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Robust and efficient approach based on Richardson extrapolation for solving badly initialised/ill-conditioned power-flow problems

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Abstract: In this study, the authors focus on solving the power-flow (PF) problems of badly initialised/ill-conditioned power systems, where the solution of these systems is a challenge for most of state-of-the-art PF techniques. This challenge is increased in case of large and very large-scale power systems which have a huge number of variables. Consequently, the PF techniques used should have a high degree of efficiency in order to handle the large vectors and matrix computations. In this study, a novel PF approach based on the Richardson extrapolation is proposed to solve these systems. The proposed approach is validated using various badly initialised/ill-conditioned systems, comparing its performance with several well known PF techniques. The obtained results prove that the proposed approach is robust and efficient enough to properly manage the badly initialised/ill-conditioned systems even if they consist of a huge number of buses, whereas the other PF techniques face different convergence difficulties.

1 Introduction

1.1 Motivation

Power systems have experienced an unstoppable change mostly between the last quarter of the 19th century and the first quarter of the 20th (past) century [1]. Power systems are tending to grow while they are interconnected each other. The advent of this new paradigm is supported by several advantages [2, 3]:

- Facilitate the massive integration of renewable energy resources.
- The opportunity to increase the capacity between interconnected electricity markets.
- Means a potential solution for bottlenecks.

Consequently, power system planning and operation tools should be steadily updated in order to consider the particularities of the very large-scale systems.

Power flow (PF) is likely the most useful tool in planning and operation of power systems [4]. PF is indispensable for power systems operators as it is used for optimisation and security tasks among others [5]. Owing to the changes experienced in power systems, PF solver needs to be adapted according to these changes. Particularly, the solution of these systems is a challenge for the most of state-of-the-art PF techniques. The challenge is due to a huge number of variables for these systems. Hence, the PF techniques used should have a high degree of efficiency in order to handle the large vectors and matrix computations. On the other hand, these systems are normally badly initialised/ill-conditioned which means that most of PF techniques fail if a flat initial guess is used.

1.2 Literature review

The origin of PF problem back to the 1950s [6] and its solution based on the Newton–Raphson's (NR's) technique was first proposed in the following decade [7]. The fast-decoupled-load flow was later developed with the aim to speed up the standard NR [8]. A linear formulation of PF problem was proposed in [9], which is known as DC-load flow and it has been very useful for years especially for security analysis. Recently, the linear version of PF problem has been revisited in order to consider the reactive problem [10, 11]. Although these approaches can be considered the most typical PF techniques, other PF formulations and solvers have been proposed in the literatures [12–16]. Alternatively, PF can be solved as an optimisation problem using metaheuristic techniques [17–19]. In this strategy, the PF state vector is taken as the control variables and minimising the power balance at buses is considered as the objective function.

Among the PF techniques, NR is typically considered as the most standard method. It may be considered quite efficient; however, this feature is strongly affected by the initial guess employed. It has a quadratic convergence (only in the vicinity of the solution), if the initial guess point is not close enough or far away from the solution, it becomes slow and weak.

With the aim to overcome this drawback of NR with respect to the initial guess, several PF techniques have been developed over decades. Robust techniques may show good performance in both ill-conditioned and badly initialised cases. Robust PF techniques can be broadly classified into second-order approaches, techniques based on the continuous Newton's method and methodologies based on the Levenberg's iteration formulation.

The second-order techniques are based on the second-order truncation of the Taylor series expansion. In [20], this formulation was first used for solving PF problem and so-called 'Iwamoto's method'. It is used to improve the performance of standard NR by calculating a multiplier each iteration. This factor is computed as a result of solving an optimisation problem. Thus, the increment vector is modified in order to avoid the divergence. The Iwamoto's method features by its outstanding robustness; however, it is normally too slow. The Iwamoto's method has been widely referenced, therefore, it can be considered as the most standard second-order approach. Nevertheless, other second-order PF techniques have been proposed in the literatures [21–25].

The continuous Newton's method was introduced in [26] and adapted for solving the PF problem in [27]. It is based on the analogy between the PF and a set of autonomous ordinary differential equations. Given this analogy, any well-assessed numerical method can be employed for solving the PF problem. In [27], the explicit-Euler method and the fourth-order Runge–Kutta have been tested for solving a large-scale test system. While standard PF methods diverged due to the initial guess and the Iwamoto's method consumed many iterations, the continuous Newton-based methods provided good performance. A combined fourth-order Runge–Kutta–Broyden's approach for PF analysis has been proposed in [28].

Recently, the Levenberg's formulation has been revisited in order to develop various robust PF techniques [29–31]. In [29], the Levenberg–Marquardt's formulation has been employed for solving the PF problem; nevertheless, it did not show good performance due to its convergence rate strongly depends on the damping factor. While the number of iterations tends to decrease when the damping factor is large, the correct convergence is not ensured. To overcome these difficulties, several high-order schemes have been proposed in [30, 31].

Recently, the holomorphic embedding method (HEM) has been applied for solving the PF problem [32]. HEM represents a distinct class of non-linear equation solvers which are recursive rather than iterative; therefore, an infinite number of formulations exist, each one with different numerical properties.

1.3 Contributions

In this paper, a novel PF approach based on the Richardson extrapolation is proposed for solving badly initialised/ill-conditioned power systems. This proposed approach has been validated using three very large-scale systems (3012 bus, 3375 bus and 13659 bus). In addition, the obtained results by the proposed PF approach have been compared with those obtained by other standard and robust PF methods (standard NR method, Iwamoto's method [20], Levenberg–Marquardt method [29], high-order Levenberg–Marquardt method [30], the non-monotone line search with corrected Levenberg–Marquardt method [31], explicit-Euler method and fourth-order Runge–Kutta formula [27]).

