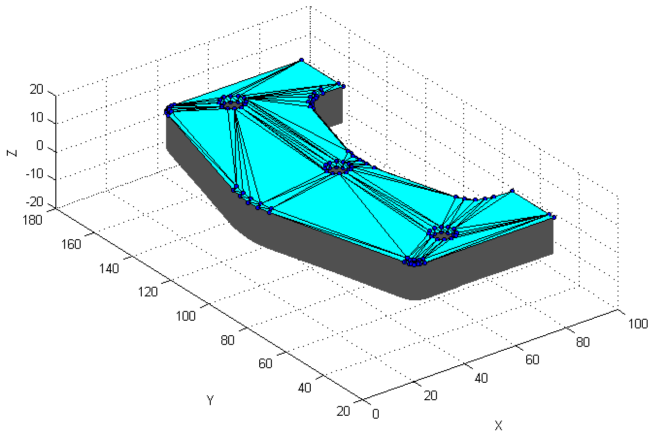


Fig 3- *Representation of facets F_i for the level k of Π_k, definition of C_k*

The F_i facets that are in the first level of the multidimensional array $\Pi_k \in \mathbb{R}^n$ (Pl_1), (those that meet the geometrical condition that at least one point $P_{ij} \in Pl_1$) are classified as demoldable facets.

$$\forall\, F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in \Pi_1 : (P_{i1} \in Pl_1) \wedge (P_{i2} \in Pl_1) \wedge (P_{i3} \in Pl_1) \rightarrow F_i \in \Omega_{f_1}$$

Where $\Omega_{f_1} \in \mathcal{B}'_f \in \mathbb{R}^3$ is defined as the locus corresponding to all demoldable facets of the upper mold cavity. For some facets $F_i \in \Pi_1$ which form a coplanar region $\dot{C_1}$, those facets $F_i \in Pl_1$ which are coplanar with each other will be included in Ω_{f_1}.

$$\text{If } \exists\, \dot{C_1} \subset C_1 : C_1 = Proj\left[F_{i,1}\right] : z = z_1 \rightarrow \dot{C_1} \in \Omega_{f_1}$$

Then, for the following levels k [2, q] a series of checkpoints or Gauss points $P_{Gauss} \in F_i$ in all facets $F_i \in C_k$ will be generated. The algorithm will project the set of points $\{(P_{ij} \in \mathcal{B}'_n) - (P_{ij} \in Pl_k)\} \rightarrow z_{ij} < z_k$ and the corresponding control points P_{Gauss} on C_{k-1} [Appendix 1]; checking whether the points P_{ij} and P_{Gauss}, can be demolded along the parting direction $(\overrightarrow{V_z})$. Thus if a point P_{ij} is strictly outside the region of study C_{k-1}, it is cataloged as an upper demoldable element $\in \Omega_{n_1}$, being Ω_{n_1} the array for storing the set of points P_{ij} in the demoldable positive direction of parting direction $(+\overrightarrow{V_z})$. On the contrary, if an element is placed inside the region C_{k-1} or at the border $Fr(C_{k-1})$, that point P_{ij} is cataloged as not demoldable $\in \beta_{n_2} \in$, being β_{n_2} the array for storing the set of non-demoldable points P_{ij} in the parting direction $(+\overrightarrow{V_z})$.

$$\mathcal{B}'_n = \{\Omega_{n_1} \cup \beta_{n_2}\}$$

$$n_1 + n_2 = n$$

A point $P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\} \in \mathbb{R}^3$ is a border point of C_k if all the surrounding zone of $P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\}$ contains at least some points $\in C_k$ and some points $\notin C_k$. The set of border points of C_k is called the border of C_k and it is denoted by $Fr(C_k)$, where $C(C_k)$ is the complementary set of C_k.

$$Fr(C_k) = \overline{C_k} \cap \overline{C(C_k)}$$

$$\text{If } \left\{\begin{matrix} \exists\ P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\} \in Pl_k \\ 1 \le k \le q \\ 1 \le i \le n \cdot 3\ ;\ 1 \le j \le 3 \end{matrix}\right\} : ([x_{ij}, y_{ij}] \notin \overline{C_k}) \wedge ([x_{ij}, y_{ij}] \notin [Fr(\overline{C_k})]) \rightarrow P_{ij} \in \Omega_{n_1}$$

$$\text{If } \left\{\begin{matrix} \exists\ P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\} \in Pl_k \\ 1 \le k \le q \\ 1 \le i \le n \cdot 3\ ;\ 1 \le j \le 3 \end{matrix}\right\} : ([x_{ij}, y_{ij}] \in [Fr(\overline{C_k})]) \wedge ([x_{ij}, y_{ij}] \in [\overline{C_k}]) \rightarrow P_{ij} \in \beta_{n_2}$$

In Fig. 4 the contour border $Fr(C_k)$, bounded by the points $P_{ij} \in Pl_k$ is shown. In Fig. 5 an example is shown of the partial analysis of demoldability for an intermediate plane Pl_k. The sub-algorithm whose function is the analysis and classification of nodes P_{ij} of the mesh $\mathcal{B}'_n$ with respect to the parting direction is called InProject (Annex I). For the successive k loops for the execution of InProject, those facets F_i that have been included in the contour C_{k-1} will not be taken into account in the boundary region C_k as a facet can be on two levels and therefore only be considered in one of them.

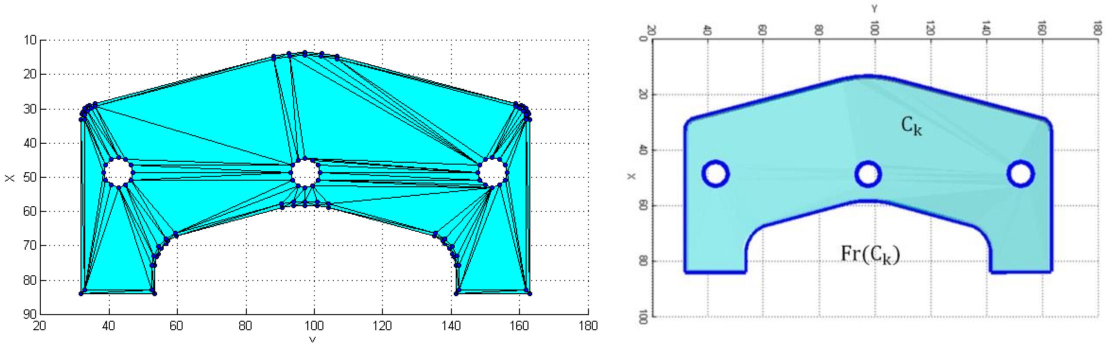


Fig 4.- *Projection of facets F_i and contour border $Fr(C_k)$ for the level k of Π_k .*

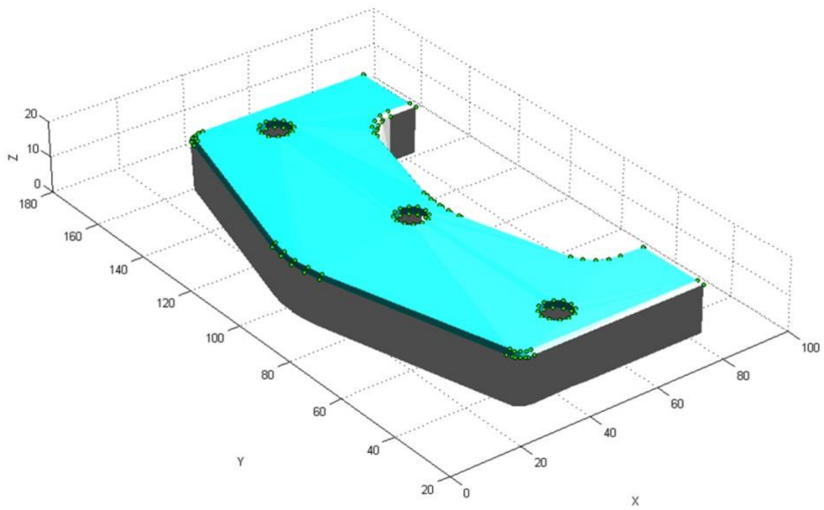


Fig5.- *Partial demoldability analysis for a plane (Pl_k)*

After the execution of the k analysis, a procedure for comparing the points stored in arrays Ω_{n_1} and β_{n_2} is carried out. It may occur that a point has been identified as demoldable $P_{ij} \in \Omega_{n_1}$ and not demoldable $P_{ij} \in \beta_{n_2}$ at different planes. The points which have this duplication are categorized as non-demoldable. In Fig. 6 the result of the algorithm InProject for solid $\mathcal{B}$ is shown.

$$\text{If } \left\{ \begin{matrix} P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\} \in \Omega_{n_1} \\ P_{qj} = \{x_{qj}, y_{qj}, z_{qj}\} \in \beta_{n_2} \end{matrix} \right\} : \left\{ \begin{matrix} x_{ij} = x_{qj} \\ y_{ij} = y_{qj} \\ z_{ij} = z_{qj} \end{matrix} \right\} \rightarrow P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\} \in \beta_{n_2}$$

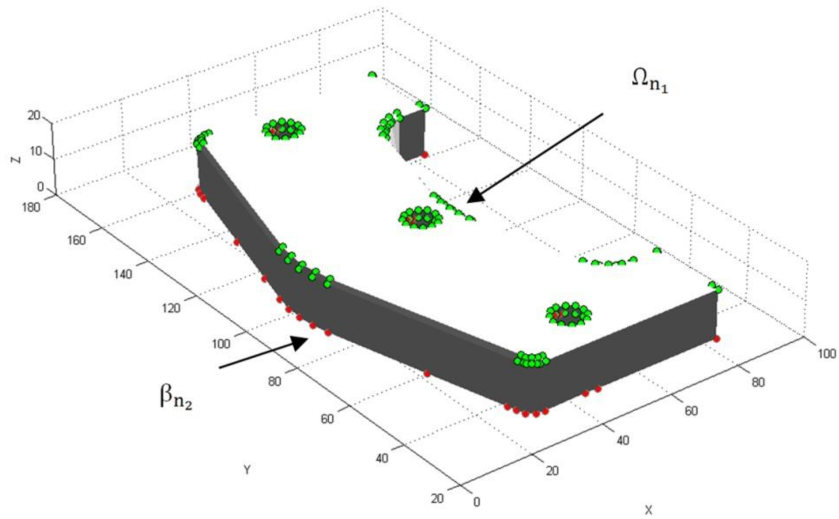


__Fig6.-__ Result of the algorithm InProject in the analysis $(+\overrightarrow{V_z})$.

3.3. Algorithm for cataloging the mesh facets, Processing.

After analyzing the demoldability of mesh nodes $P_{ij} \in \mathcal{B}'_n$ (preprocessing and classification of nodes), the objective of this phase is to catalog the facets $F_i \in \mathcal{B}'_f$, according to their demoldability, into demoldable facets, non-demoldable facets, and partially demoldable or semi-demoldable facets, and to store them in arrays with geometric information about the manufacture of the piece.

$$\mathcal{B}'_f = \{\Omega_{f_1} \cup \xi_{f_2} \cup \Gamma_{f_3} \cup \Delta_{f_4}\}$$

$$f_1+f_2+f_3+f_4=f$$

Ω_{f_1}, ξ_{f_2}: Locus corresponding to all demoldable facets, upper cavity and lower cavity respectively along $(\overrightarrow{V_z})$, being f_1 and f_2 the number of facets stored in arrays Ω_{f_1} y ξ_{f_2} .

