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A cohesion-driven consensus reaching process for large scale group decision making under a hesitant fuzzy linguistic term sets environment

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ABSTRACT

Large-scale group decision-making (LSGDM) under uncertainty modelled by comparative linguistic expressions based on a hesitant fuzzy linguistic term set (HFLTS) has recently attracted the interest of many researchers and research, due to the necessity of its function in LSGDM, and the challenges it faces such as the managing of the scalability problem, uncertainty of experts' opinions and dealing with polarized conflicting opinions. To smooth out such discrepancies and obtain agreed solutions Consensus Reaching Processes (CRPs) for LSGDM have been applied, in which experts are grouped into sub-groups according to the closeness of their opinions to deal with scalability. However, most CRPs for LSGDM are driven by a majority rule, in which larger sub-groups, where there might be internal disagreements, lead the consensus. In such processes, the internal disagreements can produce unsatisfactory solutions. Consequently, the majority view should be complemented by additional mechanisms that also measure the strength of the sub-groups' opinions. A good measurement of such strength is the cohesion among the sub-group members. Therefore, in this paper, a new cohesion measure for HFLTS based on restricted equivalence functions for measuring the experts' sub-group cohesiveness is introduced to drive the consensus process together the majority and thus reduce the impact of internal disagreements risen in majority driven CRPs. It is then integrated in a new cohesion-driven CRP approach based on LSGDM to deal with comparative linguistic expressions based on HFLTS. An experimental analysis on different large scale scenarios will show the performance and importance of cohesion in consensus based LSGDM.

1. Introduction

Decision making problems are becoming more and more complex due to the increasing technological development (social networks (Sueur, Deneubourg, & Petit, 2012), big data (Walker, 2014)), the societal demands (e-group shopping, e-democracy (Kim, 2008)) and the need of including a lot of people in the decision making processes. This kind of decision making problems are called large-scale group decision making (LSGDM) (Chen & Liu, 2006). The complexity of dealing with a large number of experts implies new challenges in GDM such as, polarization opinions, scalability and the uncertainty present in the experts' information (Wang, Rodríguez, & Wang, 2018).

Decisions that could affect groups of people have a much better chance of being accepted when there is a consensus among all experts involved in the problem (Eklund, Rusinowska, & de Swart, 2007). This usually implies including a consensus reaching process (CRP) before selecting the best alternative for the GDM problem (Cao, Wu, Chiclana, Ureña, & Herrera-Viedma, 2020; Gong, Xu, Guo, Herrera-Viedma, & Cabrerizo, 2021; Herrera-Viedma, Cabrerizo, Kacprzyk, & Pedrycz, 2014; Labella, Liu, Rodríguez, & Martínez, 2018; Palomares, Estrella, Martínez, & Herrera, 2014; Wu, Cao, Chiclana, Dong, & Herrera-Viedma, 2020; Zhang, Gao, & Li, 2020). In LSGDM, CRPs become much more complex and necessary because opinions among a large number of experts tend to be easily polarized and conflicting (Labella et al., 2018). Therefore, novel challenges in addition to the previous ones appear in CRPs for LSGDM such as: non-cooperative behaviors, supervision and majority-driven. Resulting that classical CRPs are not longer valid for LSGDM (Labella et al., 2018).

The uncertainty and vagueness of the information, inherent in the decision making problems, must be also considered in CRPs for LSGDM.

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Several approaches have been introduced in the literature to model this type of uncertainty by using single linguistic terms (Wu, Liu, & Qin, 2018), fuzzy sets (Dong, Zhao, Zhang, Chiclana, & Herrera-Viedma, 2018; Gai et al., 2020; Palomares, Martínez, & Herrera, 2014; Quesada, Palomares, & Martínez, 2015), and hesitant fuzzy sets (Rodríguez, Labella, De Tré, & Martínez, 2018; Wu & Xu, 2018). These proposals, however, do not provide expressions close to human cognitive process. Rodríguez, Martínez, and Herrera (2012); Rodríguez, Martínez, and Herrera, 2013 proposed the use of context-free grammars to generate comparative linguistic expressions (CLEs) based on hesitant fuzzy linguistic term sets (HFLTS) to model experts' hesitation. These linguistic expressions are closer to the language used by human beings in real world decision making problems and have provided successful results in CRPs (Dong, Chen, & Herrera, 2015; Wu & Xu, 2016a; Wu & Xu, 2016b; Xu, Wen, Sun, & Wang, 2018; Yu, Zhang, & Zhong, 2019; Zhang, Liang, & Zhang, 2018). Consequently, some scholars have paid attention to the use of HFLTS to propose CRP-LSGDM (Gou, Xu, & Herrera, 2018; Ren, Tang, & Liao, 2020; Zhong & Xu, 2020). Nevertheless, these proposals do not follow the computing with words scheme (Yager, 1999; Yager, 2004), in which linguistic input is translated into fuzzy sets to carry out the computations with fuzzy arithmetic and the result is retranslated to linguistic output. This is due to the fact that, they introduce an oversimplification at the beginning of the CRP, when they defuzzify the input represented by HFLTS into crisp values. This implies loss of information and lack of precision in the results.

Keeping the challenges in CRPs-LSGDM in mind, this paper is focused on *scalability*, where hundreds or thousand of experts are involved in the CRPs and the rule that drives the consensus process (usually majority driven (Zhong & Xu, 2020), though minority driven has been also used (Ren et al., 2020)). The usual majority driven rule consists in classifying experts into several sub-groups and the consensus is led by the larger sub-groups, whilst the minority rule consists in the decision making being lead by a small group of experts chosen because of their expertise and sake of operability, etc. However, in both cases the fact that experts are classified in the same sub-group does not guarantee that their preferences are totally the same even not very close to each other. Hence, it may lead to the appearance of disagreement to some extent (this is more common and obvious in large groups, but also in small groups). If we understand cohesion as a mechanism that sustains togetherness in terms of cooperative behavior, conformity to expectations, and emotional ties (McDevitt & Chaffee, 2002), the strength of the community in opinion formation comes mainly from its cohesion inside the community (Huang, Cao, Wang, & Qu, 2008), therefore not only majority/minority opinion is important to be able to form group opinions, but also the cohesion within the community (the group of individuals) is key to be able to establish the final opinion. Measures must be developed to determine the degree of social cohesion to document roles and relationships, mainly for situations in which the groups deals with reciprocal influence among their members (Pan & McLeod, 1991). In group opinions, less extreme information has a cohesive effect and a greater success rate in achieving agreements in the population (Sîrbu, Loreto, & Servedio, 2013; Sîrbu, Loreto, Servedio, & Tria, 2013). Nevertheless, the existing CRPs for LSGDM dealing with HFLTS (Gou et al., 2018; Ren et al., 2020; Zhong & Xu, 2020) are focused on majority or minority processes and do not consider cohesion, which is key to forming group opinions and reaching a consensus.

The first attempt to overcome the disagreement found in sub-groups, taking their togetherness into account, was introduced in (Rodríguez et al., 2018), where a cohesion measure for hesitant fuzzy sets in CRPs for LSGDM was defined.

Following this idea, a new proposal of a cohesion-driven CRP for LSGDM approach with CLEs based on HFLTS to make the guidance towards a solid agreed solution more flexible has been proposed. Among the different possibilities to measure the cohesiveness of group opinions modelled by CLEs represented by HFLTSs, our proposal applies restricted equivalence functions that measure the similarity between elements according to their dispersion, meaning the elements are less dispersed, and thus more similar. Besides satisfying the usual properties of similarity operators (Bustince, Barrenechea, & Pagola, 2006), such as to decrease the similarity when the inputs dispel, these functions are very restrictive when it comes to outputting extremal values (so as to facilitate the success of achieving consensus). They take values in the range between 0 and 1, where the value of 0 represents the inputs being as furthest away as possible and 1 is only obtained when the inputs are equal. This makes restricted equivalence functions, among the different similarity measures, particularity suitable for the task of measuring cohesion.

Therefore, the novelties introduced in this proposal are the following:

- A new cohesion measure for HFLTS based on restricted equivalence functions.
- A flexible cohesion-driven CRP for LSGDM defined under HFLTS context in which cohesion is considered to form opinions in addition to size. This approach mantains the fuzzy representation of HFLTS to avoid the loss of information and follows the CW scheme.
- An adaptive feedback process that guides the consensus process according to the degree of agreement reached to reduce the time cost of the CRP-LSGDM.
- A simulation of multiple LSGDM scenarios to evaluate the performance of the cohesion-driven approach proposed.

Eventually, this approach has been implemented and integrated in the CRP support system, the so-called AFRYCA (Labella, Estrella, & Martínez, 2017; Palomares, Estrella, et al., 2014) to carry out the simulations.

The remainder of this paper is structured as follows: Section 2 revises some basic concepts about LSGDM and CRP, CLEs based on HFLTS and restricted equivalence functions to easily understand the proposal. A revision of the existing approaches dealing with LSGDM is also introduced. Section 3 proposes a cohesion measure for HFLTS based on restricted equivalence functions. Section 4 presents a new cohesion-driven CRP for LSGDM dealing with CLEs. Section 5 includes a simulation to show the utility of the cohesion measure and the cohesion-driven CRP-LSGDM model using AFRYCA. Finally, Section 6 points out some conclusions and future works.

2. Preliminaries

This section reviews some concepts related to CRP-LSGDM, the modelling of the uncertainty by means of CLEs based on HFLTS and restricted equivalence functions that will facilitate the understanding of the proposal. Moreover, several CRP-LSGDM approaches dealing with linguistic information are revised.

2.1. Consensus reaching process for large scale group decision making

In the past few years, the concept of LSGDM has attracted the attention of many scholars due to current society demands in decision making processes (Eklund et al., 2007; Eklund, Rusinowska, & de Swart, 2008). This concept is similar to GDM, but differs in two main aspects, the number of experts is much bigger than in the latter and much larger than the number of the alternatives. Formally, a LSGDM problem consists of: (i) a set of alternatives $X = \{x_1, \ldots, x_n\}, (n \geqslant 2)$, which can be chosen as possible solutions for the problem, and (ii) a set of experts $E = \{e_1, \ldots, e_m\}, (m \gg n)$, who assess the set of alternatives X.

On the other hand, both kinds of problems can be solved in a similar way by means of a resolution process which consists of two phases (Roubens, 1997), (i) aggregation and (ii) exploitation. However, this process does not guarantee that the solution obtained will be accepted by all experts involved in the decision making problem, because some of them might feel that their opinions have not been considered. In order to

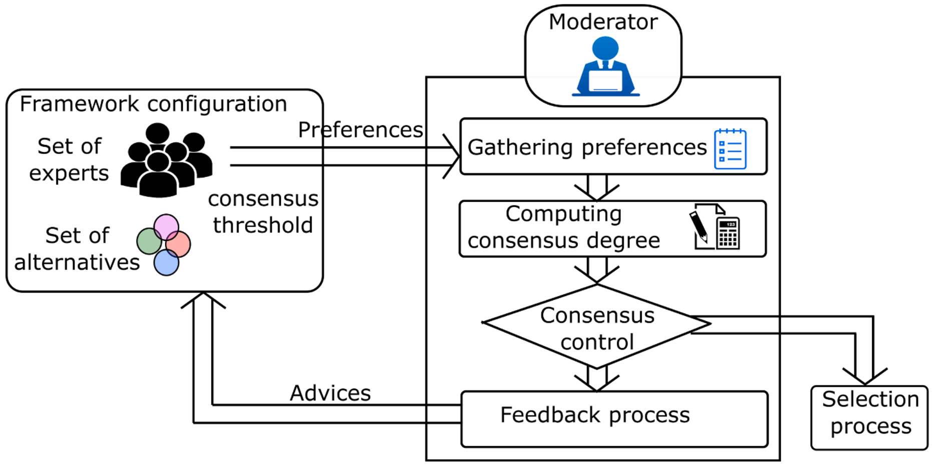

Fig. 1. General scheme of a CRP.

avoid this drawback, a CRP is included in the resolution process before obtaining the solution (Butler & Rothstein, 2006).

A CRP is an iterative process in which experts involved in the decision process discuss among them and modify their initial opinions to try to bring them closer to each other toward a collective opinion which is accepted for all them (Parreiras, Ekel, Martini, & Palhares, 2010). Fig. 1 shows a general scheme of a CRP whose phases are described as follows:

- *Framework configuration*: the set of alternatives, the set of experts and the consensus threshold desired are described.
- *Gathering preferences*: experts provide their opinions by using preference modelling options, being the most common one the use of preference relations.
- *Computing the consensus degree*: the level of agreement (cr) in the experts' group is computed.
- *Consensus control*: the consensus degree obtained is compared with a consensus threshold (μ), if the consensus threshold is achieved, a selection process starts to choose the best alternative, otherwise more discussion rounds are required.
- *Feedback process*: the opinions causing disagreement are identified and some recommendations are generated to guide experts to modify their opinions and increase the consensus degree in the next round.

In LSGDM problems, reaching a consensus is much more complex, because opinions among experts tend to be in conflict and polarized (Dong, Zhou, & Martínez, 2019), and classical CRP models do not perform well (Labella et al., 2018), consequently, it is also necessary to include a CRP before choosing the best solution for the LSGDM problems. To do so, the general scheme of a CRP is extended and modified as we have shown in Section 4.