1.4 Paper organisation

This paper is organised in eight sections, after Section 1, the main features of PF problem are outlined in Section 2.3. Badly initialised/ill-conditioned PF problems are explained in Section 3. The proposed PF approach is presented in Section 4. Section 5 provides the simple scheme to consider generators’ reactive power limits in the proposed PF approach. The robustness of the proposed PF approach with respect to the initial guess is illustrated in Section 6. Section 7 presents the cases studied and reports the obtained results. Finally, the main conclusions are presented in Section 8.

2 Outlines of PF problem

The PF problem is stated as a set of non-linear equations which relates the injected nodal powers with the nodal voltages. Generally, for each bus, the active and reactive power mismatch can be defined as [33]

$$\Delta P_i = P^{\rm sch} - \sum_{j=1}^{n_g+n_l} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{1}$$

$$\Delta Q_i = Q^{\rm sch} - \sum_{j=1}^{n_l} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{2}$$

where $P^{\rm sch}$ and $Q^{\rm sch}$ are the injected active and reactive powers at bus i, respectively; $V_i\angle\delta_i$ are the complex voltage at bus i; $Y_{ij}\angle\theta_{ij}$ is the ijth element of admittance matrix; n_g is the total number of PV buses; and n_l is the total number of PQ buses ($n = 2n_l + n_g$). Typically, the generation (PV buses) and the load buses (PQ buses) are distinguished in the PF problem. While PQ buses contribute to the problem with (1) and (2), and (1) is posed for generation buses.

The set of non-linear (1) and (2) can be rewritten in its following compact form for simplicity:

$$\boldsymbol{g}(\boldsymbol{x}) = 0 \tag{3}$$

where $\boldsymbol{x}$ ($\boldsymbol{x} \in \mathbb{R}^n$)) is the PF state vector, which is formed by the nodal voltage angles of PV buses and voltage angles and magnitudes of PQ buses. Since (3) is non-linear and cannot be explicitly inverted, an iterative technique must be used for solving it. As mentioned before, the NR might be considered the standard PF solver. A generic kth of the NR for PF problem is as

$$\begin{aligned} \Delta\boldsymbol{x}_{\rm NR}^{(k)} &= -\left[\nabla_x^{\rm T}\boldsymbol{g}(\boldsymbol{x}^{(k)})\right]^{-1}\boldsymbol{g}(\boldsymbol{x}^{(k)}) \\ \boldsymbol{x}^{(k+1)} &= \boldsymbol{x}^{(k)} + \Delta\boldsymbol{x}_{\rm NR}^{(k)} \end{aligned} \tag{4}$$

where $\left[\nabla_x^{\rm T}\boldsymbol{g}(\boldsymbol{x}^{(k)})\right]$ is the PF Jacobian matrix, which is formed by the first partial derivatives of (3) with respect to the PF state vector. The NR needs to update and invert the PF Jacobian matrix each iteration, which is considered the heaviest computational part in the solution process.

The algorithm normally ends when the following criteria is satisfied:

$$\| \boldsymbol{g}^{(k)} \|_\infty \leq \epsilon| \tag{5}$$

where ϵ is a predetermined threshold which typically is taken smaller than 10^{-3}. Also, the PF problem is ended when the number of iterations surpasses a determined limit ($k > k_{\rm max}$), but in this case, it is said that the algorithm has failed.

3 Badly initialised/ill-conditioned PF problems

Most of standard PF techniques suffer if the used initial guess is not good enough. Generally, the goodness of an initial guess is measured by its distance with respect to the correct solution. The region of attraction is considered the widely extended criteria that used to measure the effect of initial guess [34]. The region of attraction can be defined as the set of initial guess which the PF has a real solution. It mainly depends on the condition of the system and the PF technique employed. Hence, it is narrow when the condition is ill or the PF technique employed is not robust. In Fig. 1, the region of attraction of the well known IEEE 9-bus test system [35], using the standard NR, is depicted. It has been constructed by modifying the voltage magnitude and angles of bus #4, which is a PQ bus, and carrying out multiple PF problems.

From this figure, it can be observed that the number of iterations increases as the initial guess moves away from the solution. Moreover, if the initial guess is outside of the region of attraction, the PF solver diverges or achieves a wrong solution (area denoted by white colour). Therefore, if the initial guess is not close enough or far from the solution, the algorithm will likely be too slow or unstable. In this situation, robust PF techniques occasionally show wider region of attraction. Thus, the effect of the initial guess can be minimised.

In the case of very large-scale systems, the region of attraction frequently is very narrow; it may even happen that the flat initial guess is outside of the region of attraction.

It is worth mentioning that the system has different region of attraction, depending on the bus observed. Furthermore, several buses can be combined in order to analyse the effect of the initial guess. Therefore, there is not a formal criterion to measure the effect of the initial guess by the region of attraction; nevertheless, it can be properly used to compare several PF solvers.