Γ_{f_3}: Locus of the facets set partially demoldable along $(\overrightarrow{V_z})$ being f_3 the number of facets stored in the array Γ_{f_3}.

Δ_{f_4}: Locus of the set of facets non-demoldable along $(\overrightarrow{V_z})$, being $f_4$the number of facets stored in the array Δ_{f_4}.

Firstly, the demoldability of the facets $F_i \in \mathcal{B}'_f$ along the positive parting direction $(+\overrightarrow{V_z})$, is cataloged. The previously classified points P_{ij} in the array Ω_{n_1}, are the basis for cataloging the facets $F_i \in \mathcal{B}'_f$. All the facets $F_i \in \mathcal{B}'_f$ matching the condition that all their vertex $\{P_{i1}, P_{i2}, P_{i3}\}$, belongs to the demoldable region Ω_{n_1}, will be classified as demoldable along $(+\overrightarrow{V_z})$, being assigned to the array of direct demoldable facets Ω_{f_1}, and they will be removed from the array $\mathcal{B}'_f$.. In Fig 7 the location of the area Ω_{f_1} is shown on the piece example.

$$\forall\ F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in \mathcal{B}'_f$$

$$\text{If } (P_{i1} \in \Omega_{n_1}) \vee (P_{i2} \in \Omega_{n_1}) \vee (P_{i3} \in \Omega_{n_1}) \rightarrow F_i \in \Omega_{f_1}$$

$$F_i = F_{i_1}$$

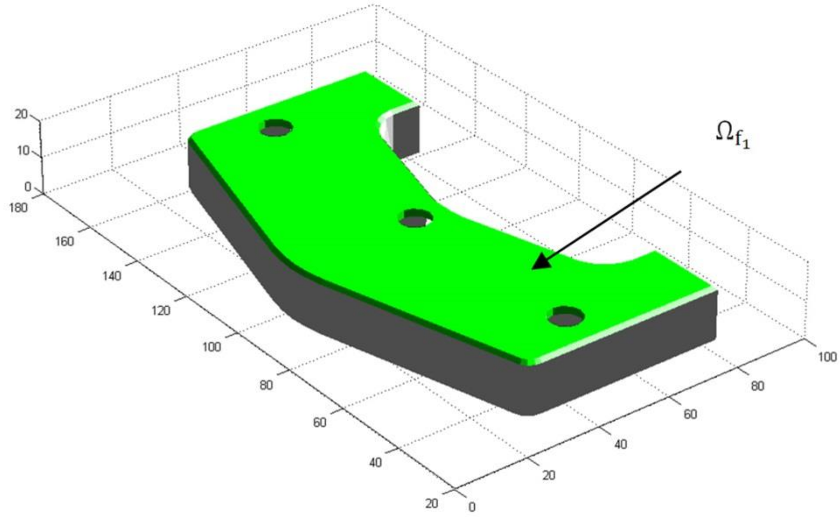


***Fig7.**- Example of location of the region* Ω_{f_1}.

The remaining facets F_i belonging to the array $\mathcal{B}'_f$ will be cataloged as non-demoldable or semi-demoldable facets along the direction $(+\overrightarrow{V_z})$, being assigned to the array formed by the union of the arrays Γ_{f_3} and Δ_{f_4}, Fig 8.

$$\forall\ F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in [\mathcal{B}'_f - (\Omega_{f_1})] \rightarrow F_i \in [\Gamma_{f_3} \cup \Delta_{f_4}]$$

$$\forall\ F_i \in [\Delta_{f_4} \cup \Gamma_{f_3}] \rightarrow F_i = F_{i_{3,4}}$$

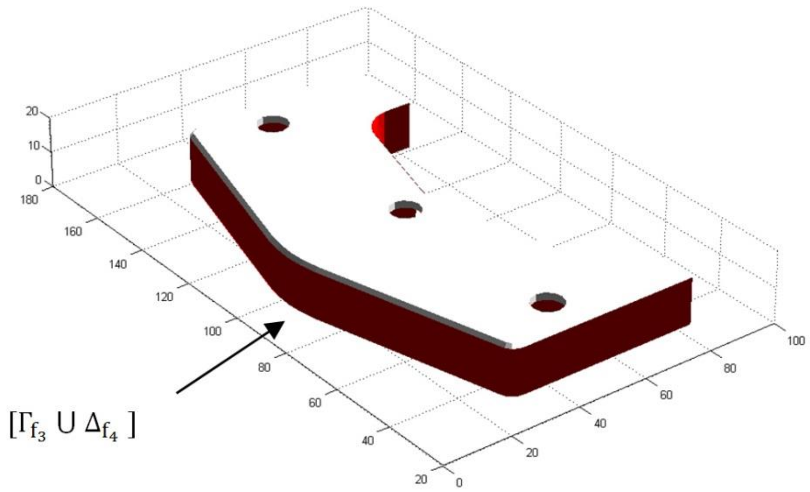


***Fig8.**- Example of location of the region* $[\Gamma_{f_3} \cup \Delta_{f_4}]$.

The algorithm for analyzing the demoldability of the facets F_i in the mesh $\mathcal{B}'_f$, from the array Ω_{f_1} is defined as Catalogfaces. For vertical sweep it is taken into account that $\mathcal{B}'_f = \{\Omega_{f_1} \cup \xi_{f_2} \cup \Gamma_{f_3} \cup \Delta_{f_4}\}$ considering facets $F_{i_2} \in \xi_{f_2}$ included in the array of non-demoldable facets Δ_{f_4}.

3.3.1.- Algorithm for obtaining the demoldability in the case of vertical facets.

As a result of applying the Catalogfaces algorithm presented in 3.3, the horizontal facets F_i and inclined facets $\in \mathcal{B}'_f$, demoldable in the upper cavity, have been classified and stored in the array Ω_{f_1}. However there is still a set of facets $\forall F_{i_{3,4}} = \{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\} \in [\Gamma_{f_3} \cup \Delta_{f_4}]$, which by their geometric configuration may be reclassified as demoldable facets. These facets fulfill the geometrical condition of being perpendicular to the parting direction $(\overrightarrow{V_z})$.

Generalmente, en geometrías faceteadas existe un cordal error relacionado con la interpolación y discretización de curvas del modelo de la pieza CAD original. A consecuencia de dicho error pueden generarse facetas near-vertical, es decir facetas que deberían ser verticales pero tienen un cierto ángulo de desviación respeto a la parting direction. En primer lugar, para su localización se propone determinar el arcocoseno del área de la faceta original con el área de la faceta proyectada ortogonalmente según the parting direction, pues dicha relación define el ángulo de desvisión de la faceta. A continuación, se establece una redefinición de la faceta near-vertical mediante el uso de afinidad/proporcionalidad, a partir de la proyección ortogonal de la faceta original, con el objetivo de desplazar virtualmente los nodos de la misma hasta generar una nueva faceta vertical virtual. Según ref[] pueden establecerse dos tipologías de near-facet: back-facing y front-facing. En el caso en el que la faceta near-vertical sea del tipo back-facing, se procede al desplazamiento virtual de los nodos con menor cota z. Y por el contrario, si se trata de una front-facing, se procede al desplazamiento virtual de los nodos con mayor cota vertical.

The facets $F_{i_{3,4}} = \{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\} \in [\Gamma_{f_3} \cup \Delta_{f_4}]$ located under the border of a facet $F_{i_1} \in \Omega_{f_1}$, so that the projection of the points $\{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\}$ that make the facet $F_{i_{3,4}}$, belong to Fr (F_{i_1}), will be demoldable. They will be repositioned in the array Ω_{f_1}, and removed from the array $[\Gamma_{f_3} \cup \Delta_{f_4}]$, Fig 9.

$$\forall F_{i_1} \in \Omega_{f_1} \rightarrow \ Fr(F_{i_1}) = \overline{F_{I_1}} \cap \overline{CF_{I_1}}$$

$$\forall F_{i_{3,4}} = \{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\} \in [\Gamma_{f_3} \cup \Delta_{f4}] : \{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\} \rightarrow \begin{bmatrix} x_{i_{3,4}\,1}, y_{i_{3,4}\,1} \\ x_{i_{3,4}\,2}, y_{i_{3,4}\,2} \\ x_{i_{3,4}\,3}, y_{i_{3,4}\,3} \end{bmatrix} \in Fr(F_{i_1}) \rightarrow F_{i_{3,4}} \in \Omega_{f_1}$$

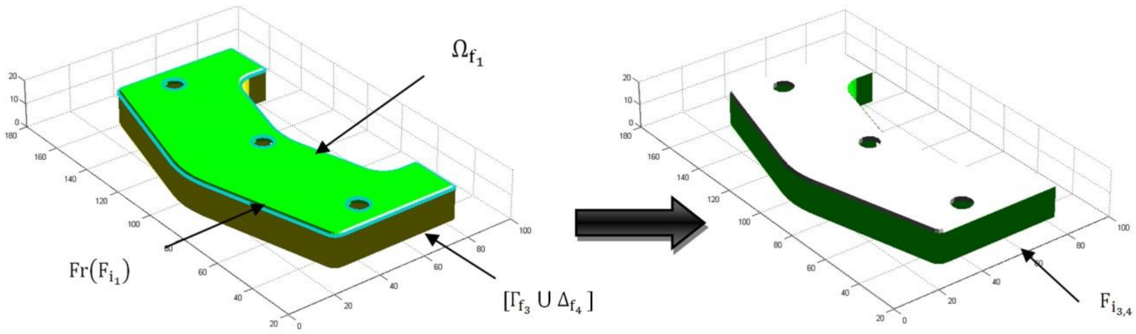


Fig 9.- *Example location of demoldable facets, perpendicular to* $\overrightarrow{V_z}$.

