Power Flow Approach based on the S-iteration Process

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Abstract*— In this paper, the application of the S-iteration process for solving the Power Flow (PF) problem is comprehensively studied. Thus, a combined approach between the S-iteration process and Newton's method (SIP-NR) is adapted for solving PF equations. Moreover, drawbacks shown by SIP-NR under heavy loading conditions due to its linear convergence are addressed by developing a simple but reliable technique called Modified SIP-NR (MSIP-NR). MSIP-NR uses a non-repeatedly updating Jacobian scheme with the aim to accelerate the convergence of SIP-NR. The introduced PF techniques are validated using well and ill-conditioned systems under several demand scenarios. The obtained results prove the robustness and efficiency of the introduced PF techniques compared with other well-known methodologies. On the light of the results obtained, the introduced PF techniques may be used for different applications in power system analysis. Some of these potential applications are enumerated and discussed.

***Index Terms*— PF analysis, S-iteration process, Newton's method, Well and ill-conditioned power Systems, Heavy loading conditions, Reactive power limits.**

I. Introduction

PF analysis is considered the most important tool for planning and operation of power systems [1], [2]. Mathematically, PF is a nonlinear problem which has to be solved using iterative techniques. Performance of different solvers is strongly affected by the conditioning of PF equations. Ill-conditioned PF problems are still challenging for most of existing techniques. This fact is sharper in large-scale systems, where an acceptable efficiency is required for properly managing large matrices and vectors calculations.

One should differentiate between the standard and robust PF techniques. The standard techniques are normally focused on improving the computational performance of PF calculation. The most popular and widespread used is the Newton-Raphson method (NR), which was firstly proposed for PF solution at late 60's [3]. Later, the Fast-Decoupled Power Flow [4] and its modified versions [5], [6] were developed in order to overcome the difficulties related with updating and factorizing the Jacobian matrix of NR. Recently, several high order Newton-like methods have been adapted for solving the PF problem [7], [8].

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On the other hand, the robust techniques are devoted to find the solution of PF problem when its equations are ill-conditioned. A formal definition of ill-conditioned systems can be found in [9]; *"The solution of the power flow problem exist, but standard solution methods fail to get this solution starting from a flat initial guess (e.g., all load voltage magnitudes equal to 1 and all bus voltage angles equal to 0)".* Therefore, ill-conditioned systems may be also called badly-initialized due to the issues encountered by the standard techniques are really related with the considered initial guess.

One of the most popular robust technique and widely considered as a benchmark in the literature is the so-called Iwamoto's method [10]. This technique is based on solving an optimization problem, and as result the corrector vector is modified in order to ensure the convergence. Based on this method, several PF techniques have been developed over decades [11]-[13]. These methods have outstanding robustness, however, they tend to be very slow (too many iterations). Moreover, since they are based on Newton's method, drawbacks related with the invertibility of Jacobian matrix still exist.

In 2009, Milano extended the Continuous Newton's principle [14], to solve the PF problem in [9]. Based on the guidelines proposed in [9], several efficient and robust PF techniques can be developed using different numerical arrangements. In [15], a combined approach based on the 4th order Runge-Kutta formula and Broyden's method has been developed.

The Levenberg's method is a popular minimization technique that finds multiple applications in optimization problems. In [16], the performance of the Levenberg-Marquardt technique for solving the PF problem has been explored. Several issues like slow convergence or inaccurate results are associated with this method. In order to overcome these issues, various high order Levenberg-like methods have been proposed in [17], [18]. Convergence characteristics of the Levenberg-like approaches strongly depend on a set of predefined parameters which should be carefully tuned. Moreover, they tend to be slow if the initial guess is far away from the solution and, sometimes, involve heavy calculations.

In [19], [20], the PF problem has been represented as a dynamical system. Using the Lyapunov theory, the PF problem is represented as a set of differential equations. Thus, they can be solved using integration techniques, for example, ode's routines in MATLAB. These techniques avoid issues related with the invertibility of Jacobian matrix and they are sensitive with respect to the initial guess. However, solving the resulting dynamic system involves heavy calculations which limits the applicability of the dynamic solution paradigm in large-scale systems.

In [21], the holomorphic theorem has been extended for solving the PF problem. This approach is quite robust, being adequate for badly-initialized cases. However, it has some disadvantages. Firstly, it involves some very heavy calculations like convolutions or Padé approximants. Secondly, numerical properties of this approach are formulation dependent, which is still an open topic [22].

Recently, several PF techniques based on numerical methods have been proposed in [23], [24]. These techniques are quite robust and efficient enough to manage large-scale badly-initialized cases.

The S-iteration process was introduced in [25]. This technique has a rate of convergence similar to the Picard iteration, but it is faster than the other fixed-point iteration algorithms. In 2012, a powerful nonlinear technique resulting from the combination of the S-iteration process and NR was proposed [26]. In this paper, this method is applied to solve the PF equations.

This paper aims to comprehensively study the application of the S-iteration process for solving the PF problem. To achieve this target, the main contributions of this work are twofold:

- Study the application of the algorithm developed in [26] for solving the PF problem and proposing a powerful PF technique (SIP-NR);
- Overcoming the potential drawbacks of SIP-NR under heavy loading conditions due to its linear convergence characteristics. This is achieved by developing a simple but reliable technique for non-repeatedly updating the Jacobian matrix. Thus, a modified version for SIP-NR PF technique is developed (MSIP-NR).

The introduced PF techniques are validated using several well-conditioned power systems ranging from 14-, to 9241-buses, and they are compared with other well-known PF solvers. In addition, the suitability of the presented PF techniques for solving ill-conditioned cases is also validated using 3012-bus, 3375-bus, 13659-bus, 27318-bus ill-conditioned power systems, comparing their performance with other standard and robust PF techniques available in the literature. In order to provide a complete analysis, several scenarios such as heavy loading conditions and variable limits enforcement are also considered. Finally, we enumerate several potential applications of the introduced PF techniques for power system analysis.

Remainder of this paper is organized as follows. Section II presents SIP-NR and MSIP-NR PF techniques. The robustness of the introduced PF techniques is compared with the standard NR in Section III. In Section IV, several numerical experiments are carried out in order to check the features of the presented PF techniques. Potential applications of the introduced PF techniques for power system analysis are enumerated in Section V. Finally, the main conclusions are presented in Section VI.

II. Introduced PF techniques

In this section, the PF problem is briefly described as well as the introduced PF techniques are presented.