2.2. Modelling uncertainty with Comparative Linguistic Expressions based on HFLTS

The appearance of uncertain information is quite common in real world problems due to the lack of information and knowledge regarding the problem. In these situations, the use of linguistic information has obtained successful results modelling the uncertainty (Martínez & Herrera, 2012; Morente-Molinera, Pérez, Ureña, & Herrera-Viedma, 2015). Rodríguez et al. pointed out in (Rodríguez, Labella, & Martínez, 2016) that most of the linguistic models limit experts to providing their opinions or preferences by using single linguistic terms, which sometimes is not enough, as experts can be hesitant and might require linguistic expressions that are more complex and flexible than a single linguistic term. To overcome this drawback, the use of context-free grammars (Rodríguez & Martínez, 2013) to generate CLEs was proposed. The following is an example of a context-free grammar:

Definition 1. (*Rodríguez et al., 2013*) Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows.

$$
\begin{array}{l}
V_N = \{\langle primary\ term\rangle,\ \langle composite\ term\rangle,\\
\langle unary\ relation\rangle,\ \langle binary\ relation\rangle, \langle conjunction\rangle\}\\
V_T = \{at\ least,\ at\ most,\ between,\ and, s_0,\ s_1, \ldots, s_g\}\\
I \in V_N\\
P = \{I\ ::=\ \langle primary\ term\rangle|\langle composite\ term\rangle\\
\langle composite\ term\rangle\ ::=\ \langle unary\ relation\rangle\\
\langle primary\ term\rangle|\ \langle binary\ relation\rangle\\
\langle primary\ term\rangle\langle conjunction\rangle\langle primary\ term\rangle\\
\langle primary\ term\rangle\ ::=\ s_0|s_1|\ldots|s_g\\
\langle unary\ relation\rangle\ ::=\ at\ least|at\ most\\
\langle binary\ relation\rangle\ ::=\ between\\
\langle conjunction\rangle\ ::=\ and\}
\end{array}
$$

These CLEs allow expert opinions to be represented in a comprehensive way and they are easy to understand. To accomplish the computing with words processes with these expressions, a transformation function that converts them into HFLTS was defined.

Definition 2. (*Rodríguez et al., 2012*) Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set, a HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S.

$$H_S = \{s_i, s_{i+1}, \ldots, s_j\}, s_k \in S, k \in \{i, \ldots, j\}$$

Definition 3. (*Rodríguez et al., 2013*) Let E_{G_H} be a function that transforms the CLEs, $ll \in S_{ll}$, obtained by G_H, into HFLTSs, H_S. Being S the linguistic term set used by G_H and S_{ll} the expression domain generated by G_H.

$$E_{G_H} : S_{ll} \rightarrow H_S$$

The transformations of the CLEs constructed by the context-free grammar G_H can be seen as follows:

$$
\begin{array}{l}
E_{G_H}(s_i) = \{s_i | s_i \in S\}\\
E_{G_H}(at\ most\ s_i) = \{s_j | s_j \leqslant s_i\ and\ s_j \in S\}\\
E_{G_H}(at\ least\ s_i) = \{s_j | s_j \geqslant s_i\ and\ s_j \in S\}\\
E_{G_H}(between\ s_i\ and\ s_j) = \{s_k | s_i \leqslant s_k \leqslant s_j\ and\ s_k \in S\}
\end{array}
$$

Several computational models have been proposed to carry out computations with HFLTS (Liu & Rodríguez, 2014; Rodríguez et al., 2013). In this proposal we will use a fuzzy representation that uses fuzzy membership functions to keep the initial fuzzy representation during the computations and obtains more precise results (Liu & Rodríguez, 2014).

Definition 4. (*Liu and Rodríguez, 2014*) The *fuzzy envelope*, $env_F(H_S)$, is defined as a trapezoidal fuzzy membership function as follows:

$$env_F(H_S) = T(a,b,c,d)$$

where H_S is a HFLTS and $T(a,b,c,d)$ is a fuzzy trapezoidal membership function (see (Liu & Rodríguez, 2014) for further detail).

2.3. Restricted equivalence functions. Definition and properties

Restricted equivalence functions are a class of aggregation functions that were introduced in (Bustince et al., 2006), based in Fodor and Roubens' equivalence functions (Fodor & Roubens, 1994). Restricted equivalence functions satisfy a series of properties that make them useful to measure similarity between objects and they have been applied in the field of computer vision (Bustince, Barrenechea, & Pagola, 2007; Sesma-Sara et al., 2018).

Let us recall the definition of fuzzy strong negation before presenting the notion of a restricted equivalence function.

Definition 5. A function $c:[0,1]\rightarrow[0,1]$ is said to be a fuzzy negation if $c(0)=1, c(1)=0$ and c is decreasing. Additionally, if c is continuous and strictly decreasing, then c is said to be strict. If, besides, c is involutive[1], c is said to be strong negation.

Example 1. The function $c_z:[0,1]\rightarrow[0,1]$ given by $c_z(x)=1-x$ is a strong negation (see (Zadeh, 1965)). Thus, we recall the definition of a restricted equivalence function.

Definition 6. Let c be a strong negation. A function $REF:[0,1]^2\rightarrow[0,1]$ is said to be a restricted equivalence function with respect to c if it satisfies the following properties:

- [(REF1)] $REF(x,y)=REF(y,x)$ for all $x,y\in[0,1]$;
- [(REF2)] $REF(x,y)=1$ if and only if $x=y$;
- [(REF3)] $REF(x,y)=0$ if and only if $\{x,y\}=\{0,1\}$;
- [(REF4)] $REF(x,y)=REF(c(x),c(y))$ for all $x,y\in[0,1]$;
- [(REF5)] For all $x,y,z\in[0,1]$, if $x\leqslant y\leqslant z$, then $REF(x,y)\geqslant REF(x,z)$ and $REF(y,z)\geqslant REF(x,z)$.

Example 2. $REF_1(x,y)=1-|x-y|$ and $REF_2(x,y)=1-(x-y)^2$ are restricted equivalence functions with respect to the strong negation c_z.

The following result is a construction method of restricted equivalence functions.

Theorem 2.1. (*Bustince et al., 2006*) *Let $\phi,\psi:[0,1]\rightarrow[0,1]$ be two continuous strictly increasing functions such that $\phi(0)=\psi(0)=0$ and $\phi(1)=\psi(1)=1$. Then, the function $REF:[0,1]^2\rightarrow[0,1]$ given by $REF(x,y)=\phi(1-|\psi(x)-\psi(y)|)$ is a restricted equivalence function with respect to the negation $c:[0,1]\rightarrow[0,1]$ given by $c(x)=\psi^{-1}(1-\psi(x))$.*

Theorem 2.1 enables more examples of restricted equivalence functions to be provided.

Example 3. These examples of restricted equivalence functions are constructed following Theorem 2.1:

- Let $p,q>0$, the function $REF_3:[0,1]^2\rightarrow[0,1]$ defined by $REF_3(x,y)=(1-|x^q-y^q|)^p$ is a restricted equivalence function with respect to the negation $c:[0,1]\rightarrow[0,1]$ given by $c(x)=(1-x^q)^{1/q}$.
- Let $p>0$, the function $REF_4\left(x,y\right)=\left(1-\left|\sin\left(\frac{\pi x}{2}\right)-\sin\left(\frac{\pi y}{2}\right)\right|\right)^p$ is a restricted equivalence function with respect to the negation given by $c\left(x\right)=\frac{2}{\pi}\arcsin(x)$.
- Let $q>0$, the function $REF_5(x,y)=(1-|x^q-y^q|)e^{(1-|x^q-y^q|)}$ is a restricted equivalence function with respect to the negation given by $c(x)=(1-x^q)^{1/q}$.
- Let $p>0$, the function $REF_6\left(x,y\right)=\left(1-\left|\frac{\log(x+1)}{\log(2)}-\frac{\log(y+1)}{\log(2)}\right|\right)^p$ is a restricted equivalence function with respect to the negation given by $c(x)=e^{\log(2)x}-1$.

In (Bustince et al., 2006), some additional construction methods for restricted equivalence functions can be found.

2.4. Related works

In the same way that GDM problems with a few experts need to include a CRP to obtain solutions more accepted by the whole group, in LSGDM is also necessary. Therefore, new CRPs for LSGDM approaches have been introduced (Zhang, Yu, Martínez, & Gao, 2020). These approaches model the uncertainty and vagueness of the information by means of fuzzy sets (Palomares, Martínez, et al., 2014; Quesada et al., 2015), hesitant fuzzy sets (Rodríguez et al., 2018; Wu & Xu, 2018) or intuitionistic fuzzy sets (Zhang et al., 2015). Nevertheless, in real world problems there are many aspects of different activities that cannot be assessed in a quantitative context, but rather in a qualitative one. Therefore, it seems logical to study the linguistic modelling in CRP for LSGDM and propose novel linguistic models: Song and Li (2019) proposed a consensus model for LSGDM that is able to deal with multi-granular probabilistic fuzzy linguistic preference relations. Several approaches have introduced the use of heterogeneous information to represent experts' preferences by using different formats (Tang et al., 2019; Zhang, Dong, & Herrera-Viedma, 2018). Li, Dong, and Herrera (2019) and Xiao, Wang, and Zhang (2020) have recently proposed CRP-LSGDM based on the personalized individual semantics model (Li, Dong, Herrera, Herrera-Viedma, & Martínez, 2017), which considers that words mean different things for different people and uses the numerical scale model (Dong, Li, & Herrera, 2016) and the 2-tuple linguistic model (Herrera & Martínez, 2000) to represent the semantics of each expert. Wang, Xu, and Huang (2019) developed a consensus-based method for LSGDM which computes the sub-groups' weights, taking into account not only its size, but also the expert's importance within the sub-group. Notwithstanding, these approaches use single linguistic terms to represent experts' preferences, and due to time pressure and a lack of information, more complex linguistic expressions should be defined to model experts' preferences. To face this challenge, Rodríguez and Martínez (2013) introduced the use of context-free grammars to generate CLEs based on HFLTS. Thus, recently, several proposals that use HFLTS to model experts' preferences in CRP for LSGDM have been introduced in the literature (Gou et al., 2018; Ren et al., 2020; Zhong & Xu, 2020). We revise them in further detail.

Gou et al. (2018) proposed a consensus model for LSGDM based on a double hierarchy HFLTS, which is an extension of HFLTS. This model introduces a clustering method based on information entropy theory to classify the experts into sub-groups and develops a feedback phase to send suggestions to each group and increase the consensus level. It is focused on the majority-driven consensus. In spite of the fact that this model tries to improve the flexibility of eliciting preferences using the double hierarchy HFLTS, it presents some important drawbacks:

- Similarly to the CLEs generated by the context-free grammar revised in Definition 1, this model obtains linguistic expressions, the difference is that experts have to select two linguistic terms from two linguistic term sets with different syntax to obtain more precise

[1] $c(c(x))=x$ for all $x\in[0,1]$.

expressions. This implies that the generated linguistic expressions are more difficult for experts to understand, for instance, "more than only a little perfect" or "between just right high and a little very high".

- In order to carry out computations with these linguistic expressions, they oversimplify the information by transforming HFLTS into crisp values, which implies the loss of their fuzzy representation and hence loss of information and precision in the final results.
- The model computes the weights of the individual experts within each sub-group and for the whole sub-group uses its size and an entropy measure. As the clustering process is applied just once, the weights of sub-groups do not change across the consensus process, because experts always belong to the same sub-group and the experts' weights of each sub-group change only a little bit (note that in the case study the changes are in the 4th decimal) because they are based on the size of the sub-group.
- In the feedback process when experts are reluctant to change their preferences, the model removes such experts or computes their preference randomly for the next round. This might provoke disagreement amongst experts in the final decision making process. Moreover, the process does not explain how experts should modify their linguistic preferences.

Ren et al. (2020) introduced a novel CRP-LSGDM that models the experts' preferences by HFLTS. This approach includes a social network analysis based clustering method to classify experts into sub-groups and proposes a method to handle minority opinions that overcomes to overemphasize their weights. The feedback process is based on the approach proposed by Herrera-Viedma, Alonso, Chiclana, and Herrera (2007). This model uses the majority driven-consensus to compute the sub-groups' weights. Their main weaknesses are the following:

- Although experts provide their preferences using HFLTS, such preferences are transformed into crisp values in the first stage of the model by using a score function. This implies the loss of the fuzzy representation from the very beginning and hence the lack of precision in the results.
- The sub-groups' weights are computed considering only their size.
- The feedback process introduces the direction rules to increase the consensus degree in the next round, but it does not describe how experts should modify their preferences.
- The case study shown reaches the desired consensus level in one round but it does not show which experts should change their opinions to increase the consensus level, nor how they modify them, therefore, the feedback process is not clear.

Zhong and Xu (2020) presented a consensus model for LSGDM in which experts elicit their preferences using HFLTS. This proposal defines a new clustering method that integrates the correlation coefficient among experts and the sub-group's consensus to find the maximum possible agreement in the group. Therefore, there are no negotiation processes in which experts change their preferences to increase the level of agreement and achieve a fixed consensus level. The sub-groups' weights are computed considering their size and consensus, since, the majority rule is used. It presents the following drawbacks:

- Although experts provide their preferences using HFLTS, they are converted into numerical values to carry out the computations by means of a hesitant degree function. In this transformation the initial fuzzy representation of HFLTS is lost, causing loss of information and precision in the results.
- This proposal does not define a CRP, but rather a clustering process that finds the maximum possible consensus by changing the clusters.

To sum up, the previous approaches oversimplify the experts' preferences represented with HFLTS into crisp values to carry out the computations, causing the loss of the initial fuzzy representation and hence the lack of precision in the results. Gou et al. (2018) and Zhong and Xu (2020) use as rule to drive the consensus process the majority driven rule and Ren et al. (2020) propose a method to deal with minority opinions, but all of them ignore the togetherness among experts that belong to the same sub-group which is key to form opinions in group and achieve a consensus. And last but not least, the proposals presented by Gou et al. (2018) and Ren et al. (2020) define in the feedback process the direction rules to suggest the direction of the changes required to increase the consensus level in the group, but they do not explain how the changes should be applied.

Taking into account the drawbacks mentioned for the existing consensus models dealing with HFLTS for LSGDM, in this paper we propose a cohesion-driven CRP-LSGDM model in which experts elicit their preferences using CLEs based on HFLTS. This model carries out the computing with words processes by using the fuzzy envelope concept that keeps the initial fuzzy representation of HFLTS and obtains more precise results. Moreover, it considers cohesion in order to form the group opinions and achieve the consensus.

3. New cohesion measure for HFLTS based on restricted equivalence functions

In this section we introduce a cohesion measure for HFLTSs based on their fuzzy envelopes and restricted equivalence functions that will be used to drive the consensus process in our proposal introduced in Section 4. Recall that the goal of this measure is to give an estimation of the cohesion of experts' preferences within a cluster regarding a set of alternatives and restricted equivalence functions that are capable of measuring the similarity between such preferences in order to construct a cohesion estimate. These preferences are given by a matrix of CLEs that are transformed into HFLTS and, in turn, the fuzzy envelope of each HFLTS is a trapezoidal membership function. Hence, we present a cohesion measure that reflects the level of cohesion among the preferences of a set of experts with respect to an alternative over another.