The other facets $F_i \in \mathcal{B}'_f$ that have not been cataloged after applying the reassignment algorithm as demoldable vertical facets will be stored in the array of non-demoldable and semi-demoldable facets $[\Gamma_{f_3} \cup \Delta_{f_4}]$.

$$\forall F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in [\mathcal{B}'_f - (\Omega_{f_1})] \rightarrow \ F_{i_{3,4}} \in [\Gamma_{f_3} \cup \Delta_{f_4}]$$

3.3.2.-Reorientation of geometry, obtaining demoldability on facets belonging to the lower cavity

$F_{i_{3,4}}$

In this section, the algorithms stated in paragraphs 3.1, 3.2 and 3.3 will be applied again, reorienting the part in the opposite direction to the parting direction $(\overrightarrow{-V_z})$, resulting in the set of demoldable facets corresponding to the lower cavity region ξ_{f_2}, Fig 11.

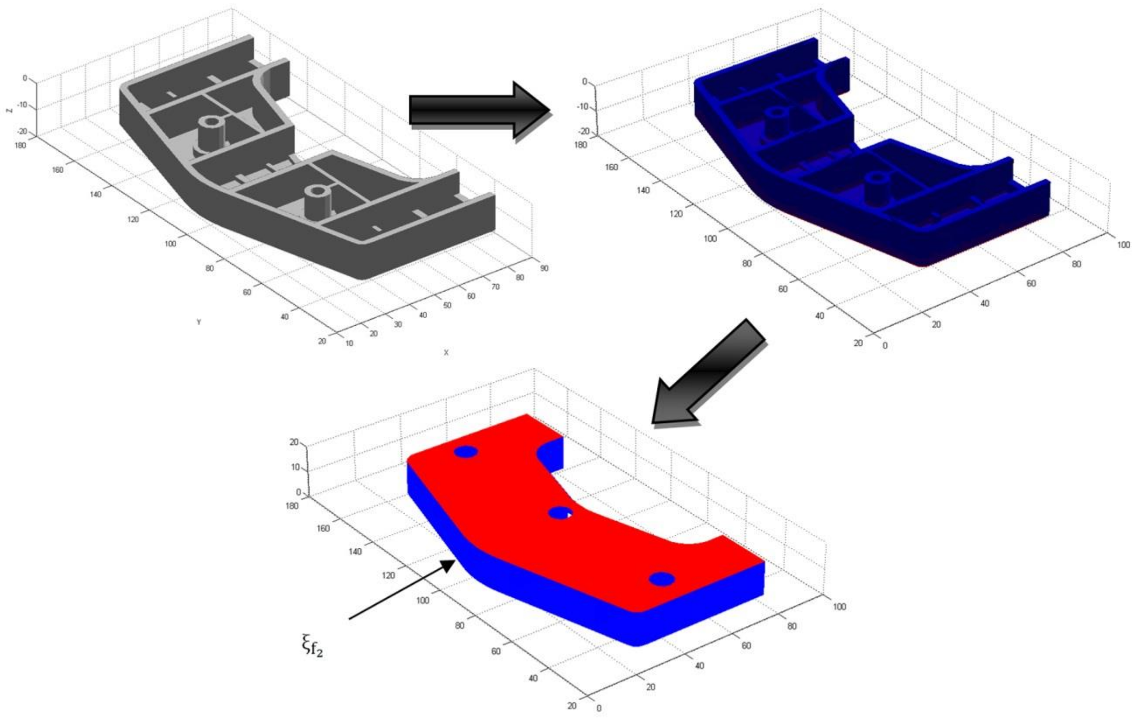


Fig 10.- *Reorientation of the geometry as the new direction of analysis, and comparison of results of different analyses performed.*

As it is noticed in Fig 10, it should be established unification criteria for those facets with duplication of results. Such criteria or boundary conditions are as follows:

- All those facets that belong to the demoldable region of the upper and lower cavity will only be stored in the array ξ_{f_2} (lower cavity) and removed from the array Ω_{f_1} (upper cavity).

$$\text{If } (F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in \Omega_{f_1}) \vee (F_i = \{P_{i1}, P_{i2}, P_{i3}\} \in \xi_{f_2}) \rightarrow F_i \in \xi_{f_2}$$

- All those facets that belong to the demoldable region corresponding to the lower cavity ξ_{f_2} and have been cataloged as non-demoldable $\in [\Gamma_{f_3} \cup \Delta_{f_4}]$ in the first analysis $(\overrightarrow{V_z})$, will be stored in the array ξ_{f_2} (lower cavity), and removed from the array $[\Gamma_{f_3} \cup \Delta_{f_4}]$.

$$\text{If } \left(F_{i_{3,4}} = \left\{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\right\} \in \left\{\Gamma_{f_3} \cup \Delta_{f_4}\right\}\right) \vee \left(F_{i_{3,4}} = \left\{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\right\} \in \xi_{f_2}\right) \rightarrow F_i \in \xi_{f_2}$$

- Similarly, all those facets that belong to the demoldable region corresponding to the upper cavity Ω_{f_1} and in turn have been cataloged as non-demoldable $\in [\Gamma_{f_3} \cup \Delta_{f_4}]$ in the second analysis $(\overrightarrow{-V_z})$, will only be stored in the array Ω_{f_1} (upper cavity), and removed from the array $[\Gamma_{f_3} \cup \Delta_{f_4}]$.

$$\text{If } \left(F_{i_{3,4}} = \left\{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\right\} \in \left\{\Gamma_{f_3} \cup \Delta_{f_4}\right\}\right) \vee \left(F_{i_{3,4}} = \left\{P_{i_{3,4}\,1}, P_{i_{3,4}\,2}, P_{i_{3,4}\,3}\right\} \in \Omega_{f_1}\right) \rightarrow F_i \in \Omega_{f_1}$$

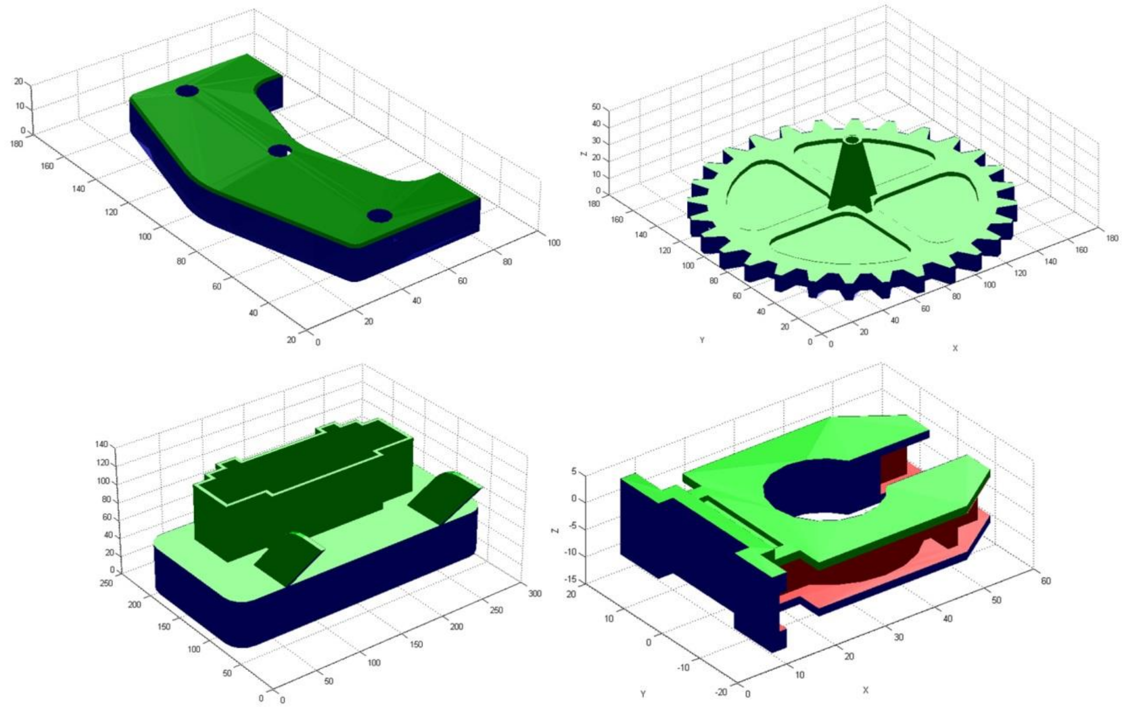


Fig 11.- *Results of the analysis of demoldability in +V_z, -V_z after the implementation of the boundary conditions 3.3.2*

3.3.3.- Algorithm of reclassification of semi-demoldable and non-demoldable facets.

At this point of the demoldability analysis of the mesh, the facets $F_{i_{3,4}} \in [\Gamma_{f_3} \cup \Delta_{f_4}]$ should be reclassified, into non-demoldable facets $F_{i_4} \in \Delta_{f_4}$ and semi-demoldable ones $F_{i_3} \in \Gamma_{f_3}$. To solve this problem the algorithm uses an analysis methodology based on the overlap between the different non-demoldable and semi-desmoldable facets.

The algorithm of reclassification of semi-demoldable and non-demoldable facets is called reclassfacet. The application of the reclassfacet algorithm will result in a reclassification of the facets in arrays, in addition to dividing the semi-demoldable facets in regions according to their demoldability. After the application of the reclassfacet algorithm a new set of facets and nodes that complement the mesh $\mathcal{B}'_f$ y $\mathcal{B}'_n$ is created

For this purpose, the reclassfacet algorithm performs a comparative facet-facet analysis $(F_{i_s} - F_{i_r}) \in [\Gamma_{f_3} \cup \Delta_{f_4}]$ in order to classify the overlapping facets. Given a pair of facets $(F_{i_s} - F_{i_r}) \in [\Gamma_{f_3} \cup \Delta_{f_4}]$, the vertical measurements $\{z_{i_s\,1}, z_{i_s\,2}, z_{i_s\,3}\}$, $\{z_{i_r\,1}, z_{i_r\,2}, z_{i_r\,3}\}$ of their nodes $\{P_{i_s\,1}, P_{i_s\,2}, P_{i_s\,3}\}$, $\{P_{i_r\,1}, P_{i_r\,2}, P_{i_r\,3}\}$ are compared, and the facets which produces overlapping are checked (located on a higher plane).