A. *Brief description of the PF problem*

The PF problem in polar coordinates can be defined as a set of n nonlinear equations as follows [8, 9]:

$$\mathbf{g}(\boldsymbol{x}) = 0 \tag{1}$$

where; $\boldsymbol{x} \in \mathbb{R}^n$ are the variables and $\mathbf{g}: \mathbb{R}^n \mapsto \mathbb{R}^n$ are smooth nonlinear equations which represent the active and reactive power balance at network buses. Since (1) are nonlinear equations, an iterative technique must be used for solving them. A generic k^{th} iteration of NR for solving (1) is defined as follows:

$$\Delta\boldsymbol{x}_k = -[\boldsymbol{J}_k]^{-1}\mathbf{g}(\boldsymbol{x}_k) \tag{2}$$
$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \Delta\boldsymbol{x}_k \tag{3}$$

where, $\boldsymbol{J}_k = \nabla^T\mathbf{g}(\boldsymbol{x}_k) \in \mathbb{R}^{n\times n}$ is the Jacobian matrix and $[\cdot]^{-1}$ represents the inverse operator.

As it is well-known, NR converges quadratically near the solution, however, its convergence is eminently local. Consequently, it is likely to fail if the starting point is not close enough to the solution.

The heaviest computational part of NR procedure is the factorization of the Jacobian matrix, which needs to be computed each iteration. In order to overcome this issue, a modified NR method (MNR) is described as follows [27]:

$$\Delta\boldsymbol{x}_k = -[\boldsymbol{J}_0]^{-1}\boldsymbol{g}(\boldsymbol{x}_k) \tag{4}$$

MNR can be written in its step-size based formulation [9]. Thus, a step size $\alpha \in (0,1)$ can be used for truncating (4). Consequently, the Truncating Modified NR (TMNR) is defined by:

$$\Delta\boldsymbol{x}_k = -\alpha[\boldsymbol{J}_0]^{-1}\boldsymbol{g}(\boldsymbol{x}_k) \tag{5}$$

MNR and TMNR are, in fact, members of the family of Frozen Jacobian Newton-like methods [28]. This kind of techniques reduce the impact of updating and factorizing the Jacobian matrix each iteration, since the initial information is repeatedly used during iterative process. Using this modification, the convergence of NR becomes linear. This fact is more noticeable when a turning point is approached, which provokes that many iterations are required to achieve the solution.

A stopping criterion should be considered in order to determine if the algorithm reached a reasonable solution or not yet. In this paper, the following criterion is used:

$$\|\mathbf{g}(\boldsymbol{x}_k)\|_\infty \le \varepsilon \tag{6}$$

where, $\|\cdot\|_\infty$ is the infinity norm and $\varepsilon \in \mathbb{R}^+$ is the convergence tolerance which is typically fixed smaller than 10^{-3}.

B. SIP-NR PF technique

The S-iteration process besides its convergence properties have been described in [25]. Briefly, let us consider C a nonempty subset of a normed space X and $T: C \mapsto C$ a mapping. Then, the generic k^{th} iteration of the S-iteration process can be defined as:

$$y_k = (1-\beta_k)x_k + \beta_k T x_k \quad (7)$$
$$x_{k+1} = (1-\alpha_k)T x_k + \alpha_k T y_k \quad (8)$$

where, $\{\alpha_k\}$ and $\{\beta_k\}$ are real sequences in $(0,1)$ which satisfy the following condition:

$$\sum_{k=1}^{\infty} \alpha_k \beta_k (1-\beta_k) = \infty \quad (9)$$

In [25], it has been demonstrated that the S-iteration process has similar convergence rate to the Picard iteration, but it is faster than other fixed-point algorithms such as the Mann and Ishikawa's iteration processes. It is worth to mention that the S-iteration process has been also applied for solving constrained minimization and split feasibility problems [29].

In [26], a combined approach between the S-iteration process and Newton's method for solving a non-linear operator equation in Banach spaces was presented. This methodology can be easily applied to solve the PF problem according to the following definitions:

$$\boldsymbol{z}_k = \boldsymbol{x}_k - [\boldsymbol{J}_0]^{-1}\mathbf{g}(\boldsymbol{x}_k) \quad (10a)$$
$$\boldsymbol{y}_k = (1-\alpha)\boldsymbol{x}_k + \alpha \boldsymbol{z}_k \quad (10b)$$
$$\boldsymbol{x}_{k+1} = \boldsymbol{y}_k - [\boldsymbol{J}_0]^{-1}\mathbf{g}(\boldsymbol{y}_k) \quad (10c)$$

In order to be consistent with [26], parameter α is taken constant instead of being defined as a real sequence.

Equation (10) presents a novel PF solution technique which has been called SIP-NR in this paper. Despite that the convergence of SIP-NR is still linear, it is demonstrated in [26] that it is faster than MNR (4). The solution process of SIP-NR PF technique can be summarized in Algorithm 1.

Algorithm #1: Solution process of SIP-NR PF technique

Step 0: Read input data, select a starting point x_0 and define the parameters $\alpha \in (0,1)$ & ε. Calculate the Jacobian matrix at $\boldsymbol{x}_0$ and factorize it using LU decomposition. Calculate $\mathbf{g}(\boldsymbol{x}_0)$. Set $k = 0$.
Step 1: Calculate $\boldsymbol{z}_k$ using (10a).
Step 2: Calculate $\boldsymbol{y}_k$ using (10b).
Step 3: Calculate $\mathbf{g}(\boldsymbol{y}_k)$, then update $\boldsymbol{x}_{k+1}$ using (10c).
Step 4: Calculate $\mathbf{g}(\boldsymbol{x}_{k+1})$. Set $k = k+1$. If the convergence criteria (6) is satisfied, then stop, Otherwise, go to Step 1.

As it can be seen, the Jacobian matrix is only factorized once at the beginning of the process, which supposes an important advantage of SIP-NR in comparison with NR. On the other hand, it implies that the convergence characteristics become linear as (4). Thus, it is expected that the SIP-NR will employ more iterations than NR. However, this drawback may be compensated by its competitive computational cost.

SIP-NR technique involves more matrix and vectors sum and products than NR, nevertheless, these computations can be efficiently addressed, and they have not a significant impact on the overall performance.

In [9], it is claimed that the operator (2) is asymptotically stable and the solution is reached when $k \mapsto \infty$ if the initial guess $\boldsymbol{x}_0$ lies inside of the Region of Attraction (ROA). Therefore, we can claim that the SIP-NR is robust if it shows a wider ROA in comparison with NR. This fact will be demonstrated in Sections III and IV. Therefore, SIP-NR can be considered a robust PF technique.