Definition 7. Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and let $\{H_S^1, \dots, H_S^k\}$ be k HFLTSs. We say that a function $\mathfrak{c} : (HFLTS)^k \rightarrow [0,1]$ is a cohesion measure for HFLTSs if it satisfies the following properties:

(c1) If $H_S^1 = \dots = H_S^k$, then $\mathfrak{c}(H_S^1, \dots, H_S^k) = 1$;
(c2) If $s_0 \in H_S^i$ and $s_g \in H_S^j$ for some $i, j \in \{1, \dots, k\}$ such that $s_0 \neg \in H_S^j$ and $s_g \neg \in H_S^i$, then $\mathfrak{c}(H_S^1, \dots, H_S^k) = 0$;
(c3) $\mathfrak{c}(H_S^1, \dots, H_S^k) = \mathfrak{c}(H_S^{\pi(1)}, \dots, H_S^{\pi(k)})$, for any permutation π of k elements.

Axioms $(c1)-(c3)$ from Definition 7 are the minimum required properties needed by a function to be considered a cohesion measure for HFLTSs. Indeed, $(c1)$ ensures that whenever the HFLTSs that are being compared are identical, the cohesion measure yields the highest possible result, axiom $(c2)$ guarantees that the cohesion measure between HFLTSs that contain the most separate linguistic terms in the given linguistic term set S is 0 and axiom $(c3)$ demands symmetry, i.e., the order in which the HFLTSs are compared should not affect their cohesion measure.

In order to present a construction method for such a cohesion mea-

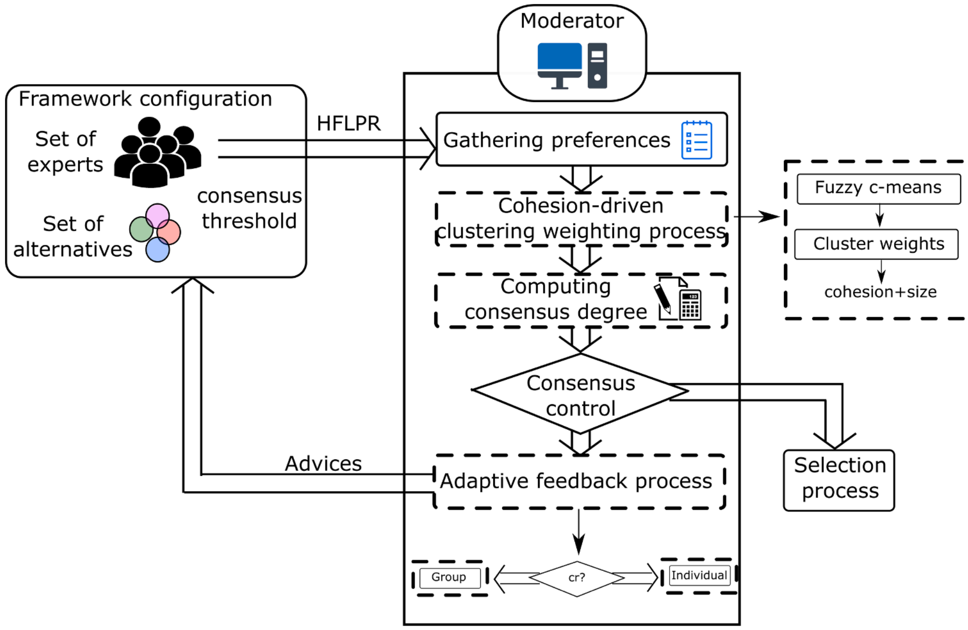

Fig. 2. General scheme of cohesion-driven CRP-LSGDM approach.

sure that is based on restricted equivalence functions, we set the following notation: Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and let $\{H_S^1, \dots, H_S^k\}$ be k HFLTSs, then we set the following points in $[0,1]$ for $i \in \{1, \dots, k\}$:

$$a_i = \min\{x \in [0,1] \,|\, env_F(H_S^i)(y) = 0 \text{ for all } y \leqslant x\}, \tag{1}$$

$$d_i = \max\{x \in [0,1] \,|\, env_F(H_S^i)(y) = 0 \text{ for all } y \geqslant x\}, \tag{2}$$

in which the fuzzy envelope, $env_F(H_S^i)$, denotes the trapezoidal membership function constructed following the methodology described in (Liu & Rodríguez, 2014).

Moreover, we set the points $a, d \in [0,1]$ to be

$$a = \min_{1 \leqslant i \leqslant k} a_i \tag{3}$$

$$d = \max_{1 \leqslant i \leqslant k} d_i \tag{4}$$

respectively.

Theorem 3.1. *Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and let $\{H_S^1, \dots, H_S^k\}$ be k HFLTSs. Let $\mathfrak{c} : (HFLTS)^k \rightarrow [0,1]$ be the function given by*

$$\mathfrak{c}\left(H_S^1, \dots, H_S^k\right) = \begin{cases} 1, & \text{if } a_i = 0,\ d_i = 1 \text{ for all } 1 \leqslant i \leqslant k, \\ \dfrac{REF(a,d)}{\max\limits_{1 \leqslant i \leqslant k} REF(a_i, d_i)}, & \text{otherwise.} \end{cases} \tag{5}$$

where a_i, d_i, a, d are the numbers obtained from Eqs. (1)–(4), REF is a restricted equivalence function and $env_F(H_S^i)$ is the fuzzy envelope of H_S^i, constructed following the methodology of (Liu & Rodríguez, 2014)[2]*. Then, $\mathfrak{c}$ is a cohesion measure for HFLTSs.*

Proof. The expression of $\mathfrak{c}$ in Eq. (5) is well defined. Indeed, note that $\max_{1 \leqslant i \leqslant k} REF(a_i, d_i) = 0$ if and only if $a_i = 0$ and $d_i = 1$ for all $1 \leqslant i \leqslant k$, and that it holds that $REF(a,d) \leqslant \max_{1 \leqslant i \leqslant k} REF(a_i, d_i)$ for all a_i, d_i, a, d obtained from Eqs. (1)–(4), therefore for any HFLTSs $H_S^1, \dots, H_S^k$, it holds that $\mathfrak{c}(H_S^1, \dots, H_S^k) \in [0,1]$.

Thus, in order to show that $\mathfrak{c}$ is a cohesion measure for HFLTSs, let us check axioms $(c1) - (c3)$.

(c1) Let $H_S^1 = \cdots = H_S^k$, then, it holds that $a = a_i$ and $d = d_i$ for all $1 \leqslant i \leqslant k$. Thus, if $a = 0$ and $d = 1$, it is clear that $\mathfrak{c}(H_S^1, \dots, H_S^k) = 1$, and if $(a,d) \neq (0,1)$, then $REF(a,d) = \max_{1 \leqslant i \leqslant k} REF(a_i, d_i) \neq 0$ and, hence, $\mathfrak{c}(H_S^1, \dots, H_S^k) = 1$.

(c2) Let $i, j \in \{1, \dots, k\}$ such that $s_0 \in H_S^i, s_g \in H_S^j, s_0 \neg \in H_S^j$ and $s_g \neg \in H_S^i$.

Note that, due to the methodology used to construct fuzzy envelopes in (Liu & Rodríguez, 2014), the fact that $s_0 \in H_S^i$ and $s_g \neg \in H_S^i$ ensures that $env_F(H_S^i)$ is a fuzzy trapezoidal membership function of the form $T(0, x_1, x_2, x_3)$ for some $0 \leqslant x_1 \leqslant x_2 \leqslant x_3 < 1$. Therefore, using Eqs. (1) and (2), it holds that $a_i = 0$ and $d_i < 1$. Similarly, since $s_g \in H_S^j$ and $s_0 \neg \in H_S^j$, $env_F(H_S^j)$ is a fuzzy trapezoidal membership function of the form $T(x_0, x_1, x_2, 1)$ for some $0 < x_0 \leqslant x_1 \leqslant x_2 \leqslant 1$ and it holds that $a_j > 0$ and $d_j = 1$. Consequently, since $a_i = 0$ and $d_j = 1$, by Eqs. (3) and (4), it holds that $a = 0$ and $d = 1$. Note that in this case, it holds that $REF(a,d) = 0$, $REF(a_i, d_i) > 0$ and $REF(a_j, d_j) > 0$. Thus, clearly, it holds that $\mathfrak{c}(H_S^1, \dots, H_S^k) = 0$.

(c3) The relative order of the HFLTSs $H_S^1, \dots, H_S^k$ does not affect the computation of Eq. (5) and, hence, the symmetry of $\mathfrak{c}$ is straightforward. □

The rationale behind the construction of a cohesion measure using Eq. (5) is that we are measuring, via fuzzy envelopes, the similarity of the furthest linguistic terms among all the HFLTSs $\{H_S^1, \dots, H_S^k\}$ and we are normalizing the result by the similarity of the furthest linguistic terms within the HFLTSs which has the narrowest envelope. Indeed, a and d from Eqs. (3) and (4) model the furthest linguistic terms among all HFLTSs and $\max_{1 \leqslant i \leqslant k} REF(a_i, d_i)$ models the similarity of the HFLTS whose envelope is the narrowest. In the context of the present study, Eq. (5) estimates the cohesion by measuring the furthest preferences among all experts, relative to the expert whose preferences are the closest. This way, HFLTSs that share a similar number of linguistic terms (and these terms are close to each other) yield high cohesion numbers, whereas

[2] We can assume that the support of the fuzzy envelopes lies within the interval $[0, 1]$, as, otherwise, they could be translated by means of a linear transformation.

HFLTSs that have fairly different linguistic terms and variety in the number of terms penalize cohesion value.

Example 4. The instances of restricted equivalence functions presented in Examples 2 and 3 (REF_1-REF_6) can be used in Eq. (5) to construct expressions of cohesion measures. Let $S=\{s_0,\dots,s_g\}$ be a linguistic term set, let $\{H_S^1,\dots,H_S^k\}$ be k HFLTSs and let a_i,d_i,a,d be the numbers obtained from Eqs. (1)–(4). The following are cohesion measures:

$$\mathfrak{c}_1\left(H_S^1,\dots,H_S^k\right)=\begin{cases}1, & \text{if } a_i=0,\ d_i=1 \text{ for all } 1\leqslant i\leqslant k,\\ \dfrac{1-|a-d|}{\max\limits_{1\leqslant i\leqslant k}1-\left|a_i-d_i\right|}, & \text{otherwise.}\end{cases}$$

$$\mathfrak{c}_2\left(H_S^1,\dots,H_S^k\right)=\begin{cases}1, & \text{if } a_i=0,\ d_i=1 \text{ for all } 1\leqslant i\leqslant k,\\ \dfrac{1-(a-d)^2}{\max\limits_{1\leqslant i\leqslant k}1-(a_i-d_i)^2}, & \text{otherwise.}\end{cases}$$

$$\mathfrak{c}_3\left(H_S^1,\dots,H_S^k\right)=\begin{cases}1, & \text{if } a_i=0,\ d_i=1 \text{ for all } 1\leqslant i\leqslant k,\\ \dfrac{(1-|a^q-d^q|)^p}{\max\limits_{1\leqslant i\leqslant k}(1-|a_i^q-d_i^q|)^p}, & \text{otherwise.}\end{cases}$$

$$\mathfrak{c}_4\left(H_S^1,\dots,H_S^k\right)=\begin{cases}1, & \text{if } a_i=0,\ d_i=1 \text{ for all } 1\leqslant i\leqslant k,\\ \dfrac{\left(1-\left|\sin\left(\frac{\pi a}{2}\right)-\sin\left(\frac{\pi d}{2}\right)\right|\right)^p}{\max\limits_{1\leqslant i\leqslant k}\left(1-\left|\sin\left(\frac{\pi a_i}{2}\right)-\sin\left(\frac{\pi d_i}{2}\right)\right|\right)^p}, & \text{otherwise.}\end{cases}$$

$$\mathfrak{c}_5\left(H_S^1,\dots,H_S^k\right)=\begin{cases}1, & \text{if } a_i=0,\ d_i=1 \text{ for all } 1\leqslant i\leqslant k,\\ \dfrac{(1-|a^q-d^q|)e^{(1-|a^q-d^q|)}}{\max\limits_{1\leqslant i\leqslant k}(1-|a_i^q-d_i^q|)e^{\left(1-|a_i^q-d_i^q|\right)}}, & \text{otherwise.}\end{cases}$$

$$\mathfrak{c}_6\left(H_S^1,\dots,H_S^k\right)=\begin{cases}1, & \text{if } a_i=0,\ d_i=1 \text{ for all } 1\leqslant i\leqslant k,\\ \dfrac{\left(1-\left|\frac{\log(a+1)}{\log(2)}-\frac{\log(d+1)}{\log(2)}\right|\right)^p}{\max\limits_{1\leqslant i\leqslant k}\left(1-\left|\frac{\log(a_i+1)}{\log(2)}-\frac{\log(d_i+1)}{\log(2)}\right|\right)^p}, & \text{otherwise.}\end{cases}$$

Remark 1. To find instances of cohesion measures it suffices to consider a restricted equivalence function and replace it in Eq. (5). For example, one could consider the possibility to construct restricted equivalence functions by means of Theorem 2.1 or any of the construction methods that can be found in (Bustince et al., 2006).

Remark 2. Note that a cohesion measure $\mathfrak{c}$ constructed by means of Theorem 3.1 can take as arguments either HFLTSs or their fuzzy envelopes directly. For clarity, from this point onwards, we consider fuzzy envelopes as arguments for the cohesion measures used in this study.

The cohesion measure introduced in Definition 7 expresses the level of togetherness of the experts' preferences regarding a specific alternative over another. Thus, we can evaluate the cohesion between experts' opinions regarding alternative i over alternative j. Such cohesion is based on the comparison of the left and right limits of the fuzzy envelopes that represent experts' opinions, the more similar the fuzzy envelopes, the greater the cohesion. However, it is essential to define a proper value of cohesion to represent the closeness among experts' opinions in order to obtain reliable results.