In order to verify whether there is overlap between a pair of facets belonging to $[\Gamma_{f_3} \cup \Delta_{f_4}]$, both facets are projected onto a plane perpendicular to the parting direction $(\overrightarrow{V_z})$ and it is checked by means of a logical Boolean operation if there is contact between these facets. That facet $F_{i_s} \in [\Gamma_{f_3} \cup \Delta_{f_4}]$ that satisfies the condition that $\{z_{i_s\,1}, z_{i_s\,2}, z_{i_s\,3}\} > \{z_{i_r\,1}, z_{i_r\,2}, z_{i_r\,3}\}$ and the intersection is not null will be assigned to the array of non-demoldable facets Δ_{f_4}. The other facet F_{i_r} is assigned to the array of semi-demoldable facets Γ_{f_3}.

$$\forall \left\{ \begin{array}{l} F_{i_s} = \{P_{i_s\,1}, P_{i_s\,2}, P_{i_s\,3}\} \rightarrow \begin{bmatrix} x_{i_s\,1}, y_{i_s\,1}, z_{i_s\,1} \\ x_{i_s\,2}, y_{i_s\,2}, z_{i_s\,2} \\ x_{i_s\,3}, y_{i_s\,3}, z_{i_s\,3} \end{bmatrix} \\ F_{i_r} = \{P_{i_r\,1}, P_{i_r\,2}, P_{i_r\,3}\} \rightarrow \begin{bmatrix} x_{i_r\,1}, y_{i_r\,1}, z_{i_r\,1} \\ x_{i_r\,2}, y_{i_r\,2}, z_{i_r\,2} \\ x_{i_r\,3}, y_{i_r\,3}, z_{i_r\,3} \end{bmatrix} \end{array} \right\} \text{If} \left\{ \begin{array}{c} \{z_{i_s\,1}, z_{i_s\,2}, z_{i_s\,3}\} > \{z_{i_r\,1}, z_{i_r\,2}, z_{i_r\,3}\} \\ F_{i_s} \cap F_{i_r} \neq \emptyset \end{array} \right\} \rightarrow \left\{ \begin{array}{l} F_{i_s} \in \Delta_{f_4} \\ F_{i_r} \in \Gamma_{f_3} \end{array} \right\}$$

Thus, it can be determined which facets belong to the set of semi-demoldable facets Γ_{f_3} and which belong to the set of non-demoldable facets Δ_{f_4}.

After the semi-demoldable facets $F_{i_3} \in \Gamma_{f_3}$ are defined, a fragmentation process is implemented, finding the demoldable upper or lower cavity regions and the non-demoldable region for each facet. The division of semi-demoldable facets is performed by applying a methodology of subtraction and intersection between each of the semi-demoldable facets $\in \Gamma_{f_3}$ and the closed set of non-demoldable facets Δ_{f_4}. The closed set $\{F_{i_4} \in \Delta_{f_4}\}$ and the semi-demoldable facets $F_{i_3} \in \Gamma_{f_3}\}$ will be projected onto a horizontal plane perpendicular to the parting direction $(\overrightarrow{V_z})$, and the intersection and subtraction resulting from these elements is calculated; This geometric information is transferred to the plane of the semi-demoldable facets under consideration.

$$\forall\, F_{i_3} = \{P_{i_3\,1}, P_{i_3\,2}, P_{i_3\,3}\} \in \Gamma_{f_3} \rightarrow F_{i_3} = F''_{i_1} + F''_{i_4} \rightarrow \begin{Bmatrix} F''_{i_1} \in \Omega_{f_1} \\ F''_{i_4} \in \Delta_{f_4} \end{Bmatrix}$$

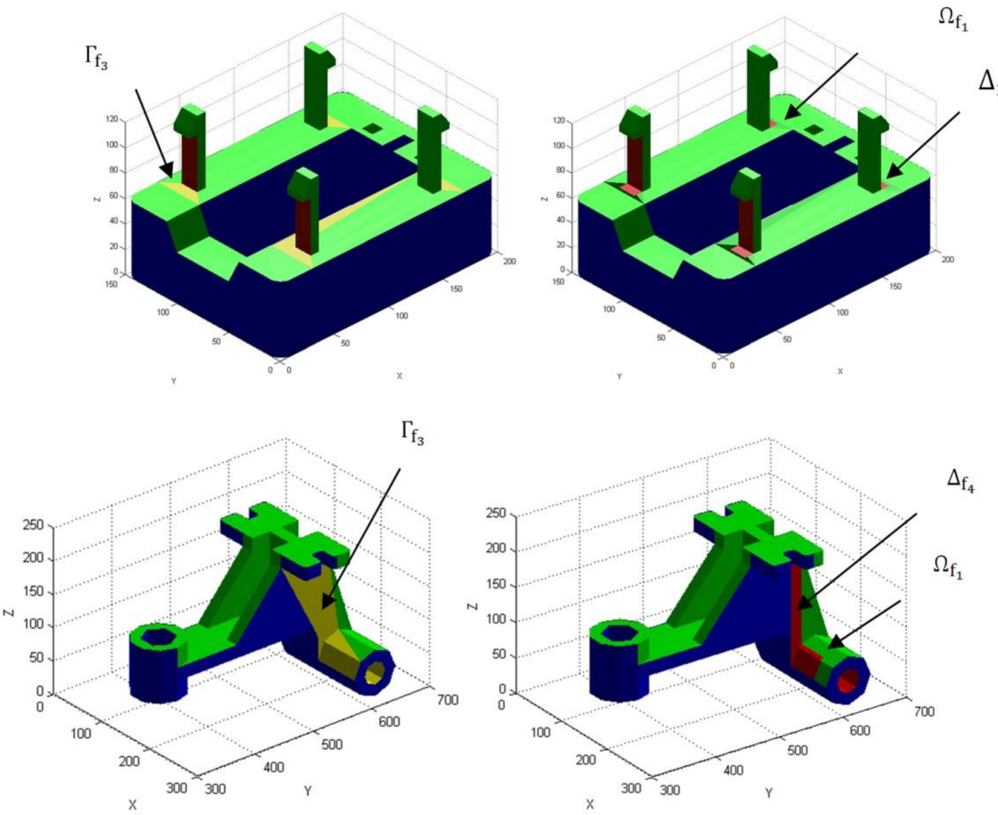


***Fig12.-** Example of resolution of semi-demoldable facets, Boolean operation (intersection and subtraction).*

As seen in Fig 12, the result of the Boolean intersection between the semi-demoldable facets F_{i_3} and non-demoldable facets F_{i_4} , for every facet causes the division thereof into two regions, the demoldable region $F''_{i_1} \in \Omega_{f_1}$ and non demoldable region $F''_{i_4} \in \Delta_{f_4}$.

$$\forall\, F_{i_3} = \{P_{i_3\,1}, P_{i_3\,2}, P_{i_3\,3}\} \in \Gamma_{f_3} \rightarrow \begin{Bmatrix} F_{i_3} \cap \Delta_{f_4} = F''_{i_4} \in \Delta_{f_4} \\ F_{i_3} - \Delta_{f_4} = F''_{i_1} \in \Omega_{f_1} \\ F''_{i_4} \cup F''_{i_1} = F_{i_3} \end{Bmatrix}$$

$$\forall\, \{F''_{i_4},\ F''_{i_1}\} \in \mathcal{B}''_f$$

The set of facets $\{F''_{i_4}, F''_{i_1}\}$ which following the reclassification algorithm have been divided according to their demoldability form a new mesh $\mathcal{B}''_f$ containing the new set of virtual polygonal regions (F''_{i_4}, F''_{i_1}) to be overlapped on mesh $\mathcal{B}'_f$. The new mesh $\mathcal{B}'''_{\mathcal{F}f} = \{\mathcal{B}''_f \cup \mathcal{B}'_f\}$ will incorporate information about the demoldability of the plastic part $\mathcal{B}$. The new mesh includes geometric information as well as manufacturing information which will be included in the files and resulting models

$$\text{Let be } \mathcal{B}'_f \in \mathbb{R}^n \,;\, \exists\ \mathcal{B}''_f \in \mathbb{R}^n \rightarrow \left\{ \begin{matrix} \mathcal{B}'''_{F_f} = \{\mathcal{B}'_f + \mathcal{B}''_F\} \\ \mathrm{F_f} \geq \mathrm{n} \end{matrix} \right\}$$

Similarly, fragmented facets create new virtual nodes that will be part of the virtual mesh $\mathcal{B}''_n$. $\mathcal{B}''_n$ is overlapped on mesh $\mathcal{B}'_n$ adding information about demoldability of the part and forming the mesh $\mathcal{B}'''_{\mathcal{N}n}$.

$$\text{Let be } \mathcal{B}'_n \in \mathbb{R}^n \,;\, \exists\ \mathcal{B}''_n \in \mathbb{R}^n \rightarrow \left\{ \begin{matrix} \mathcal{B}'''_{N_n} = \{\mathcal{B}'_n + \mathcal{B}''_N\} \\ N_n \geq 3\mathrm{n} \end{matrix} \right\}$$

After application of the algorithm reclassfacet in the positive direction of the parting direction, it will be applied analogously in the reverse direction to the parting direction in order to obtain semi-demoldable facets $\in \Gamma_{f_3}$ and $\in \Delta_{f_4}$ in this direction