C. Relationship among SIP-NR, MNR and TMNR

SIP-NR defined by (10), MNR and TMNR, belong to the family of Frozen Jacobian Newton-like methods, as Jacobian matrix is only factorized once at the beginning of the process. On the other hand, SIP-NR is closer related with TMNR since both techniques involve the usage of the step size (α). Consequently, SIP-NR and TMNR are also members of the family of Step Size-based methods. Interested readers can find other members of this family in [9]-[13], [23] or [24]

Equation (10) presents a multistep (or multipoint) scheme which brings higher convergence rate than (4) and (5) [30]. Although, this fact was pointed out in [26], it can be also deduced as follows. MNR and TMNR are the Frozen Jacobian alternatives of NR. In the same way, SIP-NR can be seen as the multistep alternative of MNR. However, NR has quadratic convergence rate while a multistep structure normally brings, at least, cubic convergence. Consequently, it may be deduced that the SIP-NR has higher convergence rate than MNR and TMNR. This idea is confirmed by the results obtained in Section IV. It is worth to mention that the higher convergence features of SIP-NR are due to its multistep structure. In this regard, an iteration of (10) is not computationally comparable with an iteration of (4) or (5). Due to its higher convergence features, SIP-NR is expected to be (at least frequently) more efficient than MNR and TMNR.

On the other hand, MNR might show convergence difficulties in ill-conditioned cases [27]. Here, parameter α plays a key role and gives a great versatility to TMNR and SIP-NR for properly managing both well and ill-conditioned systems. This parameter is used for controlling the convergence rate, hence TMNR and SIP-NR turn out to be reliable techniques as it is confirmed in Section IV.

In TMNR solution procedure, the step size plays its original role by truncating the increment vector. From the iterative procedure defined by (10), it can be observed that the parameter α is introduced in different way. In this case, the step size is used for calculating an alternating step (10b) according to the influence of $\boldsymbol{x}_k$ and $\boldsymbol{z}_k$.

This alternating step (10b) can be seen as a combination of a single and a multistep structure. Thus, it can be easily checked that algorithms (4) and (10) are equivalent at $\alpha = 0$. On the other hand, for different values of α, (10) preserves its multistep structure with its own advantages over a single step procedure. In other words, (10b) allows SIP-NR to balance between a single and a multistep structure, gathering the advantages of both schemes. This feature gives (10) great versatility for efficiently managing a wide range of cases and scenarios.

Also, closer relationship may be observed between TMNR and SIP-NR. However, some differences between them are worth mentioning:

- While TMNR has to be categorized as a single-step method, SIP-NR undoubtedly presents a multistep structure due to (10c), which normally brings higher convergence characteristics.
- Both TMNR and SIP-NR are Step Size-based methodologies. However, the parameter α plays different roles in (5) and (10). The alternating step (10b), that is original of the S-iteration process, supposes the main difference between (10) and (5). Also, importance of (10b) is remarkable since it allows to gather the advantages of a single and a multistep scheme. To sum up, we can affirm that SIP-NR belongs to four families of methodologies for solving systems of nonlinear equations. Firstly, it is a Frozen Jacobian Newton-like method. Secondly, it is a Step Size-based technique. Thirdly, it is a multistep method and fourthly it belongs to the family of those methods derived of the S-iteration process.

D. *Improving the convergence rate of SIP-NR*

As previously mentioned, SIP-NR requires only one matrix factorization. Computationally, it is an important advantage. However, it provokes linear convergence, which is considered problematic especially when a turning point (like the Maximum Loadability Point (MLP)) is approached. As it will be shown in Section IV, SIP-NR employs many iterations for converging when the solution is near to MLP. In order to overcome this issue, a modified SIP-NR (MSIP-NR) is developed. It is based on determining when the convergence is turned to be too slow, thus, the Jacobian matrix is determined to be updated for accelerating the convergence. The solution process of developed MSIP-NR PF technique is summarized in Algorithm 2.

Algorithm #2: Solution process of MSIP-NR PF technique

Step 0: Read input data, select a starting point x_0 and define the parameters; $\alpha \in (0,1)$ and ε. Calculate the Jacobian matrix at $\boldsymbol{x}_0$ and factorize it using LU decomposition. Calculate $\mathbf{g}(\boldsymbol{x}_0)$. Set $k = 0$.
Step 1: Calculate $\boldsymbol{z}_k$ using (10a) and the most updated Jacobian matrix.
Step 2: Calculate $\boldsymbol{y}_k$ using (10b).
Step 3: Calculate $\mathbf{g}(\boldsymbol{y}_k)$, then update $\boldsymbol{x}_{k+1}$ using (10c) and the most updated Jacobian matrix.
Step 4: Calculate $\mathbf{g}(\boldsymbol{x}_{k+1})$. Set $k = k + 1$. If the convergence criteria (6) is satisfied, then stop, otherwise, go to Step 5
Step 5: If $k > 1$, then calculate μ as:

$$\mu = abs(\|\Delta \boldsymbol{x}_k\|_\infty - \|\Delta \boldsymbol{x}_{k-1}\|_\infty) \tag{11}$$

and go to step 6, otherwise, go to Step 1.
Step 6: If $\mu < 10^{-2}$ go to step 7, otherwise, return to Step 1.
Step 7: Calculate and factorize Jacobian matrix at $\boldsymbol{x}_k$ and return to Step 1

In this paper, 10^{-2} is taken as threshold for determining when the Jacobian matrix should be updated. Nevertheless, this value can be freely adapted for convenience. Broadly, if this parameter is high, MSIP-NR tends to employ fewer iterations for converging, however, Jacobian matrix is frequently updated, consequently, higher computational burden is found as counterpart. On the other hand, this technique becomes similar to SIP-NR if this threshold is fixed too small, it means, the Jacobian matrix would be only factorized once at first iteration remaining constant during the whole iterative process.

Comparing SIP-NR and MSIP-NR, it is worth mentioning that these iterative techniques proceed in the same way while $\mu \geq 10^{-2}$. In the opposite case, MSIP-NR switches to the first iteration routine, conveniently updating and factorizing the Jacobian matrix.

The Jacobian matrix of PF equations is typically non-positive definite. Hence, the LU decomposition, which has a computational burden of $O(n^3)$, has to be employed. Consequently, the importance of the factorization of the Jacobian matrix cubically grows with the size of the system. Hence, this fact becomes critical in large systems. It is worth to mention that the proposed PF methods allow to use sparsity routines in the same way as NR, thus, the impact of the factorization may be reduced to $O(n^{3/2})$. However, we can affirm that SIP-NR will be, *a priori*, more efficient than NR for the same reasons that the Fast-Decoupled Load-Flow is also more efficient than NR. It is expected that MSIP-NR, typically uses less factorizations in a whole PF calculation than NR. In this case, MSIP-NR would be more efficient than NR. This is confirmed in Section IV.