Finally, in order to obtain a measure of global cohesion for each sub-group, the cohesion of the expert's preferences with respect to all alternatives, which is given by k matrices of trapezoidal membership functions, can be computed by using Theorem 3.1. For example, measuring the cohesion between the HFLTSs at each position of the matrix, and then aggregating them by means of an averaging function. This value is then used to compute the sub-group's weight (this is further detailed in Section 4.2.2).

4. A cohesion-driven CRP-LSGDM approach dealing with comparative linguistic expressions

This section introduces a novel cohesion-driven consensus model for LSGDM problems in which experts provide their assessments by means of CLEs based on HFLTS. This model follows the scheme shown in Fig. 1, but includes some additional steps in order to deal with CLEs based on HFLTS and considers the cohesion to form the groups' opinions and reach the consensus process (see dotted boxes in Fig. 2). The following subsections describe such steps in further detail.

4.1. Gathering preferences

In order to facilitate the experts' assessment elicitation, each expert provides his/her preferences by using CLEs modelled on a hesitant fuzzy linguistic preference relation (HFLPR) (Zhu & Xu, 2014), a matrix $P^i, X \times X \rightarrow S_{ll}$, where S_{ll} is the set of CLEs generated by the context-free grammar introduced in Definition 1. Considering $S = \{verybad, bad, medium, good, verygood\}$ a linguistic term set, an example of HFLPR provided by the expert e_i could be:

$$P^i = \begin{pmatrix} - & medium & good \\ medium & - & at\ most\ bad \\ bad & at\ least\ good & - \end{pmatrix}$$

In this contribution, in order to perform computations with CLEs, they are first transformed into HFLTS (see Definition 3):

$$P^i = \begin{pmatrix} - & \{medium\} & \{good\} \\ \{medium\} & - & \{verybad, bad\} \\ \{bad\} & \{good, verygood\} & - \end{pmatrix}$$

Afterwards, a fuzzy envelope (see Definition 4) for each HFLTS is obtained:

$$P^i = \begin{pmatrix} - & T(0.5, 0.75, 1) & T(0.75, 1, 1) \\ T(0.5, 0.75, 1) & - & T(0, 0, 0.32, 0.75) \\ T(0.25, 0.5, 0.75) & T(0.75, 1, 1) & - \end{pmatrix}$$

4.2. A cohesion-driven clustering weighting process

To avoid the inherent problem of scalability in LSGDM problems, this model includes a clustering process in which experts are assigned to different sub-groups. Furthermore, the influence of each sub-group in the subsequent process of consensus will be determined not only by their size, but also by the cohesion of the sub-groups' opinions which implies that group opinions with higher cohesion will have more influence on the ability to achieve the consensus. The process of clustering can be divided into two different steps: (i) the classification of the experts to different sub-groups by applying the fuzzy c-means algorithm and (ii) the sub-groups weights computation taking into account their size and cohesion.

4.2.1. Fuzzy c-means clustering process

One of the biggest problems in LSGDM problems is to manage the opinion of a large number of experts simultaneously. To tackle this problem, clustering techniques aim at dividing the initial group of experts into sub-groups or clusters and manage them independently, by reducing the problem of scalability. Whilst there are several clustering algorithms proposed in the literature, in this contribution we have used the fuzzy c-means algorithm (Bezdek, 1981) because it has provided successfull results in LSGDM (Palomares, Martínez, et al., 2014; Rodríguez et al., 2018). In short, this algorithm computes iteratively centroids (representative elements of all data objects belonging to the same cluster) and assigns a membership degree to each data object for each cluster according to the distance between said data object and the corresponding cluster centroid. A relevant step in fuzzy c-means is the initialization of the clusters and the representation of their respective centroids. In this contribution, the number of clusters, N, is fixed as the number of alternatives, $N = n$, and their respective centroids are initialized with a HFLPR, which represents the total preference of the corresponding alternative over the others. In this way, the sub-groups formed are composed by experts with similar opinions in respect to the alternatives. For each iteration, t, the centroids C^k are computed together with the membership degree of each expert preference P^i to each centroid C^k such that:

$$\mu_{C^{k,t}}\left(P^i\right) = \frac{\left(1/d_H\left(P^i, c^{k,t}\right)\right)^{2/(b-1)}}{\sum_{u=1}^{n}\left(1/d_H\left(P^i, c^{u,t}\right)\right)^{2/(b-1)}} \tag{6}$$

where $d_H(\cdot)$ is a distance measure between two HFLPRs (Zhu & Xu, 2014), t is the current iteration, and b indicates the fuzziness degree of the clusters. The larger b is, the fuzzier the clusters will be (Bezdek, 1981) (see (Rodríguez et al., 2018) for further detail).

4.2.2. Cluster weights

Once the sub-groups have been defined, the next step is to compute the importance of each one in the consensus process. Classically, such importance has been driven by the group's size, the greater the number of experts in a cluster, the greater its importance. However, the fact that a large number of experts belong to the same sub-group does not always imply implicitly that their opinions are extremely similar. In such a case, a big/small group without cohesion in their opinions should not be considered as important as a similar sized group with a higher cohesion in their opinions. For this reason, a novel cohesion measure that evaluates the degree of closeness in the experts' preferences has been proposed. Taking into account the size and the cohesion, the importance of the sub-groups is set more appropriately by obtaining fairer solutions for LSGDM problems.

As the sub-group size has been derived from the previous step, we will now focus on the clusters' cohesion computation process, which is based on the use of restricted equivalence functions (see Section 2.3). Firstly, all the experts' preferences represented by their fuzzy envelopes and belonging to the same sub-group G^k are collected in a matrix noted as S^k.

Definition 8. Let G^k be a sub-group composed by several experts $\{e_1, \ldots, e_s\}$ in which the expert e_i's preference over the pair of alternatives (l, j) is represented by the fuzzy envelope T^i_{lj}, the matrix S^k is built as follows:

$$S^k = \begin{pmatrix} - & \ldots & \{T^1_{1n}, \ldots, T^s_{1n}\} \\ \vdots & - & \vdots \\ \{T^1_{n1}, \ldots, T^s_{n1}\} & \ldots & - \end{pmatrix} \tag{7}$$

Once all the experts' preferences for the sub-group G^k are collected in S^k, the cohesion is computed for each pair of alternatives (see Eq. (5)) using pairs of fuzzy envelopes.

$$S^k_{\mathfrak{c}} = \begin{pmatrix} - & \ldots & \mathfrak{c}_{1n} \\ \vdots & - & \vdots \\ \mathfrak{c}_{n1} & \ldots & - \end{pmatrix} \tag{8}$$

where

$$\mathfrak{c}_{lj} = \frac{1}{s(s-1)/2}\sum_{i=0}^{s-1}\sum_{t=i+1}^{s}\mathfrak{c}\left(T^i_{lj}, T^t_{lj}\right) \tag{9}$$

Then, to compute the overall cohesion of the sub-group G^k, we aggregate the cohesion of each pair of alternatives from $S^k_{\mathfrak{c}}$ as follows:

$$\mathfrak{c}^k = \frac{1}{n(n-1)/2}\sum_{l=1}^{n-1}\sum_{j=l+1}^{n}\mathfrak{c}_{lj} \tag{10}$$

where $\mathfrak{c}_{lj}$ represents the average cohesion between the experts' preferences for the pair of alternatives (l,j).

Once the size and cohesion of the cluster is derived, its weighting is computed, taking into account its size and cohesion.

Definition 9. Let $Y_{G^k} = \{\mathfrak{c},\gamma\}$ the values of cohesion and size, $\mathfrak{c} \in [0,1]$, $\gamma \in [0,m]$, of the sub-group G^k respectively, the importance of such a sub-group is computed as follows

$$\varphi(Y_{G^k}) = \sqrt{\mathfrak{c}^{\alpha}\gamma^{\beta}} \tag{11}$$

where $\alpha \geqslant 0$ and $\beta \geqslant 0$ are parameters used to increase/decrease the effect of the cohesion and size in the computation of the sub-groups' weights, respectively.

Remark 3. Note that the values of the parameter α and β will be determined by the needs of each LSGDM problem. Finally, the values $\varphi(Y_{G^k})$ are normalized:

$$w_k = \frac{\varphi(Y_{G^k})}{\sum_{z=1}^{n}\varphi(Y_{G^z})} \tag{12}$$

Example 5. Let us consider a sub-group composed by two experts, $G^k = \{e_1, e_2\}$ who take part in a LSGDM with three alternatives $X = \{x_1, x_2, x_3\}$. Initially, the experts' linguistic assessments are transformed into HFLTS and then, modelled using their fuzzy envelopes:

$$P^1 = \begin{pmatrix} - & T^1_{12}(0.5,0.667,0.833) & T^1_{13}(0.667,0.833,1) \\ T^1_{21}(0.167,0.333,0.5) & - & T^1_{23}(0,0,0.15,0.5) \\ T^1_{31}(0,0.167,0.333) & T^1_{32}(0.5,0.86,1,1) & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & T^2_{12}(0.667,0.833,1) & T^2_{13}(0.667,0.833,1) \\ T^2_{21}(0,0.167,0.333) & - & T^2_{23}(0.167,0.333,0.5) \\ T^2_{31}(0,0.167,0.333) & T^2_{32}(0.5,0.667,0.833) & - \end{pmatrix}$$

First of all, the experts' preferences belonging to the same sub-group are collected in a single global matrix S^k:

$$S^k = \begin{pmatrix} - & \{T^1_{12}, T^2_{12}\} & \{T^1_{13}, T^2_{13}\} \\ \{T^1_{21}, T^2_{21}\} & - & \{T^1_{23}, T^2_{23}\} \\ \{T^1_{31}, T^2_{31}\} & \{T^1_{32}, T^2_{32}\} & - \end{pmatrix}$$

Afterwards, the cohesion for each pair of fuzzy envelopes is computed for each pair of alternatives by means of the cohesion measure defined in Eq. (5)

$$S^k_{\mathfrak{c}} = \begin{pmatrix} - & \mathfrak{c}_{12}(T^1_{12}, T^2_{12}) & \mathfrak{c}_{13}(T^1_{13}, T^2_{13}) \\ \mathfrak{c}_{21}(T^1_{21}, T^2_{21}) & - & \mathfrak{c}_{23}(T^1_{23}, T^2_{23}) \\ \mathfrak{c}_{31}(T^1_{31}, T^2_{31}) & \mathfrak{c}_{32}(T^1_{32}, T^2_{32}) & - \end{pmatrix}$$

Equivalently,

$$S^k_{\mathfrak{c}} = \begin{pmatrix} - & \frac{REF(0.5,1)}{\max(REF(0.5,0.833),REF(0.667,1))} & \frac{REF(0.667,1)}{\max(REF(0.667,1),REF(0.667,1))} \\ \frac{REF(0,0.5)}{\max(REF(0.167,0.5),REF(0,0.333))} & - & \frac{REF(0,0.5)}{\max(REF(0,0.5),REF(0.167,0.5))} \\ \frac{REF(0,0.333)}{\max(REF(0,0.333),REF(0,0.333))} & \frac{REF(0.5,1)}{\max(REF(0.5,1),REF(0.5,0.833))} & - \end{pmatrix}$$

For sake of clarity, we consider the restricted equivalence function introduced in Example 2, $REF = 1 - |x - y|$

$$S^k_{\mathfrak{c}} = \begin{pmatrix} - & 0.75 & 1 \\ 0.75 & - & 0.75 \\ 1 & 0.75 & - \end{pmatrix}$$

Remark 4. Note that any restricted equivalence function might be used.

The overall cohesion of the sub-group,

$$\mathfrak{c}^k = \frac{1}{3}\left(0.75 + 1 + 0.75\right) = 0.833$$

Finally, the weight of the sub-group is derived by considering a greater influence of the cohesion of the sub-group with respect to its size $\alpha = 1.5, \beta = 0.75$.

$$\varphi\left(Y_{G^k}\right) = \sqrt{0.833^{1.5}\cdot 2^{0.75}} = 1.131$$

Once the weights are derived for the remaining sub-groups, they will be normalized by Eq. (12).

4.3. Computing the consensus degree

The consensus degree $cr \in [0,1]$ measures the level of agreement in the group of experts and it is essential for the adaptability of the CRP. To compute the consensus degree, some operations are carried out:

1. *Similarity matrices*: a similarity matrix, $SM^{it} = (sm^{it}_{lj})_{n\times n}$, is computed for each pair of experts (e_i, e_t), $i < t$, where each similarity value $sm^{it}_{lj} \in [0,1]$ represents the agreement level among the experts e_i and e_t over the pair of alternatives (x_l, x_j):

$$sm^{it}_{lj} = 1 - d_T\left(p^i_{lj}, p^t_{lj}\right) \tag{13}$$

   where $d_T(\cdot,\cdot)$ represents the geometric distance for fuzzy numbers introduced in (Heilpern, 1997).
2. *Consensus matrix*: to aggregate the similarity values by means of an aggregation operator, ρ, which results in the consensus matrix $CM = (cm_{lj})_{n\times n}$.

$$cm_{lj} = \rho(SIM_{lj}) \tag{14}$$

where SIM_{lj} represents the set of all pairs of experts' similarities regarding the pair of alternatives (x_l, x_j) with $|SIM_{lj}| = \binom{m}{2}$, and cm_{lj} represents the consensus degree achieved by the group of experts over the pair of alternatives (x_l, x_j).

3. *Alternatives consensus degree*: to compute the consensus degree ca^l for each alternative x_l.

$$ca^l = \frac{\sum_{j=1, j\neq l}^{n} cm_{lj}}{n-1} \tag{15}$$

4. *Overall consensus degree*: to obtain the overall consensus degree, cr.

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n} \tag{16}$$

4.4. Consensus control

The overall consensus degree, cr, is compared with a predefined consensus threshold, $\mu \in [0,1]$, that represents the required consensus. If $cr \geqslant \mu$ a selection process to find the best alternative starts, otherwise, the CRP requires another discussion round. The number of rounds is usually limited by another predefined parameter, $maxrounds \in \mathbb{N}$.