However, the amplitude of region of attraction mainly depends on the PF technique employed and the condition of the system. The region of attraction is normally wider for robust PF solvers which are less sensitive with respect to the initial guess employed. On the other hand, ill-conditioning of power systems is typically determined by calculating the condition number of the Jacobian matrix. The system is considered ill-conditioned if this number is high enough. Ill-conditioned systems normally show a narrow

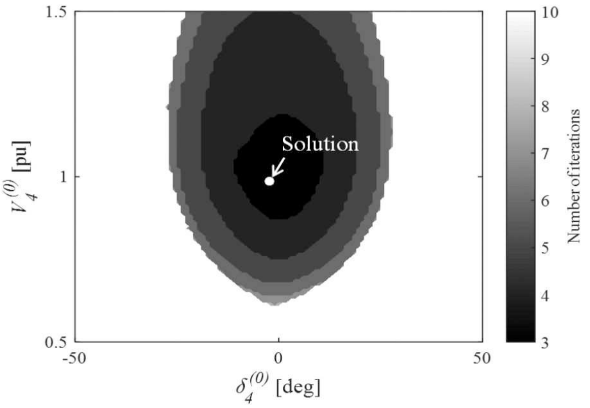

Fig. 1 *Region of attraction of IEEE 9-bus test system using the standard NR*

region of attraction, consequently, hence, they become more difficult to solve using most of PF methods.

However, ill-conditioning of PF equations does not directly imply failure of standard methods. This is rather related with the initial guess employed by initialising the iterative solution of PF problem. Specifically, a PF method diverges if the starting point lies outside of its region of attraction. Indeed, any system is considered badly initialised/ill-conditioned if the standard NR fails to find the solution with flat initial guess [27].

4 Proposed PF approach

The main contribution of this paper is to develop a novel approach for solving the PF problem of badly initialised/ill-conditioned systems, efficient enough to manage large and very large-scale cases. This approach is based on the Richardson extrapolation.

4.1 Outlines of the Richardson extrapolation

The Richardson extrapolation was first used in [36] and later embellished in [37]. Using the concept of Richardson extrapolation, very higher-order integration can be achieved using only a series of values from trapezoidal rule [38]. To explain the Richardson extrapolation, consider the problem of calculating the value of the continuous unidimensional function f, it can be estimated through the auxiliary function $\mathscr{F}$ which depends on an arbitrary step size h. Then, the quadratic behaviour of the errors can be given as follows [39]:

$$\mathscr{F} = f + \phi_2 h^2 + \phi_4 h^4 + \cdots \tag{6}$$

where the functions ϕ_n are normally unknown. If h is small enough, ϕ_4 and superior may be neglected. The main idea of Richardson extrapolation is to combine two separate discrete solutions f_1 (coarse) and f_2 (fine) on two different h_1 and h_2 ($h_1 > h_2$). This approach allows us to eliminate ϕ_2. Then, the value of f can be estimated as

$$f \simeq f_2 + \frac{f_2 - f_1}{r^{\psi} - 1} \tag{7}$$

where r is the ratio (h_1/h_2) and ψ is a real coefficient. This is the *h^2-extrapolation*; it is worth mentioning that the method may be refined by calculating more than two values of f; however, it requires the solution of a system of simultaneous linear equations, which implies an arithmetic burden [38].

The most-common procedure is with a grid doubling or halving [39]. This means that the good estimate value f_2 is calculated with ($h/2$) while the coarse value f_1 is computed with h. Therefore, (7) can be rewritten as follows:

$$f \simeq \frac{2^{\psi} f_2 - f_1}{2^{\psi} - 1} \tag{8}$$

Richardson extrapolation is the basis of some sophisticated numerical methods as the Bulirsch–Stoer [40] or the Romberg's methods [41]. Interested readers can find references to many applications of the Richardson extrapolation in [42].

4.2 Application of Richardson extrapolation for solving PF problem

The main idea of Richardson extrapolation can be easily translated to the PF problem. To do that, two guess values of the PF state vector are calculated using the Explicit-Euler method, then, the Richardson extrapolation is used to refine the value of $\boldsymbol{x}$. A generic kth iteration of the proposed Richardson PF approach is defined as follows:

$$\hat{\boldsymbol{x}}_1^{(k)} = \boldsymbol{x}^{(k)} + h\Delta\boldsymbol{x}_{\rm NR}^{(k)} \tag{9}$$

$$\hat{\boldsymbol{x}}_2^{(k)} = \boldsymbol{x}^{(k)} + \frac{h}{2}\Delta\boldsymbol{x}_{\rm NR}^{(k)} \tag{10}$$

$$\boldsymbol{x}^{(k+1)} = \frac{2^{\psi}\hat{\boldsymbol{x}}_2^{(k)} - \hat{\boldsymbol{x}}_1^{(k)}}{2^{\psi} - 1} \tag{11}$$

Parameter ψ is introduced in the proposed approach in order to ‘mimetic’ the formulation of standard Richardson extrapolation (8). Richardson extrapolation is used in a novel way (solving systems of non-linear equations); consequently, influence of ψ in the performance of the proposed approach is unknown. To properly analyse the influence of ψ, the proposed PF approach will be tested for different values of this parameter (see Section 7).

As can be seen, the Jacobian matrix is factorised only once each iteration such as NR. It is worth mentioning that the Richardson scheme can be adapted to the Levenberg–Marquardt's iteration. This can be achieved by changing the NR corrector vector by the Levenberg's one. Further analysis of this Richardson–Levenberg PF method is out of the scope of the present paper.

The distance between $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ allows to measure the reliability of the algorithm. Furthermore, the step size should be increased if the difference between $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ is small enough. Hence, the step size may be initially taken $h = 1$ and it is updated each iteration according to the following rule:

$$\zeta = \| \hat{\boldsymbol{x}}_1^{(k)} - \hat{\boldsymbol{x}}_2^{(k)} \|_{\infty} \tag{12}$$

$$\begin{aligned} &\text{if } \zeta > \varepsilon \text{ then } h \leftarrow \max\{\sigma_1 h,\ h_{\min}\} \\ &\text{if } \zeta \le \varepsilon \text{ then } h \leftarrow \min\{\sigma_2 h,\ h_{\max}\} \end{aligned} \tag{13}$$

where ε is the security threshold; σ_1 and σ_2 are the step size damping coefficients; and $h_{\min}$ and $h_{\max}$ are the minimum and maximum step sizes, respectively.