E. *Handling generators' reactive power limits*

The most popular strategy for considering the equipment limits (which is used in [31]), is carried out as follows; when a PF solution is calculated, it is checked if any reactive limit at PV buses is violated. If it occurs, the connected PV buses are converted to PQ type. Thus, the injected reactive power at this "new" PQ bus is fixed at the violated limit. Then, the PF solution is repeated. This process is repeated until no limits are violated or unfeasibility of the system is claimed. This popular strategy can be easily implemented within SIP-NR and MSIP-NR iterative processes, as it is described in the flowchart of Fig. 1.

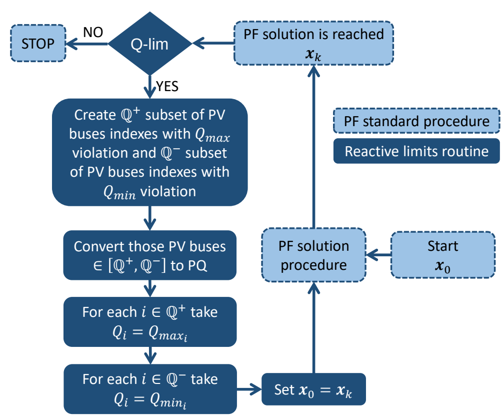

Fig. 1. The strategy implemented in SIP-NR and MSIP-NR for handling the generator's reactive power limits

Certainly, the introduced techniques are versatile enough to implement other available strategies for considering the equipment limits. For example, the generators' reactive limits are considered as inequality constraints in [19]. This strategy may be easily implemented within SIP-NR and MSIP-NR.

III. Measuring the Robustness of Introduced PF Techniques

Using a simple example, we can show that the Frozen Jacobian and Step Size-based methods may be more robust than NR. Firstly, let us analyze a 1-dimensional case for simplicity. Fig. 2 sketches the procedure of a Frozen Jacobian Newton-like method versus a standard technique which updates the Jacobian matrix each iteration (e.g. NR).

Assuming we start at an arbitrary point x_0, and we aim to find the zero of the function, both the standard and Frozen Jacobian based methods find x_1 using the information of the first derivative at first iteration. Then, NR updates the first derivative and obtains x_2 (blue trajectory). It can be easily checked that this technique diverges in following iterations. By reusing the information of first derivative at second iteration to obtain x_2 (red trajectory), it is observed that divergence is avoided since the incremental vector is reduced. This is the reason why the Frozen Jacobian methods are occasionally more robust than NR. Same effect might be achieved by simply damping the increment vector [9] as in the Step Size methodologies.

SIP-NR and MSIP-NR combine both mechanisms; therefore, they are expected to be more robust than NR. However, the Frozen Jacobian methods suffer slow convergence if the incremental step at first iteration is too small. It typically occurs around at turning point. This drawback may be overcome using MSIP-NR.

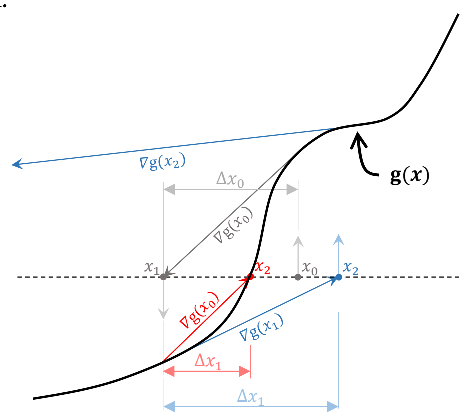

Fig. 2. Example of NR divergence (blue trajectory) and convergence of fixed Jacobian-based method (red trajectory).

In order to demonstrate that SIP-NR and MSIP-NR are more robust than NR, we have carried out a statistical analysis as follows. Taking the correct solution of the 13659-bus system from the EU PEGASE project (*13659pegase*) [32], [33], as the mean value of a Gaussian distribution, up to 50 initial guesses are built for different standard deviations. Then, NR, SIP-NR and MSIP-NR are used for solving the PF problem using the generated starting points. Fig. 3 shows the number of correct solutions achieved by each PF technique for different values of standard deviation. As it can be clearly seen, both SIP-NR and MSIP-NR outperformed NR for large standard deviations. As expected, both SIP-NR and MSIP-NR showed the same performance. For simplicity, results obtained with MSIP-NR are not included in Fig. 3.

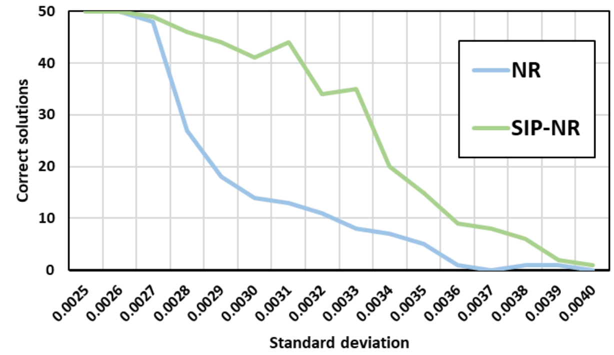

Fig. 3. Total number of correct solutions obtained by different NR and SIP-NR using different starting points (*case13659pegase*).

IV. Results and Discussion

In this section, SIP-NR and MSIP-NR are validated using several well and ill-conditioned systems ranging from 14-, to 13659-buses considering different demand scenarios. Parameter α SIP-NR and MSIP-NR is firstly empirically tuned. Several standard and robust PF techniques are considered for comparison.

All studied PF techniques have been coded in MATLAB (version R2014b) and run under Windows 10 on a 2.2 GHz Intel Core i7-8750H CPU personal laptop (16.00 GB RAM). Matpower v6.0 [31], has been used for programming all considered techniques. In this regard, Matpower-based codes of proposed SIP-NR and MSIP-NR can be found in [34].

A flat initial guess has been used in all simulations. Moreover, $\varepsilon = 10^{-5}$ has been taken as a convergence criterion. The reported computation times have been calculated as the mean value of 100 simulations, in order to avoid the influence of other computational activities.

One of the following cases may occur during the solution of PF problem:

- Convergence: in this case, the PF technique is able to obtain accurate solution within a predetermined convergence tolerance. In this regard, the solution obtained by the standard NR method and the default initial guess provided by Matpower is considered the correct one.
- No convergence (labelled NC): in this case, the value of $\|\mathbf{g}(\boldsymbol{x})\|_\infty$ grows or oscillates during a considerable number of iterations. Hence, the PF technique is not able to obtain accurate solution within a predetermined convergence tolerance.
- Low voltage solution (labelled *): due to the quadratic form of PF equations, they have two solution namely high and low voltage. The high voltage has been considered the correct (stable) solution, however, when a PF technique converges to the low voltage one, results are denoted by (*).
- Inaccurate result (labelled †): in this case, the PF technique is able to achieve a solution within the required convergence tolerance, but the obtained solution is not accurate (not high or low voltage solutions). Thus, the solution is considered inaccurate if it differs with respect

to the correct solutions greater than 0.01 pu (voltage magnitudes) or 1 deg (voltage angles).