4.5. Adaptive feedback process

When $cr < \mu$, that means that the required consensus has not been achieved in the group of experts, thus, it is necessary to increase the level of agreement in the group by modifying the experts' preferences. In order to reduce the time cost, our proposal adapts the feedback process according to the consensus level achieved and a predefined threshold σ. If $cr < \sigma$, a feedback process for groups is applied, otherwise the feedback process is applied to individual experts. The feedback process consists of:

1. *Collective opinion*: to compute a collective matrix that represents the overall opinion of all the experts involved in the LSGDM problem for each pair of alternatives by aggregating the centroids of each cluster, $C = \{C^1, \ldots C^n\}$, by means of the fuzzy weighted average operator.

$$p_{lj}^c = \sum_{k=1}^{n} c_{lj}^k w_k \tag{17}$$

where w_k are the sub-groups' weights derived from their size and cohesion.

2. *Proximity matrices*: to compute a proximity matrix PP^k between the sub-group C^k and the collective opinion P^c.

$$pp_{lj}^k = 1 - d_T\left(c_{lj}^k, p_{lj}^c\right) \tag{18}$$

pp^k values identify the sub-groups that are furthest from the collective opinion.

Depending on the consensus level reached cr, the feedback process will be focused on all experts from the furthest sub-groups or just on several experts.

- Group feedback process: when $cr < \sigma$, then consensus is considered "low" and more changes are necessary. Consequently, all experts belonging to the sub-groups that are furthest away from the collective opinion are recommended to change their preferences concerning the pair of alternatives identified as being in disagreement. To obtain the furthest sub-groups and the pair of alternatives to change, the proximity value of the sub-groups is compared with the average of the proximity values, $\overline{pp}_{lj}$ such that:

$$\overline{pp}_{lj} = \frac{1}{n}\sum_{u=1}^{n} pp_{lj}^u \tag{19}$$

If $ca^l < cr$ and $pp_{lj}^k < \overline{pp}_{lj}$ then, the preferences for the pair of alternatives (x_l, x_j) should be changed to the experts' preferences belonging to the sub-group G^k.
- Individual feedback process: when $cr > \sigma$, although consensus is then considered to be "high" it is not high enough, and individual changes must be made. Consequently, the furthest experts from the collective opinion should change their preferences. The pair of alternatives that need to be changed are identified just as they were in the *group feedback process*, but, on this occasion, the expert $e_i \in G^k$ should change his/her preferences when $\left(1 - d_T(p_{lj}^c, p_{lj}^i) \leqslant \overline{pp}_{lj}\right)$.

So far, the experts who should change their preferences have been identified, but the direction in which these changes should take place has not yet been determined. Several direction rules are applied to suggest the direction of the changes in order to increase the level of agreement in the group. To do so, an acceptability threshold $\varepsilon \geqslant 0$, a positive value close to zero, defines a margin of acceptability when p_{lj}^i and p_{lj}^c are close to each other.

- **RULE 1**
  If $\delta(p_{lj}^i) - \delta(p_{lj}^c) < -\varepsilon$ then $e_i \in G^k$, should *increase* his/her assessments p_{lj}^i on (x_l, x_j).
- **RULE 2**
  If $\delta(p_{lj}^i) - \delta(p_{lj}^c) > \varepsilon$ then $e_i \in G^k$ should *decrease* his/her assessments p_{lj}^i on (x_l, x_j).
- **RULE 3**
  If $-\varepsilon \leqslant \delta(p_{lj}^i) - \delta(p_{lj}^c) \leqslant \varepsilon$ then $e_i \in G^k$ should *not modify* his/her assessments p_{lj}^i on (x_l, x_j).

where $\delta(\cdot)$ denotes the defuzzified value of a trapezoidal fuzzy membership function $T(a, b, c, d)$ such that:

$$\delta\left(T\left(a, b, c, d\right)\right) = \frac{(a + 2b + 2c + d)}{6} \tag{20}$$

Previous rules identify the direction of change but not how the change should be applied. According to the direction of the change received and taking into account that experts accept the suggestion provided by the consensus model, the change to be applied is defined as follows:

Table 1
Simulation scenarios.

LSGDM problem	Behavior		Cohesion importance	
50 experts, 4 alternatives 75 experts, 4 alternatives	100%	75%	$\alpha = 0, \beta = 1$	$\alpha = 2, \beta = 1$

Table 2
Parameters of CRP simulation.

Parameters			
$\mu = 0.85$	$\varepsilon = 0.15$	$maxrounds = 5$	$\sigma = 0.75$

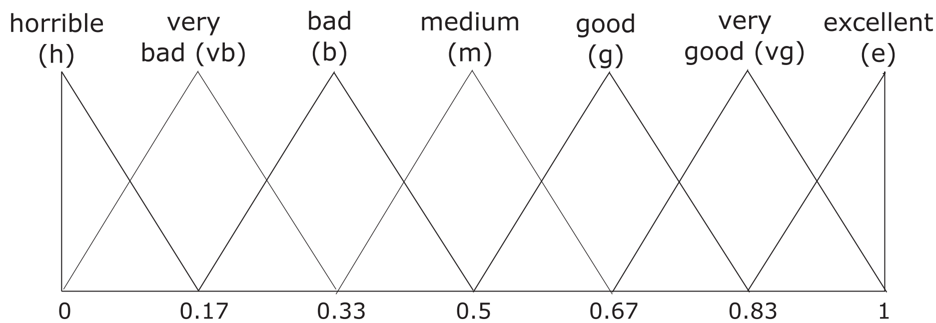

Fig. 3. Linguistic term set.

- **Expert e_i should increase his/her assessment p^i_{lj}.**
  - If $p^i_{lj} = s_p$, where s_p is a single linguistic term, then the expert is recommended to change his/her assessment so that $p^i_{lj} = s_{p+\theta}, \theta \in [1, g-1], p+\theta \leqslant g$. In case that $s_p = s_g$ no change will be applied.
  - If $p^i_{lj} = atleast\ s_p$ or $atmost\ s_p$, where s_p is a linguistic term, then the expert is recommended to change his/her assessment so that $p^i_{lj} = atleast\ s_{p+\theta}$ or $atmost\ s_{p+\theta}$ respectively, $\theta \in [1, g-1], p+\theta \leqslant g$. In case that $s_p = s_g$ no change will be applied.
  - If $p^i_{lj} = between\, s_p\ and\, s_q$, where s_p, s_q are linguistic terms $p > q$, then the expert is recommendedexpert is recommended to change his/her assessment so that $p^i_{lj} = between s_{p+\theta} and s_q, \theta \in [1, g-1], p+\theta \leqslant g$ and $p+\theta \leqslant q$. If $s_{p+\theta} = s_q$, the new assessment is $p^i_{lj} = s_q$.
- **Expert e_i should decrease his/her assessment p^i_{lj}.**
  - If $p^i_{lj} = s_p$, where s_p is a single linguistic term, then the expert is recommended to change his/her assessment so that $p^i_{lj} = s_{p-\theta}, \theta \in [1, g-1], p-\theta \geqslant 0$. If $s_p = s_0$ no change will be applied.
  - If $p^i_{lj} = atleast\ s_p$ or $atmost\ s_p$, where s_p is a linguistic term, then the expert is recommended to change his/her assessment so that $p^i_{lj} = atleast\ s_{p-\theta}$ or $atmost\ s_{p-\theta}$ respectively, $\theta \in [1, g-1], p-\theta \geqslant 0$. If $s_p = s_0$ no change will be applied.
  - If $p^i_{lj} = between\ s_p\ and\ s_q$, where s_p, s_q are linguistic terms $p > q$, then the expert is recommended to change his/her assessment so that $p^i_{lj} = between\ s_p\ and\ s_{q-\theta}, \theta \in [1, g-1], q-\theta \geqslant 0$ and $q-\theta \geqslant p$. If $s_q = s_0$, no change will be applied.

Table 3
Initial experts' sub-groups.

Sub-group	50 experts	75 experts
G^1	$e_8, e_9, e_{12}, e_{16}, e_{18}, e_{25}, e_{29}, e_{31}\ e_{35}, e_{43}, e_{46}, e_{48}$	$e_8, e_9, e_{12}, e_{16}, e_{18}, e_{25}, e_{29}, e_{31}\ e_{35}, e_{46}, e_{48}, e_{52}, e_{57}, e_{61}, e_{62}, e_{63}, e_{66}, e_{67}$
G^2	$e_1, e_{20}, e_{23}, e_{24}, e_{26}, e_{28}, e_{30}, e_{39}, e_{40}, e_{41}, e_{44}, e_{47}, e_{50}$	$e_1, e_{20}, e_{23}, e_{24}, e_{26}, e_{28}, e_{30}, e_{39}, e_{40}, e_{41}, e_{44}, e_{47}, e_{50}, e_{53}, e_{55}, e_{58}, e_{60}, e_{64}, e_{68}, e_{70}, e_{74}$
G^3	$e_4, e_5, e_6, e_{11}, e_{19}, e_{21}, e_{27}, e_{32}, e_{36}, e_{38}, e_{45}, e_{49}$	$e_4, e_5, e_6, e_{11}, e_{19}, e_{21}, e_{27}, e_{32}, e_{36}, e_{38}, e_{45}, e_{49}, e_{51}, e_{54}, e_{56}, e_{59}, e_{65}, e_{72}$
G^4	$e_2, e_3, e_7, e_{10}, e_{13}, e_{14}, e_{15}, e_{17}, e_{22}, e_{33}, e_{34}, e_{37}, e_{42}$	$e_2, e_3, e_7, e_{10}, e_{13}, e_{14}, e_{15}, e_{17}, e_{22}, e_{33}, e_{34}, e_{37}, e_{42}, e_{43}, e_{69}, e_{71}, e_{73}, e_{75}$

Remark 5. The parameter $\theta \in \mathbb{N}^{[1,g-1]}$ expresses the degree of change to apply, which can be adjusted depending on the desired degree.

To summarize the proposed consensus model, we introduce Algorithm 1.

Algorithm 1. Consensus model steps

Input: The expert preferences represented by HFLPRs, $P^i, X \times X \rightarrow S_{ll}$, the parameters α and β to compute the clusters' cohesion, the predefined consensus threshold μ, the maximum number of consensus rounds *maxrounds*, the adaptive feedback threshold σ, the acceptability threshold ε and the change degree parameter θ.

Output The overall cohesion for each sub-group c^k, the sub-groups' weights $w = (w_1, w_2, \ldots, w_n)$, the adjusted experts' preferences $\overline{P}^i, X \times X \rightarrow S_{ll}$ and the overall consensus degree cr.

For each expert $e_i, i \in \{1, 2, \ldots, m\}$, his/her preferences P^i represented by HFLPR are collected. Afterwards, the CLEs are transformed into HFLTSs and lastly the fuzzy envelopes of the latter are computed.
while $cr < \mu$ **and** $round < maxrounds$ **do**
The group of experts is divided into several sub-groups, $G^k, k \in \{1, 2, \ldots, n\}$ by using a Fuzzy c-means clustering process (Eq. (6)).
To obtain the sub-groups' weights $w = (w_1, w_2, \ldots, w_n)$, first, the matrix S^k is obtained (see Eq. (7)). Then, the cohesion for each pair of alternatives from matrix S^k is computed using Eq. (9). Afterwards, the overall cohesion of each sub-group is obtained by using Eq. (10). Finally, the sub-groups' weights are computed using Eqs. (11) and (12).
cr is derived by using the computation of the similarity matrices, SM^{it} for each pair of experts $(e_i, e_t), i < t$ (see Eq. (13)). Then, a consensus matrix, CM is obtained (see Eq. (14)). CM is used to obtain the consensus degree ca^l for each alternative x_l using Eq. (15). Finally, the overall consensus degree cr is calculated using Eq. (16).
The collective opinion, P^c, of all the experts involved in the LSGDM problem is computed with Eq. (17). Finally, a set of proximity matrices PP^k between the sub-group C^k and the collective opinion P^c are derived using Eq. (18).
  if $cr < \sigma$ **then**
    if $ca^l < cr$ **and** $p^k_{lj} \leqslant \overline{pp}^k_{lj}$ **then**
      The preferences for the pair of alternatives (x_l, x_j) should be changed for the experts belonging to the sub-group G^k.
      if $\delta(p^i_{lj}) - \delta(p^c_{lj}) < -\varepsilon$ **then**
        Increase p^i_{lj} on (x_l, x_j).
      else if $\delta(p^i_{lj}) - \delta(p^c_{lj}) > \varepsilon$ **then**
        Decrease p^i_{lj} on (x_l, x_j).
      end if
    end if
  else
    if $ca^l < cr$ **and** $1 - d_T(p^c_{lj}, p^i_{lj}) \leqslant \overline{pp}_{lj}$ **then**
      The preferences for the pair of alternatives (x_l, x_j) should be changed by the expert e_i.
      if $\delta(p^i_{lj}) - \delta(p^c_{lj}) < -\varepsilon$ **then**
        Increase p^i_{lj} on (x_l, x_j).
      else if $\delta(p^i_{lj}) - \delta(p^c_{lj}) > \varepsilon$ **then**
        Decrease p^i_{lj} on (x_l, x_j).
      end if
    end if
  end if
end while

Table 4
Cluster size, cohesion and weights for each scenario in the case with 50 experts.

50 experts		Initial			Round 1			Round 2			Round 3		
Parameters	Sub-group	$\mathfrak{c}$	γ	w	$\mathfrak{c}$	γ	w	$\mathfrak{c}$	γ	w	$\mathfrak{c}$	γ	w
					100% ACCEPTANCE								
$\alpha = 0\ \beta = 1$	G^1	-	12	0.24	-	16	0.32	-	10	0.2	-	7	0.14
	G^2	–	13	0.26	–	14	0.28	–	9	0.18	–	7	0.14
	G^3	–	12	0.24	–	9	0.18	–	17	0.34	–	18	0.36
	G^4	–	13	0.26	–	11	0.22	–	14	0.28	–	18	0.36
$\alpha = 2\ \beta = 1$	G^1	0.493	12	0.246	0.473	17	0.28	0.5	12	0.239	0.572	15	0.283
	G^2	0.499	13	0.259	0.481	16	0.276	0.537	10	0.234	0.552	10	0.223
	G^3	0.511	12	0.254	0.55	7	0.209	0.52	15	0.277	0.518	10	0.209
	G^4	0.464	13	0.241	0.515	10	0.234	0.502	13	0.25	0.574	15	0.284
					75% ACCEPTANCE								
$\alpha = 0\ \beta = 1$	G^1	–	12	0.2	–	16	0.324	–	13	0.26	–	6	0.12
	G^2	–	13	0.26	–	14	0.28	–	12	0.24	–	20	0.4
	G^3	–	12	0.24	–	12	0.24	–	16	0.32	–	21	0.42
	G^4	–	13	0.26	–	8	0.16	–	9	0.18	–	3	0.06
$\alpha = 2\ \beta = 1$	G^1	0.493	12	0.246	0.45	17	0.279	0.441	8	0.19	0.444	5	0.151
	G^2	0.499	13	0.259	0.48	14	0.27	0.484	14	0.275	0.525	16	0.319
	G^3	0.511	12	0.254	0.495	12	0.258	0.483	19	0.32	0.531	26	0.411
	G^4	0.464	13	0.241	0.485	7	0.193	0.471	9	0.215	0.454	3	0.119

Table 5
Cluster size, cohesion and weights for each scenario in the case with 75 experts.