Since it is aimed that h does not change abruptly, σ_1 and σ_2 should be around 1. On the other hand, $h_{\min}$ should not be too small in order to avoid slow convergence. Oppositely, $h_{\max}$ might be

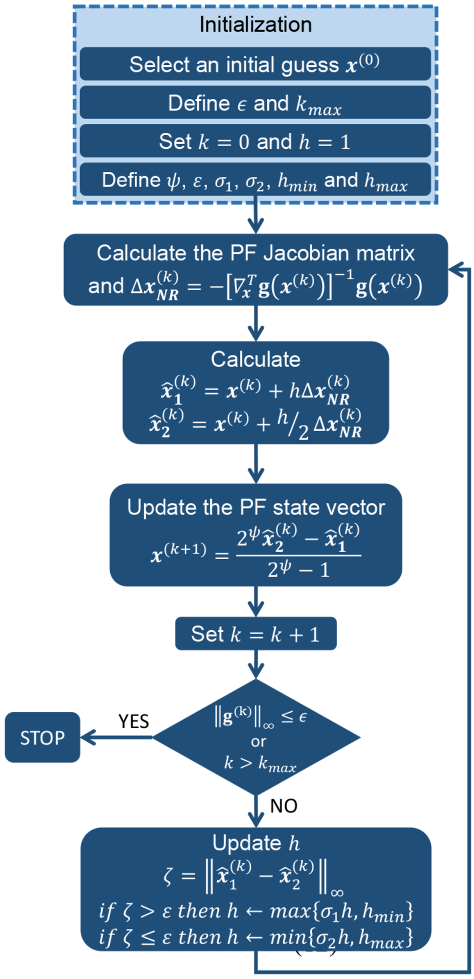

Fig. 2 *Flowchart of the proposed PF approach*

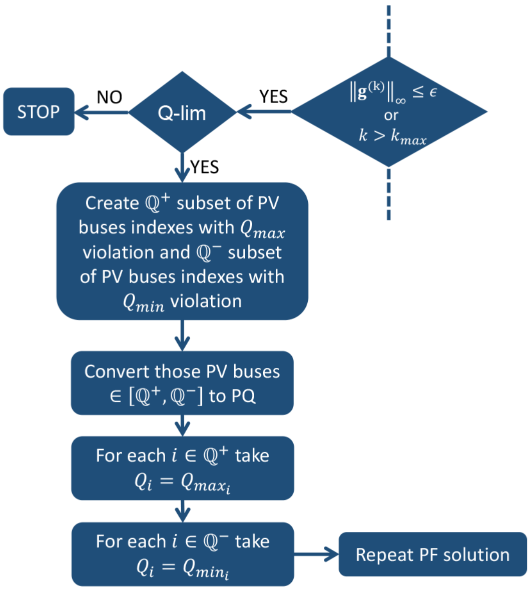

Fig. 3 *Flowchart of strategy used for considering the generator's reactive power limits*

tuned higher than 1 in order to accelerate the convergence, but not much higher with the aim to avoid the divergence ($h_{\max} \le 2$ is recommended).

On the basis of the above ideas, we consider the following: $\sigma_1 = 0.95$, $\sigma_2 = 1.05$, $h_{\min} = 0.75$ and $h_{\max} = 2$. It is difficult to know, a priori, a suitable value for ε; however, we have founded that $\varepsilon = 8$ works quite well in most cases. For the sake of clarity, the whole procedure of the proposed approach is summarised in the flowchart of Fig. 2.

5 Considering generator's reactive power limits

The generator's reactive power limits can be easily considered in the proposed PF approach based on the same strategy used in standard NR [27, 43]. In this strategy, the output reactive power of the violated generator is fixed at its limit and the connected PV bus is converted to PQ bus. Then, the PF is repeatedly solved until a feasible solution is achieved. Fig. 3 shows a partial flowchart which allows to incorporate this strategy to the main flowchart in Fig. 2.

When this strategy is employed, the total number of iterations would be the sum of iterations consumed by all the PF solutions computed.

6 Robustness of the proposed PF approach with respect to initial guess

In this section, the robustness of the proposed approach with respect to the initial guess point is proved. To demonstrate this aim, the region of attraction of the proposed approach for the well known IEEE 9-bus system [35] has been calculated and compared with that shown by the standard NR method. In Fig. 4, the region of attraction of the proposed approach and standard NR of the PQ buses of IEEE 9-bus system (i.e. buses 4–9) are shown. For the sake of completeness, results for two extreme values of the parameter ψ have been included. As can be easily noted, the region of attraction of the proposed approach is notably wider than that shown by the standard NR method. On the other hand, it should be noted that the region of attraction of the proposed approach is slightly narrower for high values of ψ.

Fig. 5 shows the region of attraction of the proposed approach for two values of ψ, indicating the number of iterations employed for different initial guesses ($\epsilon = 10^{-3}$). For the sake of simplicity, only results at bus #4 are plotted. As can be seen, the proposed approach usually employs more iterations for small values of ψ; however, the region of attraction becomes slightly narrower; consequently, the proposed approach turns to be more sensitive with respect to the initial guess for large values of ψ. These facts are deeply discussed in Section 7.

7 Tests and results

The proposed PF approach has been validated using the following three systems:

- The 3012-bus 3572-line model of the Polish system in the winter 2007–08 evening peak [44].
- The 3375-bus 4161-line model of the Polish system in the winter 2007–08 evening peak [44].
- The 13659-bus 20467-line portion of the European transmission system. Further details of this system are given in [45, 46].