A. Tuning the value of α

Iterative solution of PF equations by SIP-NR and MSIP-NR involves a parameter namely α. Hence, performance of the introduced PF techniques with different values of α is analysed in order to empirically determine the more suitable value. For the sake of brevity, only results for the *case9241pegase* [32], [33] are reported in Fig. 4 (similar results were obtained for other systems). From this figure, it can be observed that the best performance is obtained with $\alpha = 0.8$. Consequently, this value will be used in the following simulations.

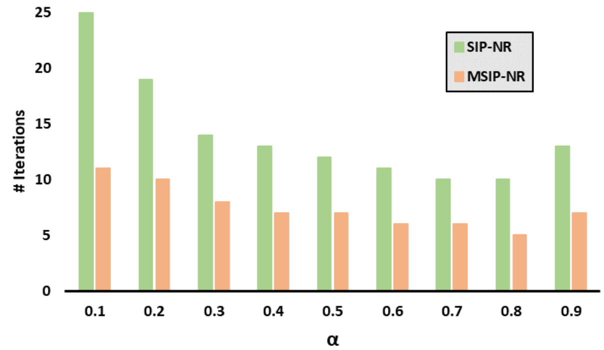

Fig. 4. Total number of iterations of introduced PF techniques with different values of α (*case9241pegase)*

B. Performance of the introduced PF techniques in well-conditioned systems

SIP-NR and MSIP-NR are validated using several well-conditioned cases. NR, Fast-Decoupled (XB version) (FDXB) [5], MNR and TMNR with $\alpha = 0.9$ are considered for comparison. Preliminary experiments carried out by the authors showed that XB and BX versions of the Fast-Decoupled Load-Flow obtained similar results. For the sake of brevity, only results for FDXB are reported.

The considered test systems are available at MATPOWER's database [35]. The total number of iterations and execution time in the considered well-conditioned systems are summarized in Fig. 5.

As expected, linear techniques (i.e. FDXB, MNR, TMNR and SIP-NR) often employed more iterations than NR and MSIP-NR. Nevertheless, FDXB and SIP-NR always gave the lowest number of iterations in comparison with MNR and TMNR. It is worth mentioning that, occasionally, NR and SIP-NR required the same number of iterations, which is remarkable due to SIP-NR has linear convergence. As previously commented, multistep structure of SIP-NR brings higher convergence features than NR. This can be more clearly observed by comparing MSIP-NR and NR, since the MSIP-NR require less iterations than NR in some cases.

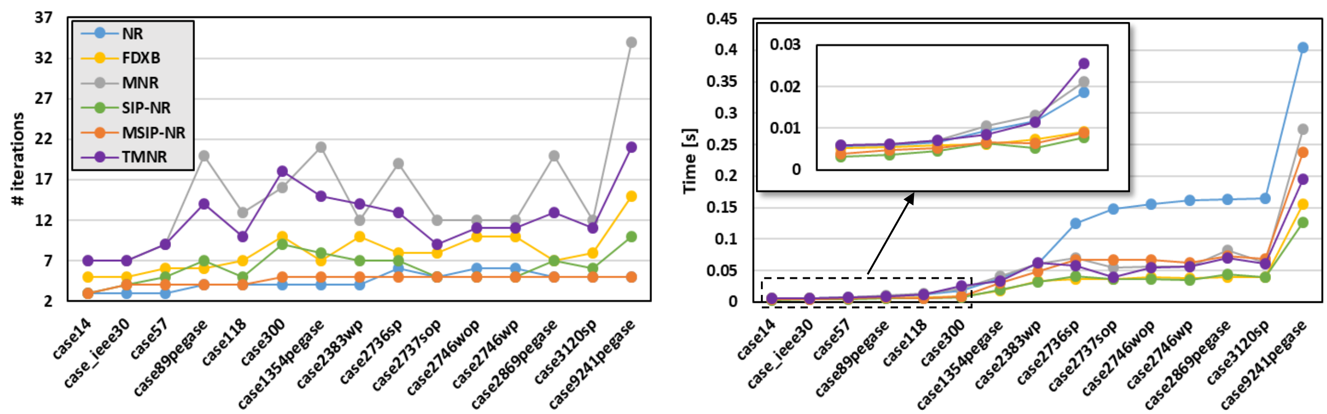

Fig. 5. Total number of iterations and execution time of different PF techniques in the base case scenario (well-conditioned systems).

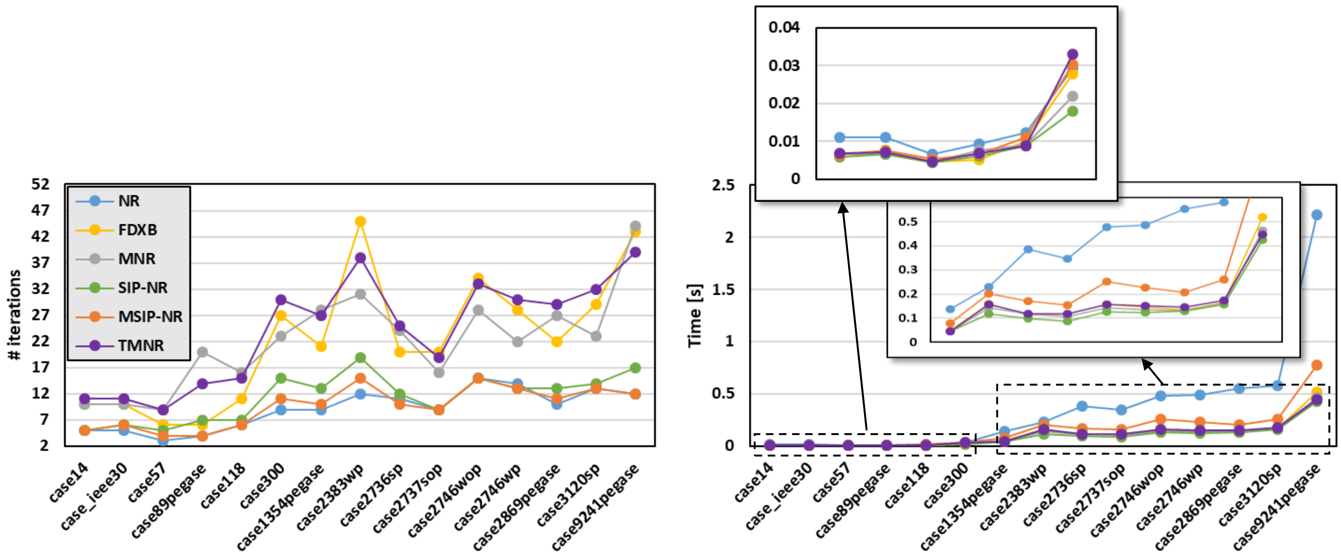

Fig. 6. Total number of iterations and execution time of different PF techniques when the generators' reactive limits are enforced (well-conditioned systems).