75 experts		Initial			Round 1			Round 2			Round 3		
Parameters	Sub-group	$\mathfrak{c}$	γ	w	$\mathfrak{c}$	γ	w	$\mathfrak{c}$	γ	w	$\mathfrak{c}$	γ	w
					100% ACCEPTANCE								
$\alpha = 0\ \beta = 1$	G^1	–	18	0.24	–	23	0.307	–	14	0.187	–	26	0.347
	G^2	–	21	0.28	–	20	0.267	–	18	0.24	–	11	0.147
	G^3	–	18	0.24	–	16	0.213	–	23	0.307	–	22	0.293
	G^4	–	18	0.24	–	16	0.213	–	20	0.267	–	16	0.213
$\alpha = 2\ \beta = 1$	G^1	0.506	18	0.245	0.465	26	0.273	0.547	13	0.223	0.588	31	0.348
	G^2	0.51	21	0.266	0.49	21	0.259	0.507	18	0.243	0.562	10	0.189
	G^3	0.535	18	0.259	0.57	12	0.227	0.518	26	0.298	0.56	27	0.309
	G^4	0.477	18	0.231	0.522	16	0.24	0.493	18	0.236	0.549	7	0.154
					75% ACCEPTANCE								
$\alpha = 0\ \beta = 1$	G^1	–	18	0.24	–	25	0.333	–	13	0.173	–	20	0.267
	G^2	–	21	0.28	–	21	0.28	–	22	0.293	–	20	0.267
	G^3	–	18	0.24	–	12	0.16	–	27	0.36	–	31	0.413
	G^4	–	18	0.24	–	17	0.227	–	13	0.173	–	4	0.053
$\alpha = 2\ \beta = 1$	G^1	0.506	18	0.245	0.455	25	0.268	0.52	13	0.222	0.53	14	0.228
	G^2	0.51	21	0.266	0.477	21	0.257	0.475	26	0.286	0.495	27	0.296
	G^3	0.535	18	0.259	0.53	15	0.242	0.487	21	0.264	0.494	21	0.26
	G^4	0.477	18	0.231	0.53	14	0.233	0.497	15	0.228	0.52	13	0.216

Table 6
Consensus level for each round and scenario in the case with 50 experts.

Parameters	Initial	Round 1	Round 2	Round 3
		100% acceptance		
$\alpha = 0, \beta = 1$	0.67	0.72	0.81	0.87
$\alpha = 2, \beta = 1$	0.67	0.71	0.79	0.88
		75% acceptance		
$\alpha = 0, \beta = 1$	0.67	0.71	0.77	0.86
$\alpha = 2, \beta = 1$	0.67	0.71	0.78	0.86

Table 7
Consensus level for each round and scenario in the case with 75 experts.

Parameters	Initial	Round 1	Round 2	Round 3
		100% acceptance		
$\alpha = 0, \beta = 1$	0.66	0.73	0.81	0.91
$\alpha = 2, \beta = 1$	0.66	0.73	0.8	0.92
		75% acceptance		
$\alpha = 0, \beta = 1$	0.66	0.72	0.79	0.86
$\alpha = 2, \beta = 1$	0.66	0.72	0.79	0.86

5. Performance Study

This section introduces a performance study of the cohesion-driven model proposed in Section 4. Therefore one of the problems with two cases caused by the number of experts has been solved in several common scenarios in LSGDM (Labella et al., 2018), and is related to the experts' behavior and influence of its cohesion and size in the weight computation. The simulations are carried out by using the CRP support system, AFRYCA (Labella et al., 2017; Palomares, Estrella, et al., 2014).

We have not included a comparative analysis with the existing models that deals with HFLTS (revised in Section 2.4) because it is not possible to carry out a fair comparison, because the consensus model for LSGDM proposed by Gou et al. (2018) uses a double hierarchy HFLTS to represent the experts' preferences that is different from the proposed cohesion-driven model that uses HFLTS. The model introduced by Zhong and Xu (2020) uses HFLTS but it does not define a consensus model, rather a clustering process to try to find the maximum possible consensus by changing the clusters. And the CRP for LSGDM proposed by Ren et al. (2020) uses HFLTS to elicit experts' preferences, but the feedback process does not describe how the experts should change their preferences to increase the consensus degree in the next round and the case study introduced does not provide details about which experts should modify their preferences, nor how they should change them. It is, therefore, not possible to repeat their case study in our proposal.

5.1. Simulation scenarios description

In order to show the usefulness of the proposed model in several decision situations, we detail two LSGDM cases composed by 50 and 75 experts and 4 alternatives. Additionally, in LSGDM problems the appa-

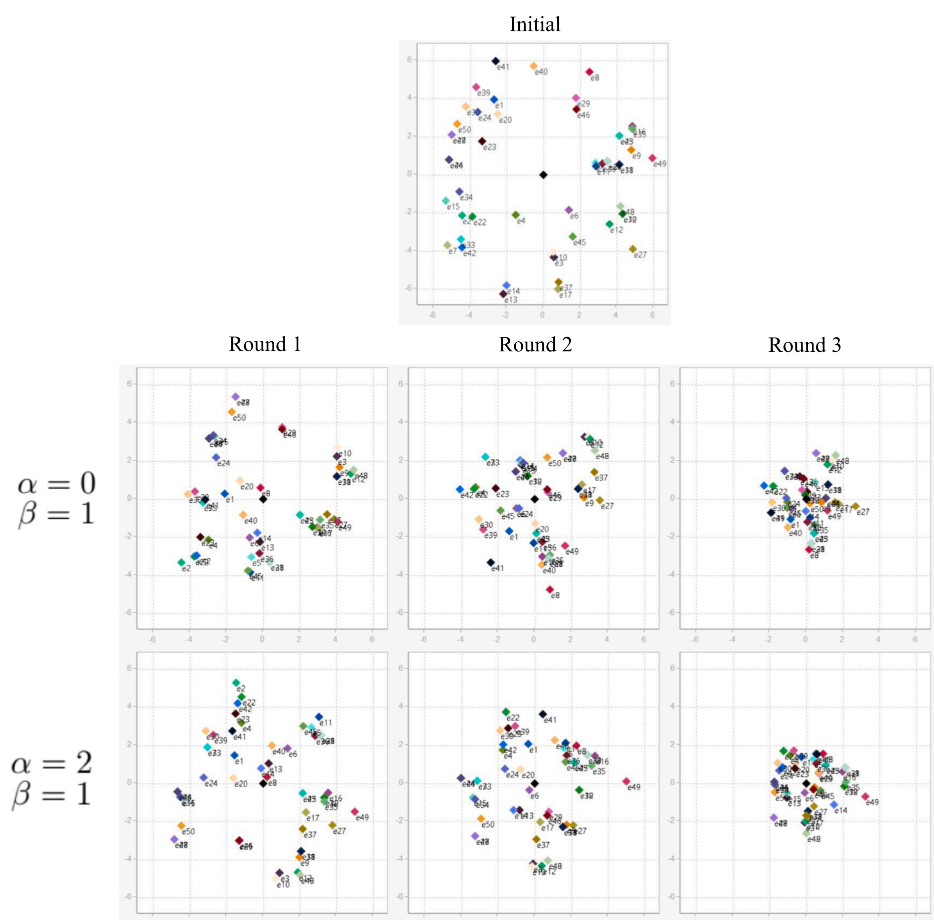

Fig. 4. MDS Visualization rounds with 100% acceptance in the case with 50 experts.

rition of different behaviors due to the participation of a large number of experts is very common, and they might initially refuse the suggestions provided by the moderator (Labella et al., 2018; Palomares, Martínez, et al., 2014). For this reason, we have considered different behavior scenarios based on the likelihood that the experts accept or reject the recommendations. Concretely, two scenarios are considered: (i) experts always accept the suggestions and (ii) 75% of experts accept recommendations. Last but not least, in order to show the importance of considering cohesion in the sub-groups' weights computation and not only the size of groups, we have derived such weights with different values of α and β for each scenario previously mentioned (see Definition 7). All the scenarios are represented in Table 1.

5.2. Resolution

The resolution process of the LSGDM cases in the different scenarios is described step by step in the following subsections according to the process introduced in Section 4 and graphically represented in Fig. 2. However, before the resolution, it is key to set the value of the parameters related to the CRP simulation. These parameters are shown in Table 2.

5.2.1. Gathering preferences

The experts compare the alternatives with each other using CLEs modelled by the linguistic term set represented in Fig. 3. Lately, HFLPRs are being built from the experts' comparisons. For the sake of space, two HFLPRs are given as examples below, the rest of the preferences are available at https://sinbad2.ujaen.es/afryca/sites/default/files/dataset/Large-scalePreferences_0.pdf.

$$e_1 = \begin{pmatrix} m & vb & vb & bt\ b\ and\ m \\ vg & m & g & vg \\ vg & b & m & bt\ g\ and\ vg \\ bt\ m\ and\ g & vb & bt\ vb\ and\ b & m \end{pmatrix}, e_2 = \begin{pmatrix} m & b & h & vb \\ g & m & vb & b \\ e & vg & m & g \\ vg & g & b & m \end{pmatrix}$$

5.2.2. Clustering process

The experts in both LSGDM cases are classified into different sub-groups according to their preferences over the alternatives by using the clustering process introduced in Section 4.2 based on the fuzzy c-means algorithm. The initial sub-groups before starting the CRP for the LSGDM cases are shown in Table 3.

Afterwards, the cohesion of each cluster is computed for each problem and scenario (100%, 75% of acceptance). In order to show the influence of the cohesion in the final results and the usefulness of the proposed linguistic cohesion measure, different values of α and β, which measure the influence of the cohesion and size in the weighting of the sub-group (see Definition 9), are considered. The cohesion ($\mathfrak{c}$), size (γ) and the resulting weights ($\boldsymbol{w}$) for each sub-group obtained from the different consensus rounds and LSGDM cases are represented in Tables 4 and 5.

In Table 4 we can appreciate the clusters evolution along the CRP. Firstly, we focus on the scenario with 100% of acceptance. If cohesion is ignored, the clusters with the same size have the same weight and, in turn, those with the largest size are considered the most important and thus, the most influential in the consensus process. However, if cohesion is taken into account, we can appreciate the differences of the weights from the initial round. On the one hand, the clusters with the same size do not have the same weight due to the cohesion influence thus, with same size, clusters whose experts have closer opinions are more highly

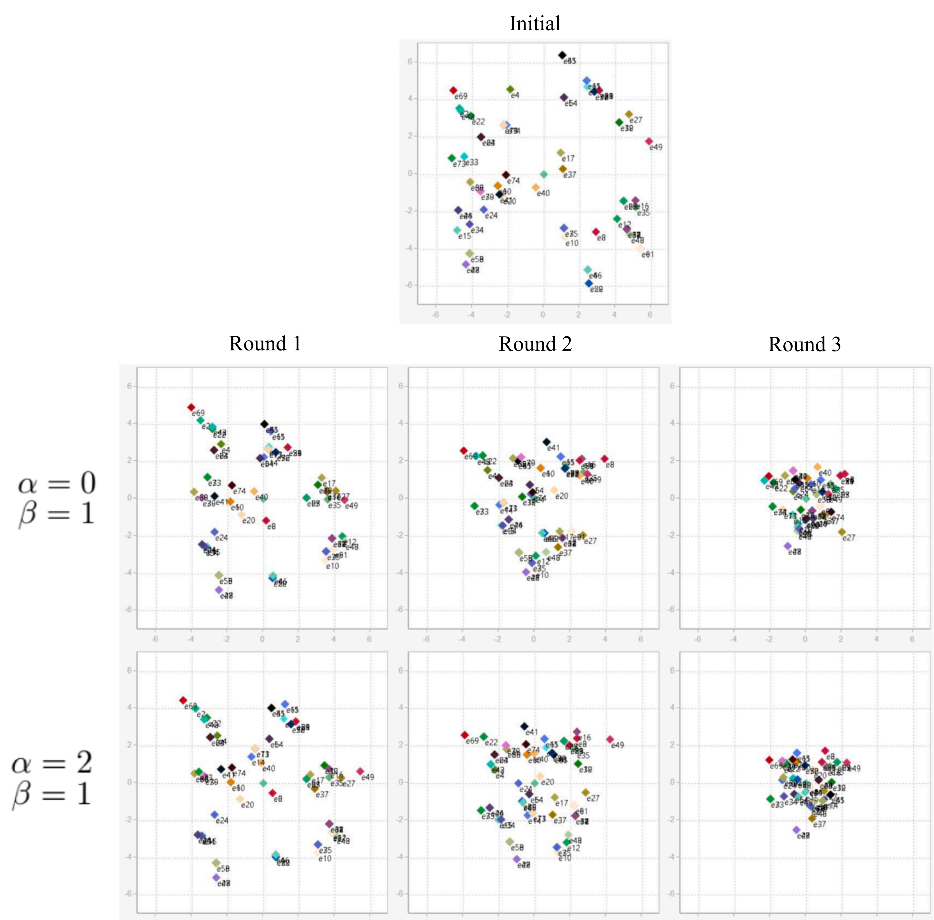

Fig. 5. MDS Visualization rounds with 100% acceptance in the case with 75 experts.

valued, which seems logical. In this way, large sub-groups with low cohesion and internal disagreements do not drive the consensus process. Additionally, smaller cluster but whose cohesion is high, are more weighted than in the first case, which guarantees that the consensus process is not only guided by those groups with the largest number of experts but means that the minority opinions are relevant too. Similar conclusions can be drawn from the scenario with 75% acceptance.