The set of facets classified as demoldable $F''_{i_4} \in \Delta_{f_4}$ incorporates a group of facets F'''_{i_5} which may be resolvable by side core along direction $(\overrightarrow{V'_z})$ perpendicular to the parting direction. At this moment the piece will be reoriented by turning 90° to axis X and then to axis Y. In this position, the preprocessing phase will be applied again by running the algorithm Inproject nodes, the facets processing algorithm by running the algorithm Catalogfaces, and the reclassification of semi-demoldable non-demoldable facets by running the reclassfacet algorithm. The regions previously classified as demoldable $\in [\Omega_{f_1} \cup \xi_{f_2}]$ will be excluded. The facets F'''_{i_5} demoldable along the direction $(\overrightarrow{V'_z})$ will be stored in the array τ_{f_5}. Assignment examples of non-demoldable facets to side core facets are shown in Fig. 13a and 13b

$$\Delta_{f_4} = \{\Delta'_{f_4} \cup \tau_{f_5}\}$$

$$(\overrightarrow{V'_z}) \perp (\overrightarrow{V_z})$$

$$\tau_{f_5} = \{\Delta_{f_4} - \Delta'_{f_4}\}$$

$$\forall F'''_{i_5} \in \tau_{f_5}$$

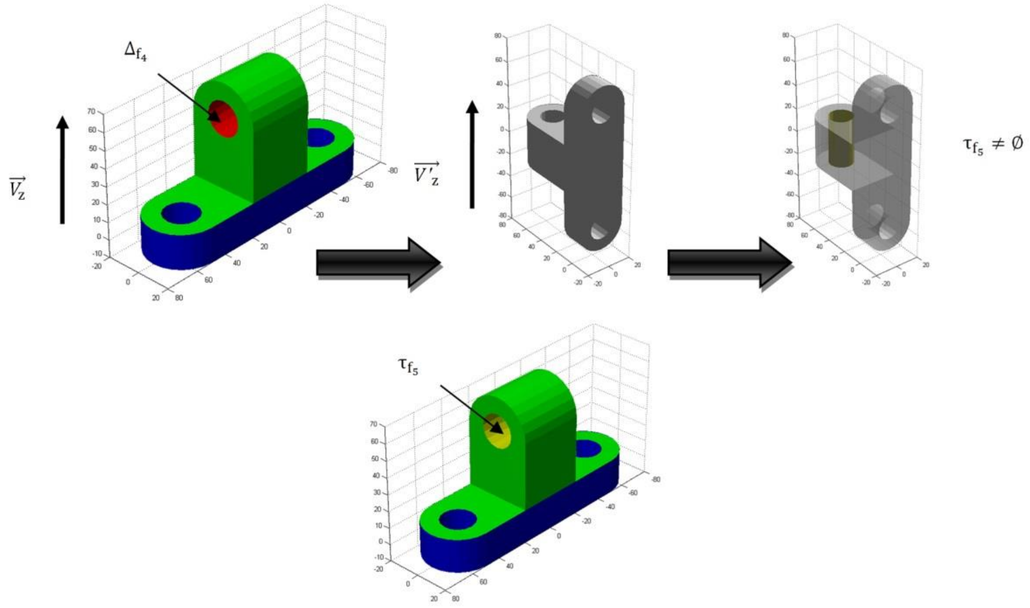


Fig 13a.- *Allocation of non-demoldable facets to side core facets, Example I.*

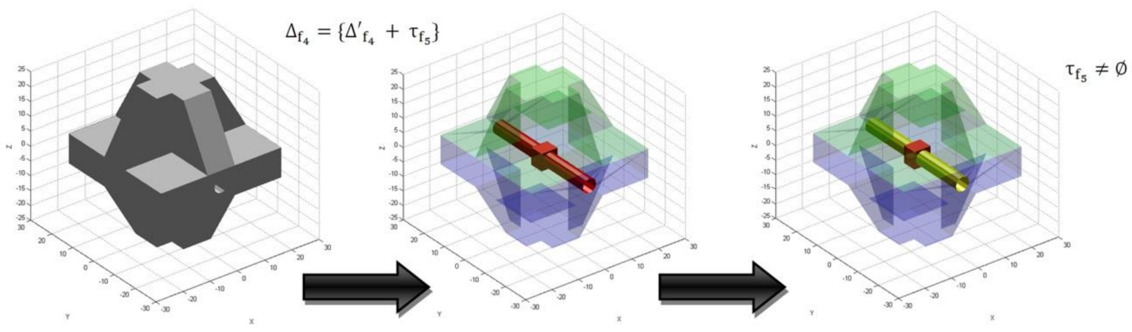


Fig13b.- *Allocation of non-demoldable facets to side core facets, Example II.*

3.4.-Algoritmo de verificación de facetas inclinadas, postprocessing.

Tras evaluar la desmoldeabilidad en el conjunto de facetas que conforman la geometría discreta, se propone incluir un algoritmo de reconocimiento y verificación de los resultados. Este algoritmo tiene por objetivo la detección de catalogaciones erróneas durante el análisis de desmoldeabilidad en facetas. Para ejecutar esta comprobación, se genera un haz de rectas a partir de los nodos y puntos de gauss para cada una de las facetas no desmoldeables. Comprobando si éstos intersectan con facetas desmoldeables, cuyos nodos poseen una coordeanada en la dirección de desmoldeo inferior a la facetas no desmoldeable de referencia. De esta forma, si existe intersección con facetas catalogadas como desmoldeables éstas dejaran de serlo y formarán parte automáticamente de la matriz de facetas no desmoldeables.

3.5.-Algorithm for defining the parting line, postprocessing.

The parting line (PL), by definition, is located in the border region between the upper cavity (Ω_{f_1}) and the lower cavity (ξ_{f_2}). The estimation of this geometric feature of the piece is made from the previous definition of the corresponding arrays upper cavity (Ω_{f_1}) and lower cavity (ξ_{f_2}). The parting line is determined as the boundary between these regions Fig 14.

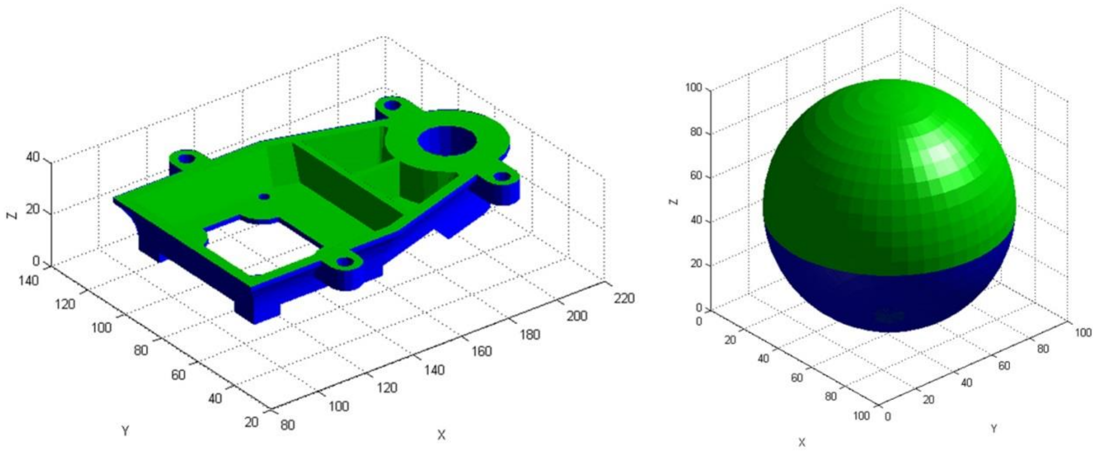


Fig 14.- *Geometric definition of the parting line.*

However, for other geometries Fig 15 whose facet model does not allow us to obtain the line of partition using this algorithm, a sub-algorithm based on the decomposition of demoldable facets is proposed. Thus a Boolean subtraction operation will be performed between the orthogonal projection of facets whose normal vector is perpendicular to the parting direction $\in [\Omega_{f_1} \cup \xi_{f_2}]$ and the facets $\in \xi_{f_2}$. As a result of this operation, virtual polygon slices $F''_{i_1} \in \Omega_{f_1}$, $F''_{i_2} \in \xi_{f_2}$ that define the optimal line are generated [28].

$$\forall F_{i_1} \in \Omega_{f_1} \; \forall F_{i_2} \in \xi_{f_2} : (F_{i_1}, F_{i_2} \perp \overrightarrow{V_z} \rightarrow \begin{Bmatrix} Proj(F_{i_1}) - \xi_{f_2} = F''_{i_1} \\ Proj(F_{i_2}) - \xi_{f_2} = F''_{i_2} \end{Bmatrix}$$

$$F''_{i_1} \in \Omega_{f_1}$$

$$F''_{i_2} \in \xi_{f_2}$$

Finally, the parting line is defined as

$$\forall \, P_{ij} = \{x_{ij}, y_{ij}, z_{ij}\} : \left(P_{ij} \in Fr\left(\Omega_{f_1}\right)\right) \vee \left(P_{ij} \in Fr\left(\xi_{f_2}\right)\right) \rightarrow P_{ij} \in [PL]$$

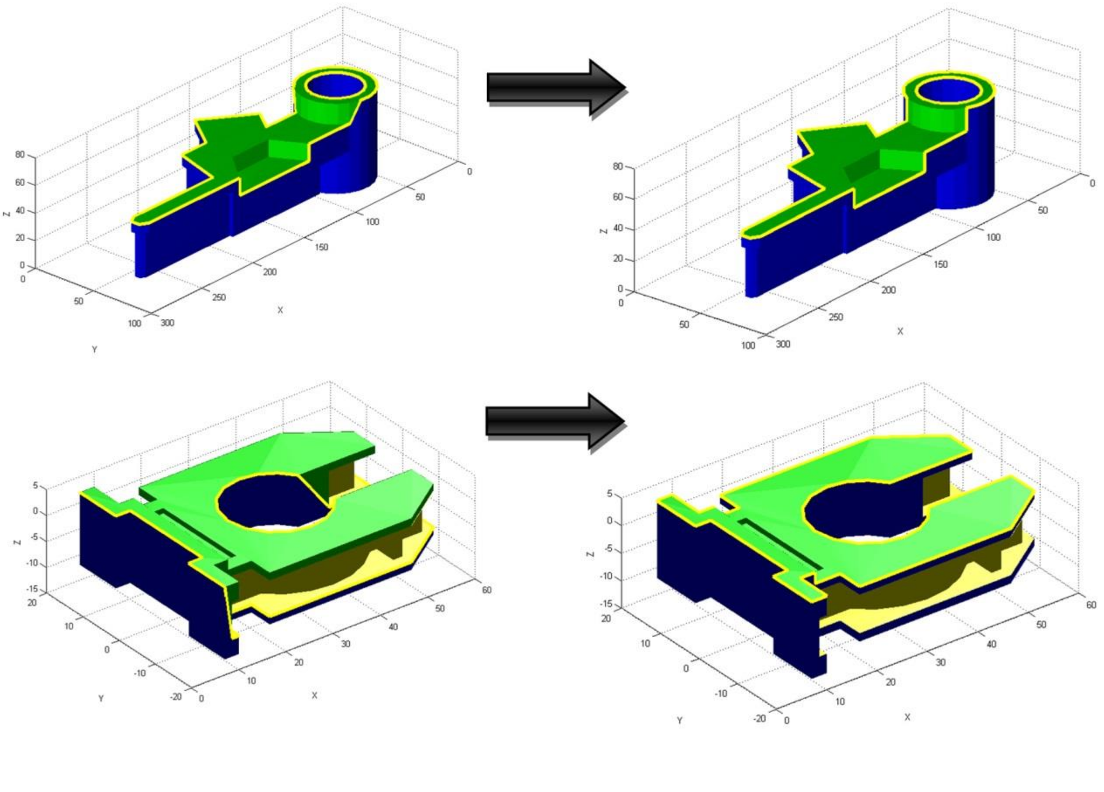


Fig 15.- *Optimizing the geometrical definition of the parting line.*

4.- Implementation & results

In order to validate the algorithm proposed, five industrial parts manufactured by injection molding have been tested. The precision of the mesh (angle and deviation) for each stage of the analysis is analogous for different geometries. A comparison of results of the algorithm, and the total computation has been performed Fig 23. The algorithms have been run with a Toshiba notebook with a Pentium (R) Dual-Core CPU T4200 @ 2.20 GHz processor and a RAM memory of 3.00 GB. It is possible to adapt to any programming language and computer.