In terms of execution time, FDXB, SIP-NR and MSIP-NR were more efficient than the remainder PF techniques in small systems (up to 300 buses). In large systems, NR becomes inefficient compared with the linear methods. In such systems, FDXB and SIP-NR typically outperformed MNR, TMNR and MSIP-NR

Following the strategy described in Section II.E, the total number of iterations and execution time of PF techniques with consideration of the generators' reactive limits are summarized in Fig. 6. From this figure, it can be observed that NR, SIP-NR and MSIP-NR typically had the highest convergence speed in comparison with the remainder PF techniques. MSIP-NR was notably more efficient than NR. The execution times showed by FDXB, MNR, TMNR and SIP-NR were very similar and not substantial differences can be appreciated. Nevertheless, these techniques were more efficient than NR and MSIP-NR.

Now, let us analyze the effect of the loading level on the performance of the introduced PF techniques. In this test, the injected active and reactive power of load buses along the injected active power of generation buses are increased in steps of 0.0001 pu until the considered PF techniques diverged. For instance, in the *case1354pegase*, the limit load is 1.3139 pu (1.3140 pu gives rise to divergence). In addition, data of these limit cases are given in [34] and the considered loading levels are collected in Appendix A (Table V). The total number of iterations and execution time of different PF techniques with heavy loading conditions for the considered well-conditioned systems are shown in Fig. 7.

From Fig. 7, it can be observed that FDXB, MNR, TMNR and SIP-NR employed many iterations in all considered systems, due to their linear convergence characteristics. Behavior of these techniques with heavy loading conditions is quite irregular, and the total number of iterations notably varies depending on the studied system. Nevertheless, SIP-NR normally converges faster than FDXB, MNR and TMNR, which is coherent with the concluding remarks of [26]. Thus, FDXB, MNR, TMNR and SIP-NR should be considered totally inefficient in this scenario. Despite that MSIP-NR always employed more iterations than NR, it still outperformed NR in terms of execution time in all considered systems.

C. Performance of the introduced PF techniques in ill-conditioned systems

In this subsection, the ability of SIP-NR and MSIP-NR for solving ill-conditioned systems is checked. In general, a system is considered ill-conditioned if, despite its solution exists, it is not reachable using NR and a flat start [9]. The following ill-conditioned cases are considered:

- The 3012-bus portion of the Polish system at winter 2007-08 evening peak (*3012wp*) [35];
- The 3375-bus portion of the Polish system at winter 2007-08 evening peak (*3375wp*) [35];
- The 13659-bus portion of the European Transmission system from PEGASE project (*13659pegase*) [33], [34],
- The duplicated version of 13659-bus system [33], [34] (*13659pegase(x2)*). This system can be found in [31].

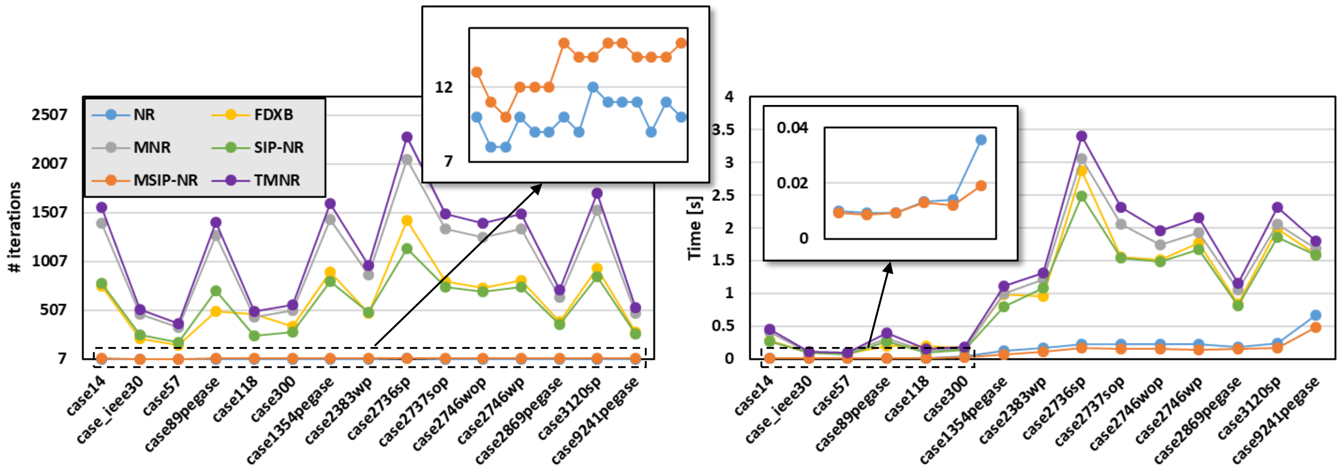

Fig. 7. Total number of iterations and execution time of different PF techniques with heavy loading conditions (well-conditioned systems).

The following robust and standard PF techniques are considered for comparison.

- Standard NR;
- Fast Decoupled Load-Flow XB version (FDXB);
- Iwamoto's method [10];
- 4th order Runge-Kutta formula (RK4) [9];
- High-Order Levenberg-Marquardt method (HLM) [17];
- Levenberg-Marquardt with non-monotone line search method (LMNM) [18];
- 2nd order Adams-Bashforth method (AB2) [23];
- Reverse Bulirsch-Stoer approach (RBS) [24].
- MNR method defined by (4).
- TMNR method defined by (5) with $\alpha = 0.9$

The parameters of the different considered PF techniques have been adjusted according to the guidelines provided in their references.

Table I reports the total number of iterations of different PF techniques for solving the considered ill-conditioned systems. From this table, it can be observed that SIP-NR and MSIP-NR PF techniques are robust enough to properly manage ill-conditioned systems.

TABLE I
TOTAL ITERATIONS NUMBER OF DIFFERENT PF TECHNIQUES FOR SOLVING ILL-CONDITIONED SYSTEMS

Method	3012wp	3375wp	13659pegase	13659pegase(x2)
NR	NC	NC	NC	NC
FDXB	9	9	10	10
Iwam.	2000	1218	43*	41*
RK4	NC	NC	21	21
HLM	113	99	908	1572
LMNM	1814	1871	1675	1675
AB2	30	31	23	50
RBS	13	13	14	14
MNR	15	15	NC	NC
TMNR	13	12	67	67
SIP-NR	6	6	29	29
MSIP-NR	6	6	13	13

From Table I, it can be observed that NR failed in all considered systems. FDXB successfully solved all systems employing a reasonable number of iterations. The Iwamoto, HLM and LMNM methods required large iterations numbers, in addition, the Iwamoto's method converged to the low voltage solution in the *13659pegase* case and its duplicated version. MNR successfully solved the *3012wp* and *3375wp* cases, however, it diverged in the *13659pegase* case and its duplicated version. TMNR was able to overcome the drawbacks exhibited by MNR and it solved all studied systems. RK4 diverged in the *3012wp* and *3375wp* cases. On the other hand, AB2, RBS, SIP-NR and MSIP-NR successfully solved all studied ill-conditioned systems.