Regarding the LSGDM problem with 75 experts (see Table 5), the differences between the weights with and without cohesion are evident again. Note that, the more experts involved in solving a LSGDM problem, the more diversity of opinion. This fact makes it even clearer that experts who are part of the same cluster may not have very similar opinions and the consensus process could be influenced by large clusters in which there might be disagreement. Taking into account that hundreds or thousands of experts may participate in a LSGDM problem, considering group cohesion seems to be a fundamental step in order to avoid leading incorrect and unreliable solutions based only on the majority.

Apart from the evolution in weights in the different cases and scenarios, we can appreciate that the necessary number of rounds to reach the consensus is 3 for any combination. However, there are significant differences related to the consensus achieved and the final ranking of the alternatives that will be analyzed in the following subsections.

5.2.3. Computing the consensus degree

The computation of the consensus degree (*cr*) in the group must be identified if the desired level of consensus has been reached and thus, the CRP finishes or, on the other hand, the feedback process will need to be carried out. The consensus degree of the group obtained from the different rounds and scenarios for each problem is represented in Tables 6 and 7.

Tables 6 and 7 show clearly that considering cohesion in the weights computation does not negatively affect the consensus process in any way. On the contrary, with 100% acceptance, the final consensus achieved in the CRP is greater if cohesion is taken into account in both cases of 50 and 75 experts by obtaining a more consensual solution which is, in turn, a more reliable one, as they are not influenced by group size and possible disagreements within the sub-groups. In both cases with 75% acceptance the number of rounds necessary to achieve the desired consensus and the final consensus degree are the same. On the other hand, if we focus exclusively on the performance of the consensus model, the desired level of consensus (0.85) is achieved (and exceeded) in just 3 rounds in all the scenarios, as it was previously pointed out, in spite of the initial consensus degree being pretty low (0.67) and the non-cooperative behavior of some experts in the 75% acceptance case.

5.2.4. Adaptive feedback process

Since the consensus level alone is not enough in the initial state of the preferences, a feedback process should be carried out. The consensus model provides two different feedback processes, using groups or individual experts. Applying one or the other depends on the σ parameter, whose value is $\sigma = 0.75$ in this case study. Therefore, when $cr < \sigma$ a group feedback process is carried out, otherwise an individual feedback process starts when $cr > \sigma$. The performance evolution of the CRP over the different scenarios and cases is represented in Figs. 4–7, in which the experts' preferences are also represented using the software tool AFRYCA (Labella et al., 2017; Palomares, Estrella, et al., 2014) and its multi-dimensional scaling (MDS) representation (Kruskal & Wish, 1978).

In Figs. 4 and 5 it is very easy to identify the receptive experts'

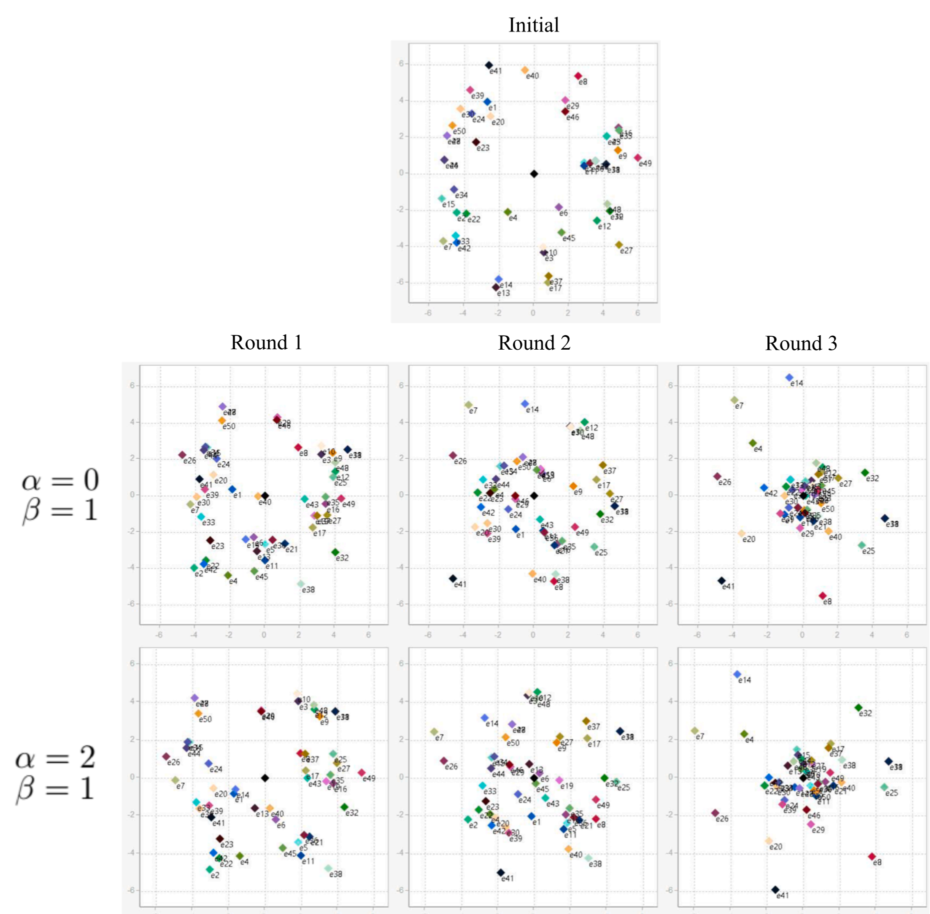

Fig. 6. MDS Visualization rounds with 75% acceptance in the case with 50 experts.

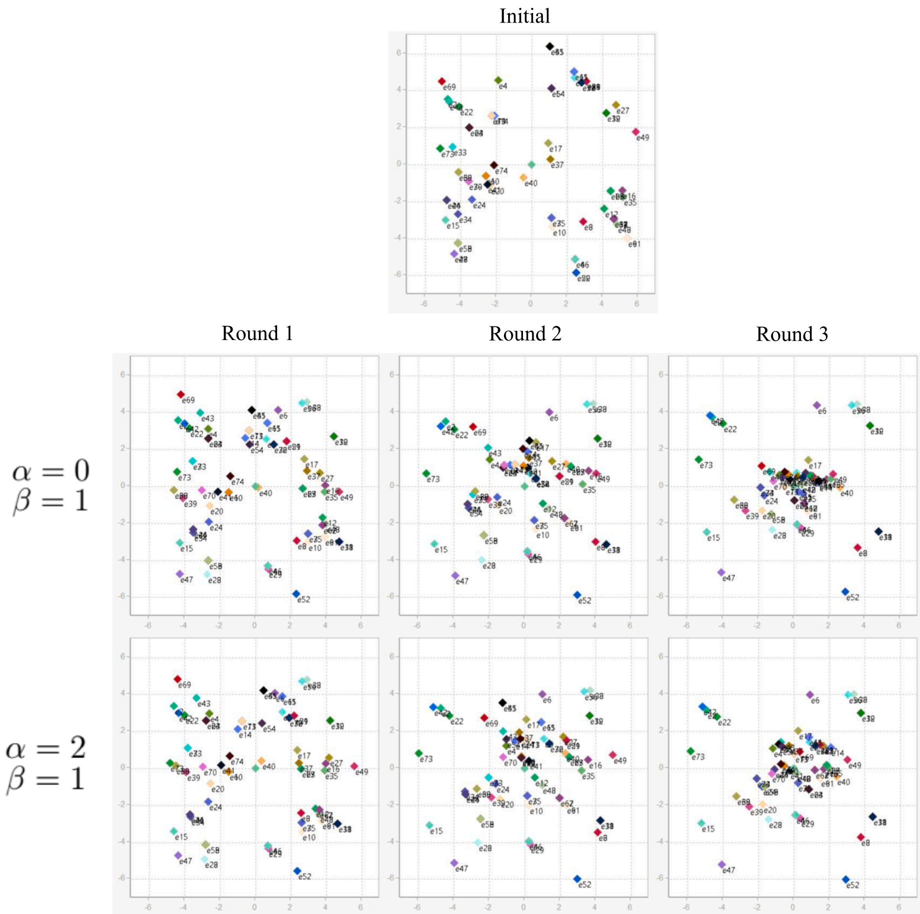

Fig. 7. MDS Visualization rounds with 75% acceptance in the case with 75 experts.

behavior, since all of them always move onto the group collective opinion through the CRP rounds according to the suggestions provided. All the experts modify their preferences in favor of consensus and none have a position that is far from the group. However, taking cohesion into account has a positive influence on the consensus process as it allows us to obtain a higher final consensus degree.

On the other hand, Figs. 6 and 7 clearly show the experts that are not receptive to the idea of changing their initial preferences. This obviously makes it difficult to reach the desired consensus, but by taking cohesion into account we can guarantee that the CRP obtains a solution in which the togetherness in the sub-groups is essential and disagreements within the groups are not ignored, otherwise, the solutions reached may not correspond to reality.

Finally, Tables 8 and 9 show the rankings obtained by following the group decision solving process presented in (Orlovsky, 1978), Depending on the influence of the cohesion in sub-groups' weights computation, the selection of the best alternative shows differences that are due to the cohesion-driven process that weights more the cohesive group opinions. Initially, the ranking is the same for all the cases, $x_3 \succ x_2 \succ x_1 \succ x_4$.

In the case with 50 experts and 100% of acceptance, if cohesion is ignored the best alternative selected is x_2, whereas with cohesion, it would be x_4. Here the difference is quite significant, since initially the worst alternative appears to be x_4 which whilst it only moves up one position in the first scenario, in the second one it reaches first place. With 75% acceptance, there are also evident differences in the rankings, x_2 is selected as the best alternative in the first case but it x_1 would be the best in the second one. Again, although initially x_1 ranks second to last in the solution obtained without cohesion, comes first in the solution provided by the cohesion-driven model.

A similar situation arises in the case with 75 experts, and although x_3 is selected as the best solution with and without cohesion (but rankings are different), with 75% acceptance the solution is different again and exactly the same situation occurs as we observed in the previous case with 75% acceptance.

Table 8
Rankings in the case with 50 experts.

Scenario	Ranking		Solution
	Initial	Final	
	100% acceptance		
$\alpha = 0, \beta = 1$	$x_3 \succ x_2 \succ x_1 \succ x_4$	$x_2 \succ x_3 \succ x_4 \succ x_1$	x_2
$\alpha = 2, \beta = 1$		$x_4 \succ x_3 \succ x_2 \succ x_1$	x_4
	75% acceptance		
$\alpha = 0, \beta = 1$	$x_3 \succ x_2 \succ x_1 \succ x_4$	$x_2 \succ x_3 \succ x_1 \succ x_4$	x_2
$\alpha = 2, \beta = 1$		$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1

Table 9
Rankings in the case with 75 experts.

Scenario	Ranking		Solution
	Initial	Final	
	100% acceptance		
$\alpha = 0, \beta = 1$	$x_3 \succ x_2 \succ x_1 \succ x_4$	$x_3 \succ x_2 \succ x_4 \succ x_1$	x_3
$\alpha = 2, \beta = 1$		$x_3 \succ x_2 \succ x_1 \succ x_4$	x_3
	75% acceptance		
$\alpha = 0, \beta = 1$	$x_3 \succ x_2 \succ x_1 \succ x_4$	$x_2 \succ x_3 \succ x_4 \succ x_1$	x_2
$\alpha = 2, \beta = 1$		$x_1 \succ x_3 \succ x_2 \succ x_4$	x_1

However, when evaluating which solution is the best or most reliable, we must take into account that the strength of the groups in the opinion formation comes mainly from their cohesion, therefore not only the majority/minority opinions are important, but also the cohesion within the group (Huang et al., 2008).

6. Conclusions

Nowadays it is necessary to deal with LSGDM problems in which scalability, uncertainty and polarization of opinions are common challenges in their resolution. A common and useful way to deal with scalability is grouping together by similar opinions the large number of experts involved in the LSGDM problem. Uncertainty in multiple real-world problems has been modelled using CLEs which in turn are modelled by HFLTs. In spite of that, the polarize opinions in large groups keeps present and majority based consensus processes are usually applied for achieving agreed solutions. However, such majority rules in large groups can still imply disagreements on the solution obtained, producing unsatisfactory results (Huang et al., 2008).

Therefore, in this paper has been studied if the inclusion of a cohesion measure in addition to the majority rule to drive the consensus in LSGDM problems defined under HFLTs frameworks can overcome the previous inadequate results. Among the different possibilities, the use of a cohesion measure based on restricted equivalence functions for HFLTSs has been proposed because it has suitable properties and achieves reasonable results. Such a cohesion measure has been then integrated into a new cohesion based CRP approach for LSGDM problems dealing CLEs. Such a new CRP approach models the influence of the sub-groups, considering the togetherness of the experts' opinions that form the sub-groups, and its size, to be able to properly weight the sub-groups opinions and obtain solutions that are more reliable. To show the usefulness of this new proposal, a complete performance study on different LSGDM cases and scenarios has been carried out.

As future research, we plan to study how to improve the flexibility of the experts' opinions and adapt the changes of the feedback process to a continuous expression domain instead of a discrete domain. Therefore, the use of ELICIT (Dutta, Labella, Rodríguez, & Martínez, 2019; Labella, Rodríguez, & Martínez, 2020) could facilitate the CRP and the precision of the results. Another interesting topic to study is the use of linguistic distribution assessments (Wu et al., 2021; Zhang, Guo, & Martínez, 2017) to model experts' preferences in CRPs for LSGDM.

CRediT authorship contribution statement

Rosa M. Rodríguez: Writing - original draft, Writing - review & editing, Investigation. **Álvaro Labella:** Software, Validation, Formal analysis, Writing - original draft, Visualization. **Mikel Sesma-Sara:** Writing - original draft, Writing - review & editing, Formal analysis. **Humberto Bustince:** Methodology, Investigation, Supervision. **Luis Martínez:** Conceptualization, Methodology, Formal analysis, Supervision, Project administration, Funding acquisition.

References

Bezdek, J. (1981). *Pattern Recognition with Fuzzy Objective Function Algorithms*. Springer.