4.1.- Case Studies

The results of the application of the algorithm are shown below. They have been classified according to the degree of demoldability. The first, A and B (Fig 16), are direct demoldability parts, all the surfaces are demoldable towards top and bottom cavities, along the parting direction ($\overrightarrow{V_z}$).Part A is composed of 306 facets and part B is composed of 1211 facets.

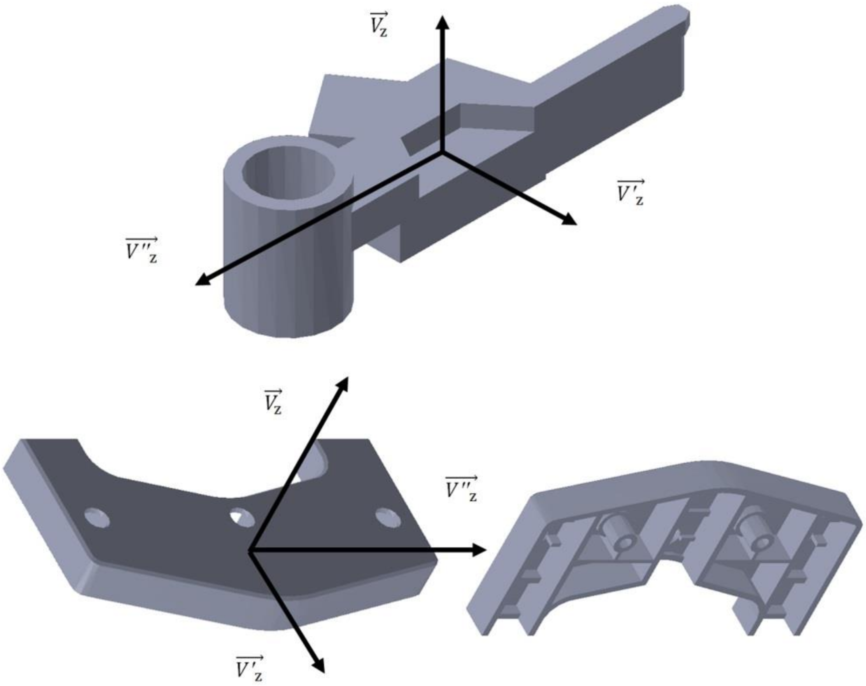


Fig 16.- *Case A and B, demoldable parts by means of upper and lower cavities.*

In Fig. 17 the test pieces C, D are shown. These pieces, unlike parts A and B, have undercuts along the given parting direction ($\overrightarrow{V_z}$). Their manufacture is possible by means of side cores that move perpendicular to the proposed main axis. This allows the demolding of these regions according to the directions ($\overrightarrow{V'_z}$, $\overrightarrow{V''_z}$). Part C has 400 facets in its definition, and part D consists of 696 facets, Table 1.

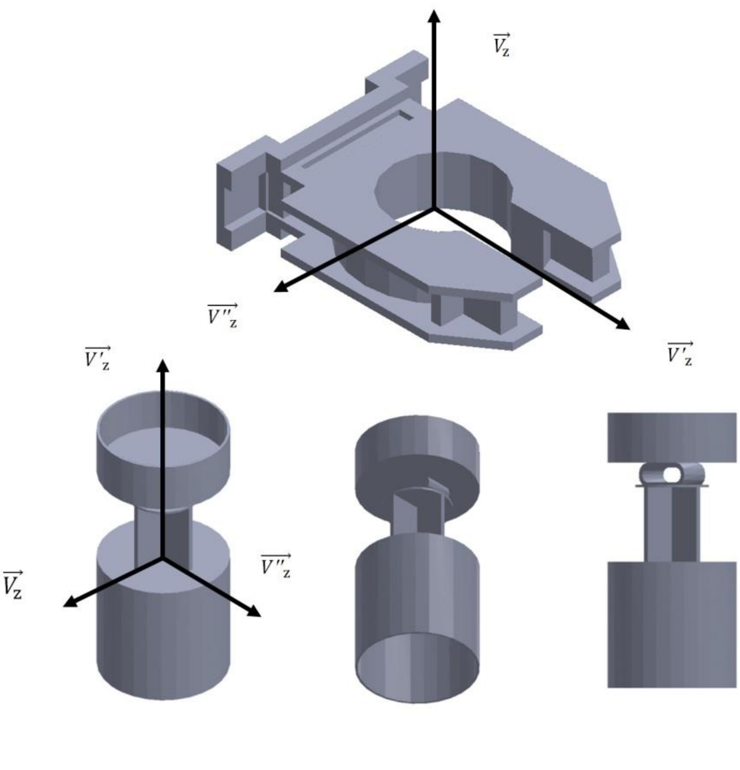


Fig 17.- *Parts C, D with resoluble demolding regions by means of side cores.*

Fig. 18 shows part E. This part has an inner conical revolution drain region which prevents the part being demoldable. Part E consists of 1110 facets, Table1.

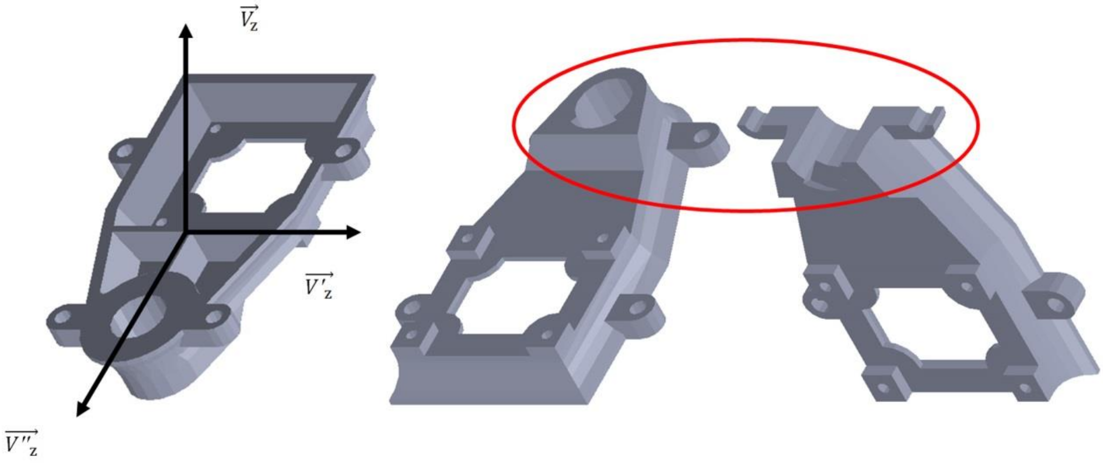


Fig18.- *Part E, parts with non-demoldable regions.*

Fig. 19 shows parts F and G. These parts, as with parts C and D, have undercuts resolvable by side cores. However, once problematic manufacturing areas have been detected the part design has changed, eliminating undercuts and enabling all surfaces to be demoldable as upper and lower cavity. Part F consists of 474 facets and part F' of 506. Moreover, parts G and G' consist of 1334 and 1556 facets respectively, Table 2

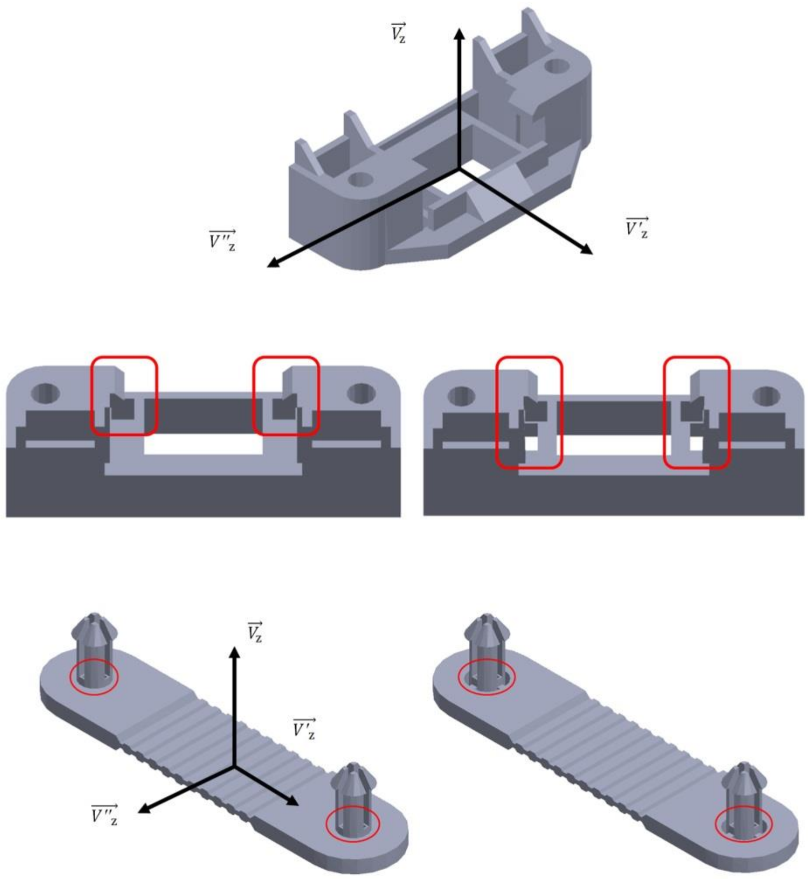


Fig 19.- *Cases F and G. Parts with non demoldable regions, geometrically modified to improve their demoldability.*

In Table 1 and in Fig. 20 the data and graphic results of the demoldability analysis performed over parts A, B, C, D, E are collected. In Table 1 the reallocating of facets after applying the demoldability analysis presented in this paper is shown, as well as the time spent on demoldability analysis.