The results obtained by the proposed approach have been compared with those obtained by the following seven robust and standard PF methods:

- Standard NR method.
- Iwamoto's method [20].
- Levenberg–Marquardt method [29].
- High-order Levenberg–Marquardt method [30].
- The non-monotone line search with corrected Levenberg–Marquardt method [31].
- Explicit-Euler method [27].
- Fourth-order Runge–Kutta formula [27].

All simulations have been carried out using an Intel Core i5-7500 3.4 GHz personal computer and software package MATPOWER 6.0 [43]. The computation time has been calculated as the average value of 50 simulations, in order to minimise the effect of other computation activities.

Table 1 presents the computation time and number of iterations of different PF methods for all studied systems, whereas Figs. 6–8 show the convergence characteristics of the proposed PF approach (for the sake of brevity, only results for $\epsilon = 10^{-10}$ are presented). On the other hand, Table 2 presents the computation time and number of iterations of different PF methods for all studied systems in case of considering the reactive power limits as explained in Section 5.

The proposed PF approach has successfully converged in all studied systems, while most of remainder methods have faced different convergence difficulties as:

- *Divergence*: In this case, the value of $\boldsymbol{g}_\infty$ has grown during the iterative procedure. Therefore, the algorithm has tended to move far from the correct solution. This failure is due to the flat initial guess which lied outside of the region of attraction.
- *Slow convergence*: The algorithm has not converged during a specified number of iterations (considered 50 in this case). In this case, the flat initial guess may be inside the region of attraction, but it is not close enough to the final solution.
- *Computational burden*: It is normally produced by heavy computations which might be attributable to the size of the studied systems.
- *Low-voltage (unstable) solution*: In this case, the algorithm fails to find the stable solution.

The proposed approach can overcome these issues. Problems related with the flat initial guess are also addressed due to the high robustness of the proposed approach with respect to badly initialised cases. On the other hand, the proposed approach has proved its capability to properly handle large matrixes and vectors computations.

Regarding the effect of parameter ψ, it can be observed that the number of iterations decreases for high values of ψ. However, it seems that the performance does not significantly change for $\psi \geq 6$. Moreover, the value of $\boldsymbol{g}_\infty^{(k)}$ becomes smaller during the iterative process as the value of ψ grows. The proposed approach does not consume much more time for small values of ψ, if the size of system is not too large (around 3000 buses), since in these kinds of systems, one iteration is not much heavy. For larger systems, it may be recommendable to use $\psi = 4$, since it has been observed as the number of iterations is reduced for more than ten with respect to $\psi = 2$, and the robustness is still quite acceptable. The number of iterations is not considerably reduced beyond $\psi = 4$.

The fourth-order Runge–Kutta has also converged in the 13659-bus system; however, the proposed approach ensures higher efficiency. Also, the number of iterations is smaller for all values of ψ. Similar conclusions can be observed when reactive limits are considered.

Next, let us suppose that the initial voltage magnitudes of some PQ buses are perturbed by adding an error to the flat initial guess. It is not a frequent situation, but it allows us to evaluate the robustness of the proposed approach. Fig. 9 shows the maximum error allowed by the proposed approach for all studied systems. As

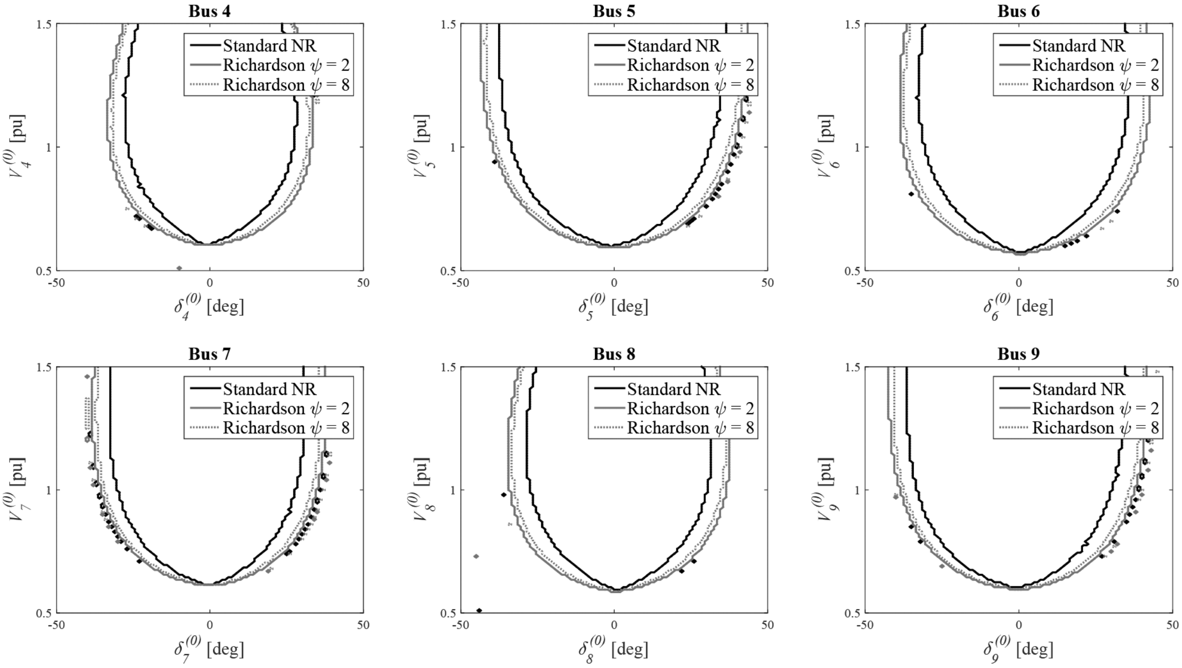

Fig. 4 *Region of attraction of IEEE 9-bus test system using standard NR and the proposed approach*