Bustince, H., Barrenechea, E., & Pagola, M. (2006). Restricted equivalence functions. *Fuzzy Sets and Systems, 157*(17), 2333–2346.

Bustince, H., Barrenechea, E., & Pagola, M. (2007). Image thresholding using restricted equivalence functions and maximizing the measures of similarity. *Fuzzy Sets and Systems, 158*(5), 496–516.

Butler, C. T. L., & Rothstein, A. (2006). On Conflict and Consensus: A Handbook on Formal Consensus Decision Making. *Takoma Park*.

Cao, M., Wu, J., Chiclana, F., Ureña, R., & Herrera-Viedma, E. (2020). A personalized consensus feedback mechanism based on maximum harmony degree. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*. https://doi.org/10.1109/TSMC.2019.2960052

Chen, X. H., & Liu, Rong (2006). Improved clustering algorithm and its application in complex huge group decision-making. *Systems Engineering and Electronics, 28*(11), 1695–1699.

Dong, Y., Chen, X., & Herrera, F. (2015). Minimizing adjusted simple terms in the consensus reaching process with hesitant linguistic assessments in group decision making. *Information Sciences, 297*, 95–117.

Dong, Y., Li, C., & Herrera, F. (2016). Connecting the linguistic hierarchy and the numerical scale for the 2-tuple linguistic model and its use to deal with hesitant unbalanced linguistic information. *Information Sciences, 367*, 259–278.

Dong, Y., Zhao, S., Zhang, H., Chiclana, F., & Herrera-Viedma, E. (2018). A self-management mechanism for noncooperative behaviors in large-scale group consensus reaching processes. *IEEE Transactions on Fuzzy Systems, 26*(6), 3276–3288.

Dong, Q., Zhou, X., & Martínez, L. (2019). A hybrid group decision making framework for achieving agreed solutions based on stable opinions. *Information Sciences, 490*, 227–243.

Dutta, B., Labella, Á., Rodríguez, R. M., & Martínez, L. (2019). Aggregating interrelated attributes in multi-attribute decision-making with elicit information based on bonferroni mean and its variants. *International Journal of Computational Intelligence Systems, 12*, 1179–1196.

Eklund, P., Rusinowska, A., & de Swart, H. (2007). Consensus reaching in committees. *Decision Support, 178*(1), 185–193.

Eklund, P., Rusinowska, A., & de Swart, H. (2008). A consensus model of political decision-making. *Annals of Operations Research, 158*(1), 5–20.

Fodor, J., & Roubens, M. (1994). *Fuzzy Preference Modelling and Multicriteria Decision Support*. Dordrecht: Kluwer.

Gai, T., Cao, M., Cao, Q., Wu, J., Yu, G., & Zhou, M. (2020). A joint feedback strategy for consensus in large-scale group decision making under social network. *Computers & Industrial Engineering, 147*, 106626.

Gong, Z., Xu, X., Guo, W., Herrera-Viedma, E., & Cabrerizo, F. J. (2021). Minimum cost consensus modelling under various linear uncertain-constrained scenarios. *Information Fusion, 66*, 1–17.

Gou, X., Xu, Z. S., & Herrera, F. (2018). Consensus reaching process for large-scale group decision making with double hierarchy hesitant fuzzy linguistic preference relations. *Knowledge-Based Systems, 157*, 20–33.

Heilpern, S. (1997). Representation and application of fuzzy numbers. *Fuzzy Sets and Systems, 91*(2), 259–268.

Herrera, F., & Martínez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems, 8*(6), 746–752.

Herrera-Viedma, E., Alonso, S., Chiclana, F., & Herrera, F. (2007). A consensus model for group decision making with incomplete fuzzy preference relations. *IEEE Transactions on Fuzzy Systems, 15*(5), 863–877.

Herrera-Viedma, E., Cabrerizo, F. J., Kacprzyk, J., & Pedrycz, W. (2014). A review of soft consensus models in a fuzzy environment. *Information Fusion, 17*, 4–13.

Huang, G., Cao, J., Wang, G., & Qu, Y. (2008). The strength of the minority. *Physica A: Statistical Mechanics and its Applications, 387*(18), 4665–4672.

Kim, J. (2008). A model and case for supporting participatory public decision making in e-democracy. *Group Decision and Negotiation, 17*(3), 179–192.

Kruskal, J. B., & Wish, M. (1978). *Multidimensional scaling*. Springer.

Labella, Á., Estrella, F. J., & Martínez, L. (2017). Afryca 2.0: an improved analysis framework for consensus reaching processes. *Progress in Artificial Intelligence, 6*, 181–194.

Labella, Á., Liu, Y., Rodríguez, R. M., & Martínez, L. (2018). Analyzing the performance of classical consensus models in large scale group decision making: A comparative study. *Applied Soft Computing, 67*, 677–690.

Labella, Á., Rodríguez, R. M., & Martínez, L. (2020). Computing with comparative linguistic expressions and symbolic translation for decision making: Elicit information. *IEEE Transactions on Fuzzy Systems*. https://doi.org/10.1109/TFUZZ.2019.2940424

Li, C. C., Dong, Y., & Herrera, F. (2019). A consensus model for large-scale linguistic group decision making with a feedback recommendation based on clustered personalized individual semantics and opposing consensus groups. *IEEE Transactions on Fuzzy Systems, 27*(2), 221–233.

Li, C. C., Dong, Y., Herrera, F., Herrera-Viedma, E., & Martínez, L. (2017). Personalized individual semantics in computing with words for supporting linguistic group decision making. an application on consensus reaching. *Information Fusion, 33*, 29–40.

Liu, H., & Rodríguez, R. M. (2014). A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. *Information Sciences, 258*, 220–238.

Martínez, L., & Herrera, F. (2012). An overview of the 2-tuple linguistic model for computing with words in decision making: extensions, applications and challenges. *Information Sciences, 207*(1), 1–18.

McDevitt, M., & Chaffee, S. (2002). From top-down to trickle-up influence: Revisiting assumptions about the family in political socialization. *Political Communication, 19*(3), 281–301.

Morente-Molinera, J. A., Pérez, I. J., Ureña, M. R., & Herrera-Viedma, E. (2015). On multi-granular fuzzy linguistic modeling in group decision making problems: A systematic review and future trends. *Knowledge-Based Systems, 74*, 49–60.

Orlovsky, S. A. (1978). Decision-making with a fuzzy preference relation. *Fuzzy Sets and Systems, 1*(3), 155–167.

Palomares, I., Estrella, F. J., Martínez, L., & Herrera, F. (2014). Consensus under a fuzzy context: Taxonomy, analysis framework AFRYCA and experimental case of study. *Information Fusion, 20*, 252–271.

Palomares, I., Martínez, L., & Herrera, F. (2014). A consensus model to detect and manage noncooperative behaviors in large-scale group decision making. *IEEE Transactions on Fuzzy Systems, 22*(3), 516–530.

Pan, Z., & McLeod, J. M. (1991). Multilevel analysis in mass communication research. *Communication Research, 18*(2), 140–173.

Parreiras, R. O., Ekel, P. Y., Martini, J. S. C., & Palhares, R. M. (2010). A flexible consensus scheme for multicriteria group decision making under linguistic assessments. *Information Sciences, 180*(7), 1075–1089.
Quesada, F. J., Palomares, I., & Martínez, L. (2015). Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators. *Applied Soft Computing, 35*, 873–887.
Ren, R., Tang, M., & Liao, H. (2020). Managing minority opinions in micro-grid planning by a social network analysis-based large scale group decision making method with hesitant fuzzy linguistic information. *Knowledge-based Systems*, 189.
Rodríguez, R. M., Labella, Á., De Tré, G., & Martínez, L. (2018). A large scale consensus reaching process managing group hesitation. *Knowledge-Based Systems*.
Rodríguez, R. M., Labella, A., & Martínez, L. (2016). An overview on fuzzy modelling of complex linguistic preferences in decision making. *International Journal of Computational Intelligence Systems, 9*(81–94), 2016.
Rodríguez, R. M., & Martínez, L. (2013). An analysis of symbolic linguistic computing models in decision making. *International Journal of General Systems, 42*(1), 121–136.
Rodríguez, R. M., Martínez, L., & Herrera, F. (2012). Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems, 20*(1), 1109–1119.
Rodríguez, R. M., Martínez, L., & Herrera, F. (2013). A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. *Information Sciences, 241*(1), 28–42.
Roubens, M. (1997). Fuzzy sets and decision analysis. *Fuzzy Sets and Systems, 90*(2), 199–206.
Sesma-Sara, M., De Miguel, L., Pagola, M., Burusco, A., Mesiar, R., & Bustince, H. (2018). New measures for comparing matrices and their application to image processing. *Applied Mathematical Modelling, 61*, 498–520.
Sîrbu, A., Loreto, V., & Servedio, V. D. P. (2013). Opinion dynamics with disagreement and modulated information. *Journal of Statistical Physics, 151*, 218–237.
Sîrbu, A., Loreto, V., Servedio, V. D. P., & Tria, F. (2013). Cohesion, consensus and extreme information in opinion dynamics. *Advances in Complex Systems, 16*(06), 1350035.
Song, Y., & Li, G. (2019). Consensus constructing in large-scale group decision making with multi-granular probabilistic 2-tuple fuzzy linguistic preference relations. *IEEE Access, 7*, 56947–56959.
Sueur, C., Deneubourg, J. L., & Petit, O. (2012). From social network (centralized vs. decentralized) to collective decision-making (unshared vs. shared consensus). *PLoS One, 7*(2), e32566.
Tang, M., Zhou, X., Liao, H., Xu, J., Fujita, H., & Herrera, F. (2019). Ordinal consensus measure with objective threshold for heterogeneous large-scale group decision making. *Knowledge-Based Systems, 180*, 62–74.
Walker, S. J. (2014). Big data: A revolution that will transform how we live, work, and think. *International Journal of Advertising, 33*(1), 181–183.
Wang, L., Rodríguez, R. M., & Wang, Y. M. (2018). A dynamic multi-attribute group emergency decision making method considering experts hesitation. *International Journal of Computational Intelligence Systems, 11*, 163–182.
Wang, P., Xu, X., & Huang, S. (2019). An Improved Consensus-Based Model for Large Group Decision Making Problems Considering Experts with Linguistic Weighted Information. *Group Decision and Negotiation, 28*(3), 619–640.
Wu, J., Cao, M., Chiclana, F., Dong, Y., & Herrera-Viedma, E. (2020). An optimal feedback model to prevent manipulation behaviours in consensus under social network group decision making. *IEEE Transactions on Fuzzy Systems*. https://doi.org/10.1109/TFUZZ.2020.2985331
Wu, T., Liu, X., & Qin, J. (2018). A linguistic solution for double large-scale group decision-making in e-commerce. *Computers & Industrial Engineering, 116*, 97–112.
Wu, Z., & Xu, J. (2016a). An interactive consensus reaching model for decision making under hesitation linguistic environment. *Journal of Intelligent & Fuzzy Systems, 31*(3), 1635–1644.
Wu, Z., & Xu, J. (2016b). Managing consistency and consensus in group decision making with hesitant fuzzy linguistic preference relations. *Omega-International Journal of Management Science, 65*, 28–40.
Wu, Z., & Xu, J. (2018). A consensus model for large-scale group decision making with hesitant fuzzy information and changeable clusters. *Information Fusion, 41*, 217–231.
Wu, Y., Zhang, Z., Kou, G., Zhang, H., Chao, X., Li, C. C., Dong, Y., & Herrera, F. (2021). Distributed linguistic representations in decision making: Taxonomy, key elements and applications, and challenges in data science and explainable artificial intelligence. *Information Fusion, 65*, 165–178.
Xiao, J., Wang, X., & Zhang, H. (2020). Managing personalized individual semantics and consensus in linguistic distribution large-scale group decision making. *Information Fusion, 53*, 20–34.
Xu, Y. J., Wen, X., Sun, H., & Wang, H. (2018). Consistency and consensus models with local adjustment strategy for hesitant fuzzy linguistic preference relations. *International Journal of Fuzzy Systems, 20*(7SI), 2216–2233.
Yager, R. R. (1999). Approximate reasoning as a basis for computing with words. In *Computing with words and information/intelligent systems 2: applications* (pp. 50–77). Physica Verlag.
Yager, R. R. (2004). On the retranslation process in Zadeh's paradigm of computing with words. *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics), 34*(2), 1184–1195.
Yu, W., Zhang, Z., & Zhong, Q. (2019). Consensus reaching for MAGDM with multi-granular hesitant fuzzy linguistic term sets: a minimum adjustment-based approach. *Annals of Operations Research*.
Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338–353.
Zhang, H., Dong, Y., & Herrera-Viedma, E. (2018). Consensus building for the heterogeneous large-scale gdm with the individual concerns and satisfactions. *IEEE Transactions on Fuzzy Systems, 26*(2), 884–898.
Zhang, Z., Gao, Y., & Li, Z. (2020). Consensus reaching for social network group decision making by considering leadership and bounded confidence. *Knowledge-Based Systems, 204*, 106240.
Zhang, Z., Guo, C., & Martínez, L. (2017). Managing multigranular linguistic distribution assessments in large-scale multiattribute group decision making. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 47*(11), 3063–3076.
Zhang, F., Ignatius, J., Zhao, Y., Peng Lim, C., Ghasemi, M., & Sang Ng, P. (2015). An improved consensus-based group decision making model with heterogeneous information. *Applied Soft Computing, 35*, 850–863.
Zhang, B., Liang, H., & Zhang, G. (2018). Reaching a consensus with minimum adjustment in MAGDM with hesitant fuzzy linguistic term sets. *Information Fusion, 42*, 12–23.
Zhang, Z., Yu, W., Martínez, L., & Gao, Y. (2020). Managing multigranular unbalanced hesitant fuzzy linguistic information in multiattribute large-scale group decision making: A linguistic distribution-based approach. *IEEE Transactions on Fuzzy Systems, 28*, 2875–2889.
Zhong, X., & Xu, X. (2020). Clustering-based method for large group decision making with hesitant fuzzy linguistic information: Integrating correlation and consensus. *Applied Soft Computing, 87*.
Zhu, B., & Xu, Z. (2014). Consistency measures for hesitant fuzzy linguistic preference relations. *IEEE Transactions on Fuzzy Systems, 22*(1), 35–45.