Case Studies	Nº Facets	Nº Upper Cavity Facets	Nº Lower Cavity Facets	Nº Side Core Facets	Nº Undercut Core Facets	Simulation Time (s)
A	306	123	183	-	-	30,282
B	1211	153	1058	-	-	583,199
C	400	81	151	168	-	85,233
D	696	244	316	136	-	254,073
E	1110	318	758	-	34	653,927

Table 1. *Cases A, B, C, D, E.*

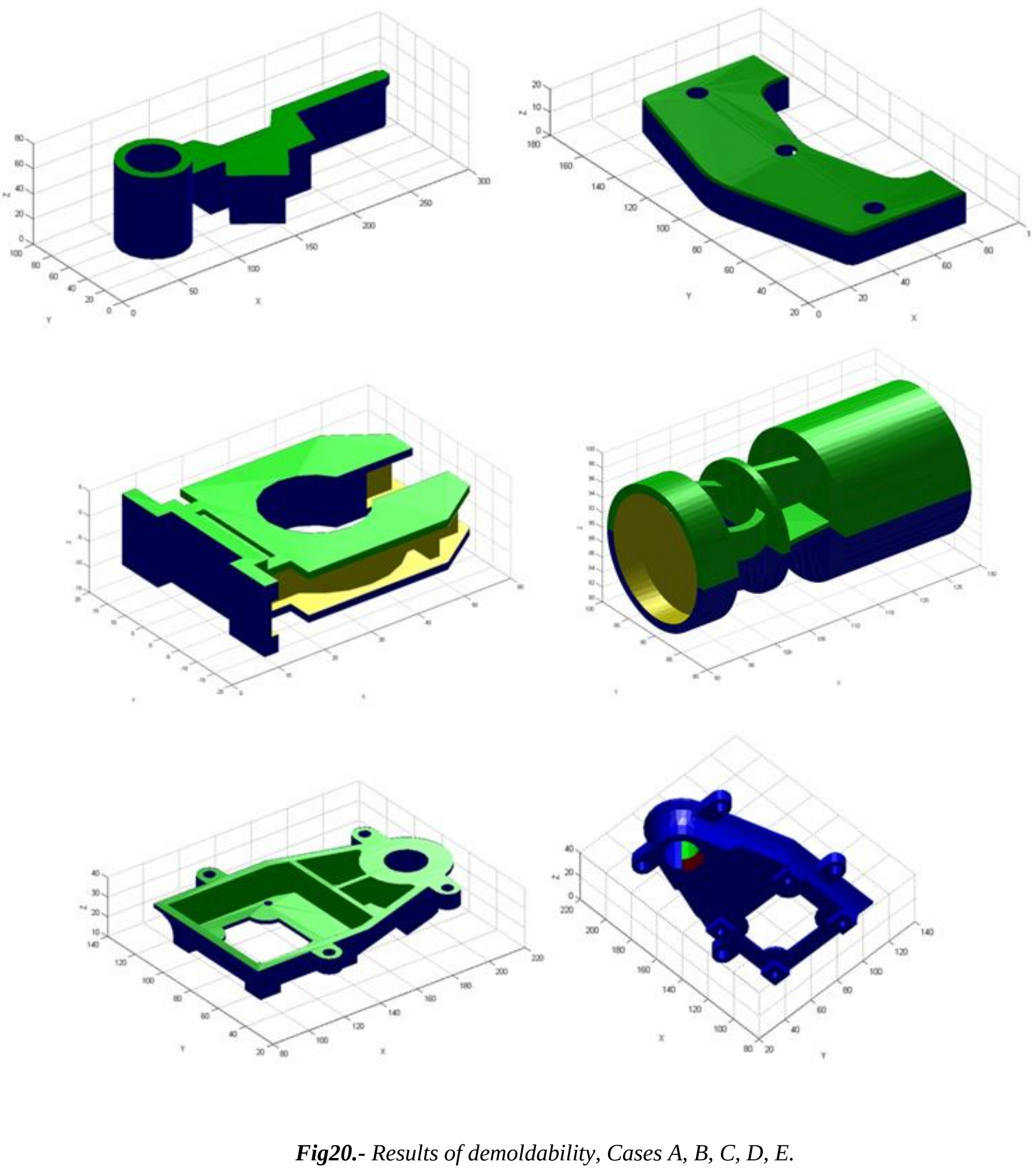


Fig20.- *Results of demoldability, Cases A, B, C, D, E.*

Parts F and G have been used as an example of optimizing the geometry of the part as a methodology in order to enhance its demoldability, Table 2 shows the results of applying the demoldability analysis algorithm over both parts. The graphic demoldability analysis results are indicated in Fig. 21.

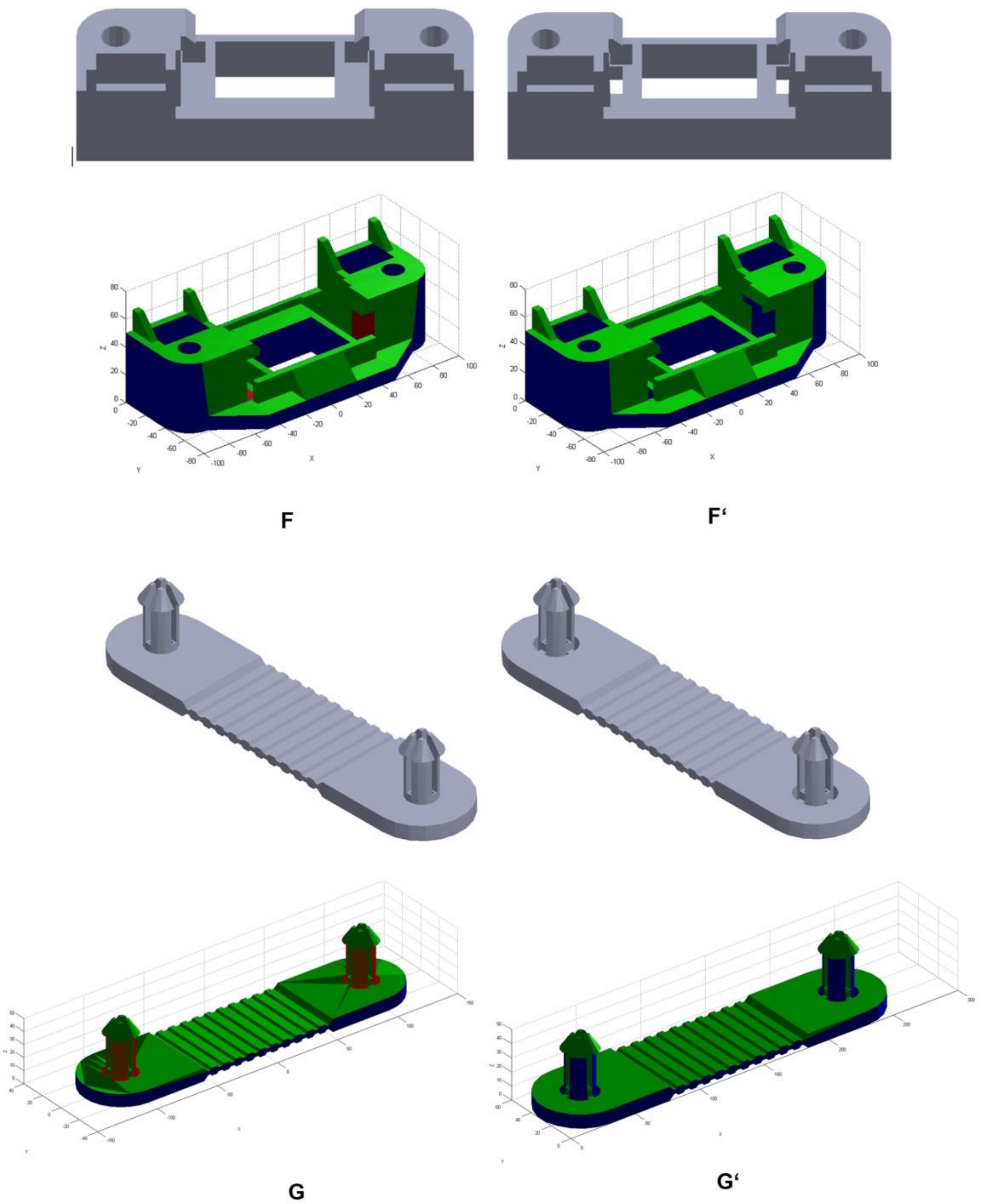


Fig21.- *Results of demoldability, Cases F, G original part and Cases F',G' optimized part..*

Case Studies	Nº Facets	Nº Upper Cavity Facets	Nº Lower Cavity Facets	Nº Side Core Facets	Nº Undercut Core Facets	Simulation Time (s)
F	474	245	205	-	24	65,433
F'	506	250	256	-	-	69,850
G	1334	484	772	-	78	835,267
G'	1556	433	1123	-	-	1107,703

Table 1. *Cases F , G, F', G'.*

In order to evaluate the computational cost of the algorithm, Fig. 22 shows a graph relating the number of facets for each application of the algorithm to the total time of execution. In the pieces studied a linear relationship between the number of facets of the piece and the time of completion of the analysis is observed, the ratio time / number of facets being 0.85.