Both SIP-NR and MSIP-NR converged faster than the remainder methods in the *3012wp* and *3375wp* cases. However, in the two largest cases, some techniques employed fewer iterations than SIP-NR. This is due to these systems are operated near the MLP, which affected the convergence features of SIP-NR. Nevertheless, MSIP-NR was able to overcome this drawback employing a reasonable number of iterations in the *13659pegase* case and its duplicated version (only FDXB employed fewer iterations than MSIP-NR in these systems).

On the other hand, Table II reports the execution time (in seconds) of different PF techniques for solving the considered ill-conditioned systems. From this table, it can be observed that SIP-NR ad MSIP-NR were more efficient than the Iwamoto's method, RK4, HLM, LMNM, AB2 and RBS. FDXB was more efficient than MSIP-NR in all studied systems and faster than SIP-NR in the *13659pegase* case and its duplicated version, SIP-NR outperformed FDXB in the *3012wp* and *3375wp* cases. SIP-NR was always more efficient than MNR and TMNR, however, these techniques outperformed MSIP-NR in the *3012wp* and *3375wp* cases. It is worth mentioning that the solution times exhibited by the linear methods (i.e. FDXB, MNR, TMNR and presented methods), were actually very similar which, can be considered as insignificantly different for practical purposes.

TABLE II
EXECUTION TIME [S] OF DIFFERENT PF TECHNIQUES FOR SOLVING ILL-CONDITIONED SYSTEMS

Method	3012wp	3375wp	13659pegase	13659pegase(x2)
NR	NC	NC	NC	NC
FDXB	0.05	0.06	0.20	0.51
Iwam.	39.35	26.42	3.82*	7.45*
RK4	NC	NC	6.80	14.12
HLM	4.27	4.96	246.80	878.06
LMNM	44.27	60.11	289.54	601.04
AB2	0.57	0.65	2.01	8.78
RBS	0.80	0.89	4.03	8.35
MNR	0.06	0.07	NC	NC
TMNR	0.06	0.05	0.61	0.88
SIP-NR	0.04	0.04	0.34	0.60
MSIP-NR	0.07	0.08	0.37	0.75

Table III presents the total number of iterations and execution time of different PF techniques for solving the studied ill-conditioned systems when the generators' reactive limits are enforced. The strategy described in Section II.E has been employed in this case. In this scenario, the Iwamoto's method diverged in the *13659pegase(x2)* case. On the other hand, AB2 diverged in the *13659pegase* case and its duplicated version. For remainder cases, similar conclusions as for the base case scenario (Tables I and II) can be extracted.

Table IV reports the total number of iterations and execution time of different PF techniques for solving the ill-conditioned systems with heavy loading conditions. The loading level is tuned following the same procedure described for the well-conditioned cases. In addition, the considered loading levels are collected in Appendix A (Table VI). As for the well-conditioned systems, limit cases of the studied ill-conditioned systems are given in [34].

In this case, the Iwamoto's method successfully solved all studied systems. On the other hand, NR converged to the low voltage solution and LMNM obtained an inaccurate solution, in the *13659pegase* case and its duplicated version. MNR failed in all studied systems. As in the base case scenario, RK4 diverged in the *3012wp* and *3375wp* cases, in addition, it converged to the low voltage solution in the *13659pegase* case and its duplicated version. HLM, AB2 and RBS successfully solved the *3012wp* and *3375wp* cases, however, they diverged in the *13659pegase* case and its duplicated version. FDXB, TMNR SIP-NR and MSIP-NR successfully solved all considered cases.

As expected, the Iwamoto and Levenberg-like methods employed many iterations. FDXB, TMNR and SIP-NR required many iterations due to their linear convergence characteristics. On the other hand, MSIP-NR was able to successfully solve all systems employing a reasonable number of iterations. NR methods neede less iterations than MSIP-NR in the *13659pegase* case and its duplicated version, however, it was not able to obtain the high voltage solution in these systems.

MSIP-NR was the most efficient method in the *3012wp* and *3375wp* cases. TMNR gave the best results in the *13659pegase* case and its duplicated version. Nevertheless, in the two largest systems, FDXB, TMNR, SIP-NR and MSIP-NR performed similarly.

V. POTENTIAL APPLICATIONS OF THE INTRODUCED PF TECHNIQUES IN POWER SYSTEM ANALYSIS

The introduced PF techniques have been validated using several well and ill-conditioned cases under different demand conditions. These techniques were competitive in comparison with other available PF methodologies.

Based on the obtained results, SIP-NR and MSIP-NR might be widespread used in power system analysis. In this paper, only the deterministic PF problem has been addressed. However, the introduced PF techniques can be used in several applications such as:

- In different continuation or homotopy methods, the PF problem has to be solved repeatedly even near of the MLP. Due to the outstanding trade-off between efficiency and robustness showed by SIP-NR and MSIP-NR, their usage in some of these tools like the Continuation Power Flow [36] may offer good results;
- When stochastic character of generation and loads are taken into account, the PF analysis becomes a probabilistic problem. One standard solution procedure in this tool is the Monte Carlo method which computes a large number of PF problems in order to obtain an accurate solution. SIP-NR and MSIP-NR may be used for this purpose and saving in execution time can be substantially achieved. Other methodologies for solving the probabilistic PF have been recently proposed [37], [38]. They also solve various PF problems and, therefore, the introduced PF techniques can be straightforward used within these applications;
- PF analysis via Quasi-Static Time-Series has been addressed in several papers (see [39] and references therein). Indeed, this methodology solves several PF problems (frequently for each hour during a day). Saving in terms of execution time may be considerable if SIP-NR or MSIP-NR are employed within this tool;
- In security analysis, multiple PF problems for several contingencies are carried out. SIP-NR and MSIP-NR offer good characteristics in terms of robustness and efficiency to be successfully adapted for this tool;
- Generally, power system analysis involves multiple online applications. In this kind of tasks, a huge quantity of computations must be run quickly. SIP-NR and MSIP-NR offer an optimal trade-off between efficiency and reliability to be used for multiple online applications.