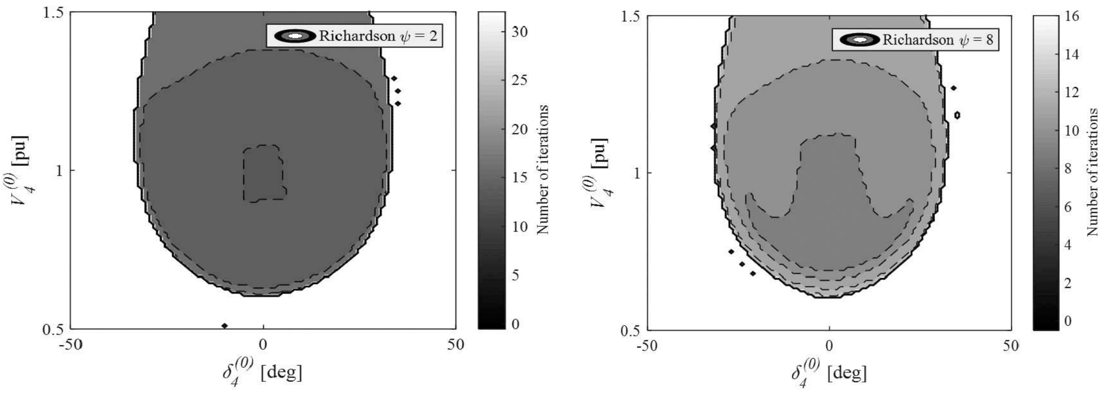

Fig. 5 *Region of attraction of IEEE 9-bus test system using the proposed PF approach with $\psi = 2$ (left) and $\psi = 8$ (right)*

Table 1 Computation time and number of iterations of PF methods for all studied systems

Method		$\epsilon = 10^{-3}$					
		3012 Bus		3375 Bus		13659 Bus	
		Time, s	Number of iterations	Time, s	Number of iterations	Time, s	Number of iterations
Standard NR		diverge	—	diverge	—	diverge	—
Iwamoto [20]		fail[a]	—	fail[a]	—	fail[a]	—
Levenberg's methods	[29]	fail[a]	—	fail[a]	—	fail[a]	—
	[30]	fail[a]	—	fail[a]	—	fail[b]	—
	[31]	fail[a]	—	fail[a]	—	fail[b]	—
Explicit-Euler [27]		fail[a]	—	fail[a]	—	fail[c]	—
fourth-order Runge–Kutta	[27]	diverge	—	diverge	—	5.431	15
Richardson extrapolation	$\psi = 2$	0.389	19	0.435	19	1.788	19
	$\psi = 4$	0.267	13	0.322	14	1.328	14
	$\psi = 6$	0.267	13	0.302	13	1.236	13
	$\psi = 8$	0.248	12	0.302	13	1.236	13

Method		$\epsilon = 10^{-3}$					
		3012 Bus		3375 Bus		13659 Bus	
		Time, s	Number of iterations	Time, s	Number of iterations	Time, s	Number of iterations
Standard NR		diverge	—	diverge	—	diverge	—
Iwamoto [20]		fail[a]	—	fail[a]	—	fail[a]	—
Levenberg's methods	[29]	fail[a]	—	fail[a]	—	fail[a]	—
	[30]	fail[a]	—	fail[a]	—	Fail[b]	—
	[31]	fail[a]	—	fail[a]	—	Fail[b]	—
Explicit-Euler [27]		fail[a]	—	fail[a]	—	fail[a]	—
fourth-order Runge–Kutta	[27]	diverge	—	diverge	—	7.593	21
Richardson extrapolation	$\psi = 2$	0.467	23	0.521	23	2.153	23
	$\psi = 4$	0.308	15	0.365	16	1.517	16
	$\psi = 6$	0.288	14	0.345	15	1.423	15
	$\psi = 8$	0.288	14	0.323	14	1.423	15

Method		$\epsilon = 10^{-3}$					
		3012 Bus		3375 Bus		13659 Bus	
		Time, s	Number of iterations	Time, s	Number of iterations	Time, s	Number of iterations
Standard NR		diverge	—	diverge	—	diverge	—
Iwamoto [20]		fail[a]	—	fail[a]	—	fail[a]	—
Levenberg's methods	[29]	fail[a]	—	fail[a]	—	fail[a]	—
	[30]	fail[a]	—	fail[a]	—	fail[b]	—
	[31]	fail[a]	—	fail[a]	—	fail[b]	—
Explicit-Euler [27]		diverge	—	fail[a]	—	fail[a]	—
fourth-order Runge–Kutta	[27]	diverge	—	diverge	—	13.38	37
Richardson extrapolation	$\psi = 2$	0.662	33	0.756	34	3.194	33
	$\psi = 4$	0.425	20	0.448	20	1.967	21
	$\psi = 6$	0.350	17	0.406	18	1.779	19
	$\psi = 8$	0.350	17	0.383	17	1.690	18

[a]It did not converge in 50 iterations.
[b]Fail for computational burden.
[c]Low-voltage (unstable) solution is obtained.

can be seen, when a value of $\psi = 2$ is used, the maximum error allowed is higher. This has been already commented in Section 5. On the other hand, it is remarkable that the 13659-bus system barely allows error while the region of attraction in the studied 3375-bus system is notably narrower than in the 3012-bus system. It is worth mentioning that these results are only indicative.