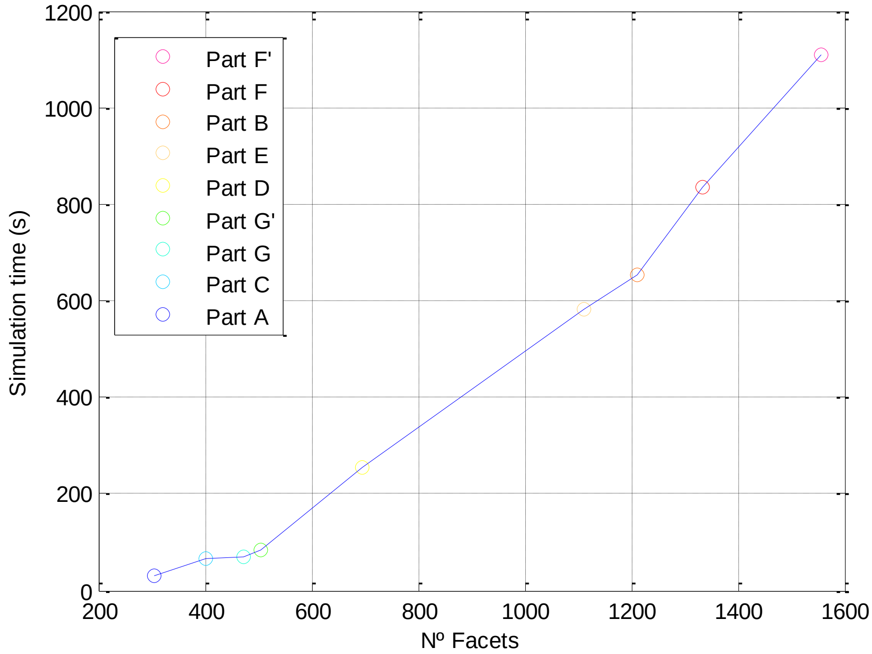


Fig22.- *CPU Computational Cost of Demoldability Algorithm.*

5.- Conclusions

This paper proposes a new method for performing the analysis of demoldability and the calculation of parting line for a given parting direction; this methodology poses a hybrid analysis of the discrete mesh of the plastic part, analyzing the demoldability of both the mesh nodes and the facets. The developed algorithm works with an initial file with discrete surface geometry, and obtains, after analyzing, a new virtual discrete geometry which incorporates information regarding the manufacture of the piece. It is valid for all kinds of parts and CAD systems, regardless of the modeler used. The algorithm detects not only undercuts resolvable by side cores, but also non-demoldable areas, allowing the designer to modify the design in the early stages.

Three subalgorithms for analyzing demoldability of the plastic part (Inproject, Catalogfaces and Reclasssfacet) have been developed. The first sub-algorithm Inproject executes a mesh preprocessing, resulting in a classification of the nodes by means of arrays according to their demoldability. The Inproject preprocessing phase uses a series of checkpoints or Gauss points on the analyzed facets as a means of discrete checking of the demoldability. Inproject Algorithm (PGauss) was compared with the Isoobstructing algorithm (Boolean operations) [37] resulting in the conclusion that the Inproject algorithm takes only 6% of the time taken by the Isoobstructing algorithm.

The second sub-algorithm Catalogfaces catalogs facets belonging to the mesh according to their demoldability into demoldable facets, non demoldable facets, and semi-demoldable facets, based on the analysis of mesh nodes (Inproject). Finally, the third sub-algorithm Reclassfacet reclassifies and fragments the semi-demoldable facets, obtaining demoldable and non demoldable facets. Applying the sub-algorithm Reclassfacet has resulted in the creation of a new virtual mesh made up of nodes and facets that complement the initial mesh of the plastic part, providing information not only on the geometry but also on manufacture. The side core demoldable areas were calculated by reorienting the part in the direction perpendicular to the parting direction and reapplying the developed subalgorithms.

The algorithm has been validated through its application to five variable demoldability parts and two real parts, eliminating after analysis the areas classified as non-demoldable in order to improve their manufacture. All analyses were carried out using the same precision of the mesh (deviation and angle). We evaluated the relationship between the number of facets and the total execution time of the algorithm, resulting in a linear relationship between the number of facets of the part and the running time of the analysis.

The proposed method improves the methods developed to date in that it allows the realization of demoldability analysis independently of the CAD modeler, no utiliza el vector normal para el reconocimiento y clasificación de las zonas que componen la superficie de la pieza de plástico, it is valid for any geometry of the plastic part, and it does not need access to internal information of the part. The geometry of the solid remains stored in arrays for later use in other applications.

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7.- Appendices

7.1.- InProject algorithm

The goal of the Inproject algorithm is the discrete checking of demoldability in every facet under study. First, given a facet $F_i \in \mathcal{B}'_f$ a series of checkpoints or Gauss points $P_{Gauss} \in F_i$ are generated on the surface. Gauss points are control points that run and belong to the surface of the facet under study. During the gauss points generation, a set of equidistant lines to edges of facets separated a distance Dp are defined. Distance Dp is equal to the length of the minimum edge that belongs to upper boundary. The contour is varying in each plane, adapting the value of D_p to each new level of study. Next, these equidistant lines are segmented in order to get nodes separated a distance Dp. Thus, it is ensuring that the algorithm will be able to identify the minimum detail of a plastic part in successive scans along the parting direction.

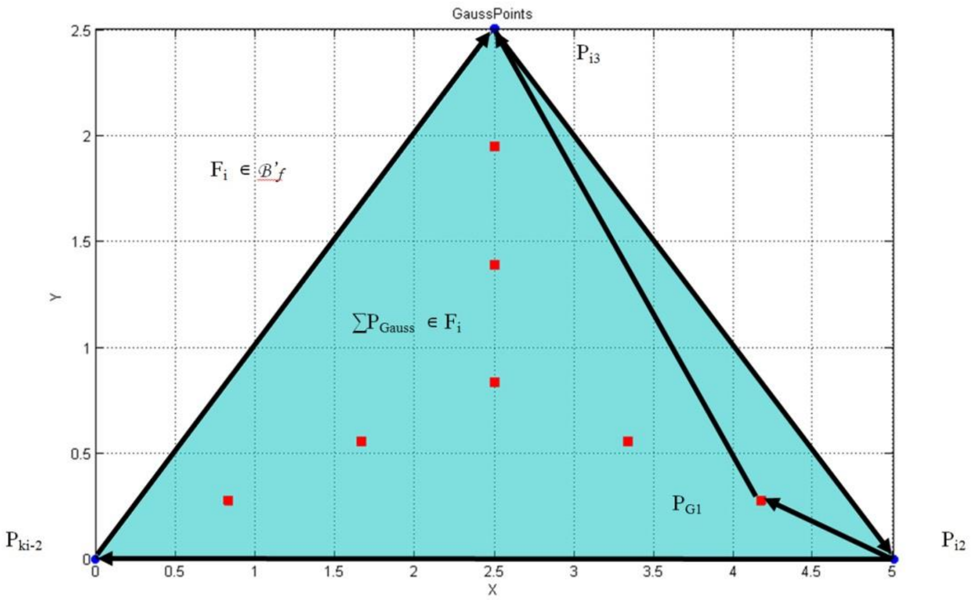

Fig23.- *Formal definition of Gauss points of a facet.*

Thus, and according to paragraph 3.2, given a facet under study $F_i \in \mathcal{B}'_f$ and a closed control contour $C_k \in Pl_k$, the algorithm is executed sequentially to check whether a point is inside the triangular facets of the closed contour C_k.

The orientation of a triangular region is defined by the orientation of the 3 vertices, considering the facet $F_i = \{P_{i1}, P_{i2}, P_{i3}\}$ Fig 23 and the checkpoint P_{G1}.

$$D = [(P_{i1})_x - (P_{i3})_x] \cdot [(P_{i2})_y - (P_{i3})_y] - [(P_{i1})_y - (P_{i3})_y] \cdot [(P_{i2})_x - (P_{i3})_x]$$

$$D' = \left[(P_{i2})_x - (P_{G_1})_x\right] \cdot \left[(P_{i3})_y - (P_{G_1})_y\right] - \left[(P_{i2})_y - (P_{G_1})_y\right] \cdot [(P_{i3})_x - (P_G)_x]$$

$$\text{If } [(D \geq 0) \wedge (D' \geq 0)] \vee [(D < 0) \wedge (D' < 0)] \rightarrow P_{G_1} \in F_i = \{P_{i1}, P_{i2}, P_{i3}\}$$

Applying this methodology, which is computationally faster, the overlap between different facets can be checked, and also whether the set of nodes and control points of a facet are demoldable against a contour C_k type such as that set forth in section 3.2. In Table 3 the comparison between Iso-obstructing facets overlap algorithms and the Inproject algorithm is indicated.

Methodology (Overlap Resolution)	Unioperational computational cost(s)
IsObstructing [37]	0,006137
InProject (8 Gauss points)	0,000432

Table 2. *Comparison of algorithms for solving overlapping of facets.*

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Vitae

Jorge Manuel Mercado Colmenero received his B.Eng and M.Eng. in Mechanical Engineering in 2012 and 2014 respectivily from the University of Jaén in Spain. In 2014 he joined the Design, Engineering Graphics and Project departament at Jaen university as a PhD student. He is currently developing his thesis about automatic analyses and design of injection plastic mold, including demoldability, ejection system, cooling system design and CAD-CAE design and manufacturing.

Dr. Miguel Angel Rubio Paramio received his MEng and Ph.D in Mechanical Engineering in 1992 and 2000 respectively from Polytechnic University of Madrid in Spain. In 1994 he joined the University of Jaen and currently he teaches courses of Computer-aided Design, Manufacturing and Engineering in the Engineering Graphics, Design and Projects Department. His thesis was focused on automatic analysis of injection plastic part. His research interests include CAD-CAM-CAE, Design for Manufacturing, computational geometry, constraint-based parametric modeling, and their applications on plastic injection mold design and manufacturing.

Dr. Jesus M. Perez is a professor in the Department of Mechanical and Manufacturing Engineering at Polytechnic University of Madrid. He has led research projects and publication studies in topics related to forming processes, dimensional metrology, process improvement and control, quality, and instrumentation. He held a position of Vice Director for Research in the Industrial Engineering School. His areas of interest include improvement of manufacturing processes, quality control and instrumentation

Dr. Cristina Martín Doñate received her B.Eng. and M.Eng. in Electrical - Electronics Engineering and Industrial Engineering (Polytechnic University of Valencia - Spain) and Ph.D in Industrial Engineering (University of Jaen - Spain). She performed her master final project at the Technical University of Graz in Austria, in the field of manufacturing systems. She worked for several years as project manager on developing new products in automotive manufacturing industries. She is currently university teacher at Jaen University where she develops research and conduct research projects in manufacturing systems, plastic injection mold design and technology, CAD systems and new products development.