TABLE III
TOTAL NUMBER OF ITERATIONS AND EXECUTION TIME [S] (IN PARENTHESIS) REQUIRED BY PF TECHNIQUES FOR SOLVING THE ILL-CONDITIONED SYSTEMS WITH GENERATORS' REACTIVE LIMITS ENFORCEMENT

Method	3012wp	3375wp	13659pegase	13659pegase(x2)
NR	NC	NC	NC	NC
FDXB	18 (0.10)	22 (0.15)	16 (0.33)	16 (0.65)
Iwam.	4175 (83.00)	2811 (61.90)	73 (6.44)*	Diverge
RK4	NC	NC	34 (11.77)	34 (22.89)
HLM	119 (4.64)	106 (6.49)	911 (247.85)	1575 (874.91)
LMNM	3541 (86.72)	3856 (143.82)	2838 (497.22)	2838 (1×10^3)
AB2	34 (0.71)	34 (0.78)	NC	NC
RBS	29 (1.49)	32 (2.36)	24 (6.97)	24 (14.66)
MNR	18 (0.10)	19 (0.16)	NC	NC
TMNR	22 (0.13)	22 (0.18)	73 (0.74)	73 (1.23)
SIP-NR	10 (0.08)	10 (0.13)	31 (0.44)	31 (0.84)
MSIP-NR	10 (0.17)	10 (0.19)	15 (0.57)	15 (1.12)

TABLE IV
TOTAL NUMBER OF ITERATIONS AND EXECUTION TIME [S] (IN PARENTHESIS) REQUIRED BY PF TECHNIQUES FOR SOLVING THE ILL-CONDITIONED SYSTEMS WITH HEAVY LOADING CONDITIONS

Method	3012wp	3375wp	13659pegase	13659pegase(x2)
NR	NC	NC	7 (0.63)*	7 (1.30)*
FDXB	355 (0.83)	341 (0.84)	58 (0.50)	58 (1.01)
Iwam.	1847 (36.35)	1100 (23.76)	41 (3.65)	41 (7.40)
RK4	NC	NC	20 (6.41)*	20 (13.61)*
HLM	1647 (63.10)	1897 (115.39)	NC	NC
LMNM	1814 (44.27)	1871 (60.11)	1675 (289.54)†	1833 (655.84)†
AB2	34 (0.63)	35 (0.71)	NC	NC
RBS	13 (0.80)	13 (0.89)	NC	NC
MNR	NC	NC	NC	NC
TMNR	492 (0.81)	462 (0.76)	66 (0.47)	66 (0.96)
SIP-NR	247 (0.77)	232 (0.71)	41 (0.52)	41 (1.03)
MSIP-NR	12 (0.14)	12 (0.16)	17 (0.48)	17 (0.97)

VI. CONCLUSIONS AND FUTURE WORKS

This paper has studied the application of the S-iteration process for PF analysis. Thus, the iterative nonlinear solver SIP-NR proposed in [26] has been considered for solving PF problems in well and ill-conditioned power systems. SIP-NR is very efficient but its convergence is linear, which is problematic in some cases. With the aim of overcoming this issue, a modified version of SIP-NR (MSIP-NR) has been developed to notably improve the convergence characteristics of SIP-NR.

A statistical analysis using different initial guesses has been carried out in order to demonstrate that SIP-NR and MSIP-NR are less affected by the starting point compared with the standard NR method.

The introduced PF techniques have been tested in a wide range of systems and scenarios. They have been firstly validated using several well-conditioned systems ranging from 14-, to 9241-buses. Their suitability for solving ill-conditioned cases have been also checked using 4 naturally ill-conditioned systems. In addition, different demand scenarios such as heavy loading conditions and variable limits enforcement have been studied.

Based on the obtained results, SIP-NR and MSIP-NR are able to notably outperform NR in well-conditioned systems, due to their low computational burden. In terms of robustness, the proposed SIP-NR and MSIP-NR showed high reliability successfully solving all studied ill-conditioned systems. On the other hand, they notably outperformed other robust techniques such as the Iwamoto's method, Levenberg-based techniques, RK4, AB2 and RBS.

In comparison with other linear techniques such as FDXB, MNR or TMNR, despite that SIP-NR typically showed the highest convergence rate (undoubtedly due to its multistep structure), this was not always reflected in the most competitive execution time. One should note that one iteration of SIP-NR is, approximately, twice heavier that one iteration of MNR or TMNR. Nevertheless, differences among these linear techniques were not substantial, and their performance was normally very similar.

However, an important conclusion should be remarked. In ill-conditioned systems, difficulties encountered by MNR can be properly overcome by introducing the effect of a step size (this was also outline in Section III). Thus, the obtained results demonstrated that TMNR and SIP-NR are, in fact, robust techniques.

As expected, FDXB, MNR, TMNR and SIP-NR were inefficient in heavy loading systems due to their linear convergence characteristics. In such scenario, MSIP-NR was normally the most efficient PF technique. This is undoubtedly due to the updating rule (11) notably improves the convergence features of SIP-NR. This idea can be also applied to MNR and TMNR. This topic should be covered in future works.

Finally, on the light of the results obtained, SIP-NR and MSIP-NR may find multiple applications in power system analysis. We have commented some of these potential applications in Section V. In future works, suitability of the introduced PF techniques for these tools should be studied.

Appendix A
Loading Levels considered in results section

In this paper, studied systems have been considered at base case and heavy loading conditions. Furthermore, the loading level of the system has been modified using the following expressions:

$$P_i^{sch} = \lambda P_i^{base} \quad \text{for PQ and PV buses} \qquad (12)$$
$$Q_i^{sch} = \lambda Q_i^{base} \quad \text{for PQ buses} \qquad (13)$$

where, P_i^{sch} and Q_i^{sch} are the nodal active and reactive power injected at i^{th} bus and $\lambda \in \mathbb{R}^+$ is the loading level. Tables V and VI report the loading levels considered in the studied well and ill-conditioned systems, respectively. In addition, the studied systems with heavy loading conditions can be found in [34].

Table V
Loading Levels considered in simulations (well-conditioned systems)

System	λ [pu]	System	λ [pu]
case14	4.0045	*case2736sp*	1.6671
case_ieee30	2.9524	*case2737sop*	2.1097
case57	1.7855	*case2746wop*	1.6129
case89pegase	1.8689	*case2746wp*	1.4516
case118	1.8164	*case2869pegase*	1.1418
case300	1.0360	*case3120sp*	1.5653
case1354pegase	1.3139	*case9241pegase*	1.0767
case2383wp	1.3463		

Table VI
Loading Levels considered in simulations (Ill-conditioned systems)

System	λ [pu]	System	λ [pu]
3012wp	1.2734	*13659pegase*	1.0017
3375wp	1.1586	*13659pegase(x2)*	1.0017

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