Now, let us study the effect of loading level on the performance of the proposed approach. It is expected that the total number of iterations grows with the loading conditions of the system; moreover, the convergence can be putted at risk. While the 13659-bus systems show its maximum loadability point at base case conditions, the loading level in the 3012-bus and 3375-bus systems can be increased as follows:

$$P_i^{\text{sch}} = \mu P_i^{\text{sch}}, \ \ \forall i \in [\boldsymbol{PV}, \boldsymbol{PQ}] \tag{14}$$

$$Q_i^{\text{sch}} = \mu Q_i^{\text{sch}}, \ \forall i \in [\boldsymbol{PQ}] \tag{15}$$

where μ is a real positive coefficient which represents the loading level. Table 3 presents the total number of iterations employed by the proposed approach for solving the PF problems of 3012-bus and 3375-bus systems at heavy loading levels, considering $\epsilon = 10^{-10}$. From the results obtained, it can be concluded that the overall performance of the proposed approach is not much affected by the loading level. The number of iterations has been kept with respect to the base case for all loading levels.

The proposed approach has also tested at high *R/X* ratios. In this test, the real part of branch admittances of the studied systems has been multiplied by a real factor ρ. Results for different *R/X* ratios are presented in Table 4 ($\epsilon = 10^{-10}$). Regarding to the 13659-bus case, it has been observed that it has no solution for a value of ρ slightly higher than 1; therefore, we have considered that this system does not provide considerable results in this aspect.

As already mentioned above, the robustness of the proposed approach is affected when the parameter ψ is high. The proposed approach has successfully converged even for $\rho = 1.50$; nevertheless, it has diverged in all cases for $\psi = 8$. This is due to the region of attraction becomes narrower when ρ is high, as it shown in Section 6, the region of attraction shown by the proposed approach is narrower for high values of ψ. In this case, the flat initial guess has lied outside of the region of attraction of proposed approach, except for $\psi = 2$. However, a value of $\psi = 8$ should be used in most cases since the total number of iterations is drastically reduced with respect to the case of $\psi = 2$.

8 Conclusions

In this paper, a novel efficient approach based on the Richardson extrapolation has been proposed for solving the PF problems of badly initialised/ill-conditioned power systems. The proposed approach is efficiently able to manage large vectors and matrix computations and, consequently, it is also suitable for large and very large-scale systems. The proposed approach has been validated using three very large-scale systems (3012 bus, 3375 bus

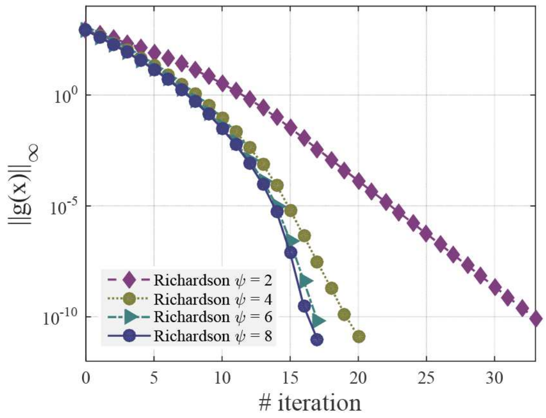

Fig. 6 *Convergence characteristics of the proposed approach without considering reactive power limits (3012-bus system)*

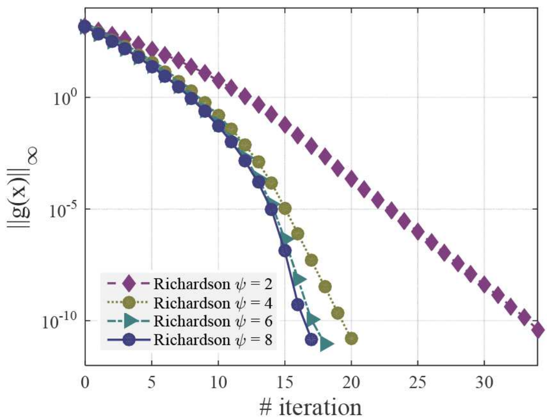

Fig. 7 *Convergence characteristics of the proposed approach without considering reactive power limits (3375-bus system)*

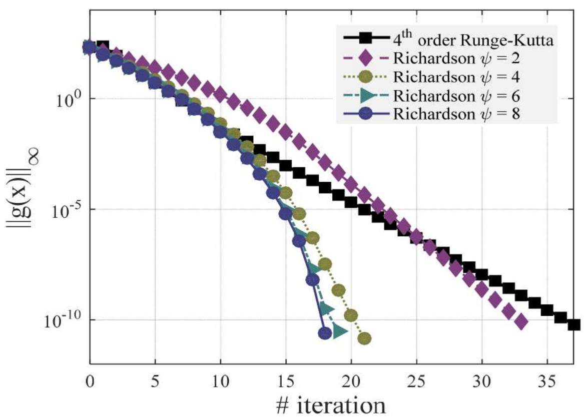

Fig. 8 *Convergence characteristics of the proposed approach and the fourth-order Runge–Kutta without considering reactive power limits (13659-bus system)*

Table 2 Computation time and number of iterations of PF methods for all studied systems in case of considering reactive power limits

Method		3012 bus		3375 bus		13659 bus	
		Time, s	Number of iterations	Time, s	Number of iterations	Time, s	Number of iterations
				$\epsilon = 10^{-3}$			
standard NR		fail[a]	—	fail[a]	—	fail[a]	—
Iwamoto [20]		fail[a]	—	fail[a]	—	fail[a]	—
Levenberg's methods	[29]	fail[a]	—	fail[a]	—	fail[a]	—
	[30]	fail[a]	—	fail[a]	—	fail[a]	—
	[31]	fail[a]	—	fail[a]	—	fail[a]	—
Explicit Euler [27]		fail[a]	—	fail[a]	—	fail[a]	—
fourth-order Runge–Kutta [27]		fail[a]	—	fail[a]	—	8.365	23
Richardson extrapolation	$\psi = 2$	0.745	36	0.796	34	2.911	31
	$\psi = 4$	0.528	25	0.571	24	2.176	23
	$\psi = 6$	0.528	25	0.552	23	1.987	21
	$\psi = 8$	0.485	23	0.552	23	1.987	21
				$\epsilon = 10^{-5}$			
Standard NR		fail[a]	—	fail[a]	—	fail[a]	—
Iwamoto [20]		fail[a]	—	fail[a]	—	fail[a]	—
Levenberg's methods	[29]	fail[a]	—	fail[a]	—	fail[a]	—
	[30]	fail[a]	—	fail[a]	—	fail[a]	—
	[31]	fail[a]	—	fail[a]	—	fail[a]	—
Explicit Euler [27]		fail[a]	—	fail[a]	—	fail[a]	—
fourth-order Runge–Kutta [27]		fail[a]	—	fail[a]	—	12.74	34
Richardson extrapolation	$\psi = 2$	1.050	51	1.312	57	3.735	40
	$\psi = 4$	0.726	35	0.954	41	2.634	28
	$\psi = 6$	0.686	33	0.910	39	2.544	27
	$\psi = 8$	0.686	33	0.866	37	2.452	26
				$\epsilon = 10^{-10}$			
Standard NR		fail[a]	—	fail[a]	—	fail[a]	—
Iwamoto [20]		fail[a]	—	fail[a]	—	fail[a]	—
Levenberg's methods	[29]	fail[a]	—	fail[a]	—	fail[a]	—
	[30]	fail[a]	—	fail[a]	—	fail[a]	—
	[31]	fail[a]	—	fail[a]	—	fail[a]	—
Explicit Euler [27]		fail[a]	—	fail[a]	—	fail[a]	—
fourth-order Runge–Kutta [27]		fail[a]	—	fail[a]	—	23.77	66
Richardson extrapolation	$\psi = 2$	1.624	83	2.234	102	5.613	60
	$\psi = 4$	1.028	52	1.435	65	3.555	38
	$\psi = 6$	0.937	47	1.350	61	3.281	35
	$\psi = 8$	0.937	47	1.307	59	3.174	34

[a]Fail for solving the case without reactive power limits.

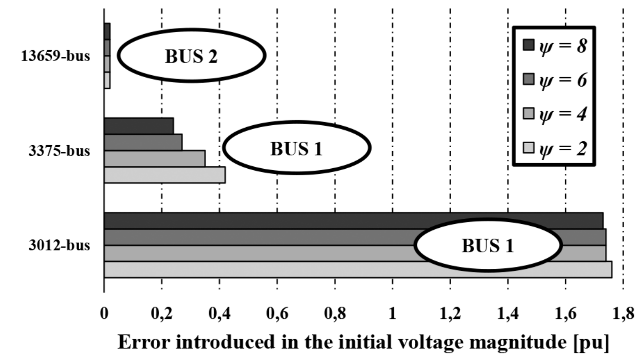

Fig. 9 *Highest error in the initial voltage magnitude for all studied systems*

and 13659 bus) under different scenarios. While standard NR has diverged and other robust PF methods faced different convergence issues in these systems, the proposed approach has been able to successfully solve them. In addition, the proposed approach only requires one matrix factorisation each iteration; therefore, it has similar computational cost of NR method.

The effect of parameter ψ on the performance of the proposed approach has been explored. Broadly, it has been concluded that the number of iterations is reduced while ψ grows. However, it seems that its limit is around $\psi = 6$, for values higher than this value, the performance of the proposed approach is not much different. On the other hand, it has been demonstrated that the proposed approach is more robust when the value of ψ is small, especially in the case of high R/X ratios.

In this current work, we have only studied the halving Richardson formula. Nevertheless, other approaches can be used for solving the PF problem. Any other value of $r = (h_1/h_2)$ should be studied in future works, comparing its performance with the grid-doubling approach ($r = 2$) used in this paper.

Table 3 Total number of iterations employed by different PF techniques at heavy loading levels ($\epsilon = 10^{-10}$)

Method		3012 bus $\mu = 1.27$	3375 bus $\mu = 1.15$
standard NR		diverge	diverge
Iwamoto [20]		fail[a]	fail[a]
Levenberg's methods	[29]	fail[a]	fail[a]
	[30]	fail[a]	fail[a]
	[31]	fail[a]	fail[a]
Explicit Euler [27]		fail[a]	fail[a]
fourth-order Runge–Kutta [27]		diverge	diverge
Richardson extrapolation	$\psi = 2$	33	34
	$\psi = 4$	20	20
	$\psi = 6$	17	18
	$\psi = 8$	17	17

[a]It did not converge in 50 iterations.

Table 4 Total number of iterations employed by the proposed approach at different *R/X* ratios ($\epsilon = 10^{-10}$)

	$\rho = 1.20$	$\rho = 1.30$	$\rho = 1.40$	$\rho = 1.50$
—		3012-bus		
$\psi = 2$	33	33	33	33
$\psi = 4$	20	20	diverge	diverge
$\psi = 6$	18	diverge	diverge	diverge
$\psi = 8$	diverge	diverge	diverge	diverge
—		3375-bus		
$\psi = 2$	34	34	34	34
$\psi = 4$	20	diverge	diverge	diverge
$\psi = 6$	diverge	diverge	diverge	diverge
$\psi = 8$	diverge	diverge	diverge	diverge

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