



# Mechanisms and evidence of prospective teachers' learning through enquiry-oriented practices: the case of a lesson study intervention

F. J. García<sup>1</sup> · E. M. Lendínez<sup>1</sup> · A. M. Lerma<sup>1</sup> · A. M. Abril<sup>1</sup>

Accepted: 30 July 2024 / Published online: 15 August 2024  
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## Abstract

Existing research has highlighted the key role of observation and reflection in teacher learning during lesson study interventions, and how challenging it can be for pre-service teachers. This study focuses on the post-lesson discussion phase and on the observation of the research lesson of a lesson study process. It is carried out with early childhood education pre-service teachers who studied the Theory of Didactical Situations. The aim of our research is two-fold: finding evidence of the activation of pre-service teacher knowledge during the post-lesson discussion, and identifying mechanisms that may explain said activation as well as the potential development of their professional knowledge. To model pre-service teacher knowledge, the notion of teachers' praxeological equipment from the Anthropological Theory of the Didactic, specified in the Theory of Didactical Situations, is used. It is hypothesised that potential teacher learning is mainly connected to the identification of particularly relevant episodes during the observation phase, henceforth referred to as critical events. The analysis of a group discussion on three critical events shows the activation and development of mathematical and didactic components of pre-service teachers' praxeological equipment. However, it is found that it depends on the degree of dissonance these events can provoke, and on the quality and depth of the discussion that takes place around them. It is therefore concluded that the identification of critical events followed by productive discussion is a mechanism that explains teacher learning through lesson study, which can be enhanced by the intervention of teacher educators as mediators.

**Keywords** Lesson study · Initial teacher education · Theory of Didactical situations · Teachers' praxeological equipment · Post-lesson discussion

## 1 Introduction

Lesson study (LS) is a practice-based, research-oriented, student-focused, collaborative model of teacher professional development (Huang et al., 2019; Murata, 2011). It has been gaining popularity as a professional practice and as a research topic for the past 25 years, mainly since the publication of Stigler and Hiebert (1999). Their book contributed to spreading both LS and how Japanese teachers conceive the teaching and learning of mathematics.

There are multiple interpretations of LS in research that could lead to different "models" (see, for instance, Takahashi & Wake, 2024, or Dudley, 2019). Given the diversity,

Seleznyov's (2018) literature review summarises what the critical components of Japanese LS are: (1) identifying the focus, which leads to formulating the research theme; (2) collaborative planning based on the study of material relevant to the research theme, which gives rise to a detailed written plan for a research lesson; (3) the research lesson, taught by a member of the group, and observed by the other members who collect evidence of pupil learning; (4) a post-lesson discussion, to talk about evidence gathered in relation to the research theme, and that could inform subsequent cycles; (5) repeated cycles of research that draw on the findings from the post-lesson discussion, and should not be interpreted as revisions or re-teachings of previous research lessons; (6) input from an external expert regarding the planning process and the research lesson; and (7) mobilising knowledge through disseminating and sharing findings.

Research has highlighted several benefits associated with LS. In Japan, Murata (2011) shows that LS has been used

✉ F. J. García  
fjgarcia@ujaen.es

<sup>1</sup> Universidad de Jaén, Jaén, Spain

to improve teaching; to examine and better understand new educational approaches, curriculum content and instructional sequences; to make teaching approaches more practical and understandable; to enable teachers to develop a deeper understanding of content and student thinking; and to help teachers to connect theory and practice. Outside Japan, Lewis et al. (2019) argue that LS can influence instruction and student learning through changes in teachers' knowledge and beliefs, professional norms and routines, and instructional materials. Seleznyov (2019) reveals that studies show positive changes in teachers' professional learning (subject knowledge, pedagogical content knowledge, or teacher confidence) and changes in teachers' practice.

LS has also been adopted in the education of pre-service teachers (PSTs), as part of methods courses, or in other training activities (Shinno & Yanagimoto, 2023). PST learning can take place in and from the process of engaging in LS (Larssen et al., 2018), leading to an increase in PSTs' subject matter knowledge and pedagogical skills (Baumfield et al., 2022). LS could trigger the activation of knowledge that PSTs learnt in pedagogical courses, but that, to a certain extent, remained inert (Leavy & Hourigan, 2016), contributing also to developing new mathematical and pedagogical knowledge (Hourigan & Leavy, 2019; Murata & Pothen, 2011; Ni Shuilleabhain & Clivaz, 2017). The actual implementation of the research lesson gives authenticity to the process (Lewis, 2019), contributing to reducing the theory-practice divide (Baumfield et al., 2022; Leavy & Hourigan, 2016; Murata & Pothen, 2011).

This paper is based on an LS experience with early childhood education PSTs that includes Seleznyov's (2018) seven critical components. It is linked to a methods course the PSTs took previously. In this course, they gained professional knowledge about the Theory of Didactical Situations (TDS, Brousseau, 1997). Since LS is a complex practice, this study mainly focuses on the post-lesson discussion phase, and on what was noticed while observing the research lesson.

## 2 Literature review of lesson study with pre-service teachers

What differentiates LS from other professional development programmes is the live research lesson (Murata, 2011; Lewis, 2019) as a bridge between the enquiry and planning phases, and the observation and discussion phases. As mentioned earlier, this research focuses on the post-lesson discussion and on the observation of the research lesson, which are key levers of PST learning. Issues like PSTs' lack of experience with children, their poor knowledge of the curriculum (Lewis, 2019), or their unfamiliarity with LS (Rasmussen, 2016) can hinder thorough observation and

productive discussion, jeopardising PST learning. Larssen's et al. (2018) literature review of LS in initial teacher education suggests that the observation process in LS deserves further research. The connection between observation and the post-lesson discussion also requires more attention. Existing research shows how challenging these processes can be for PSTs.

Leavy and Hourigan (2016) reported growth in PSTs' pedagogical content knowledge. They also identified particular features of LS that supported it, such as PSTs' engagement in all aspects of teacher noticing, a shift in the focus from teacher's actions to children's learning, and a focus on observation. They argue that LS provides a structure that supports PSTs in noticing noteworthy events. They also maintain that group dialogue and reflection during the post-lesson discussion, with the support of the mentor, allows PSTs to start reasoning about those events, giving rise to making informed teaching decisions. Nonetheless, PSTs might need extra support due to their lack of knowledge and/or experience (Hourigan & Leavy, 2019; Lewis, 2019).

Sims and Walsh (2009) analysed two LS cycles with PSTs. In the first one, the lesson was collectively designed, but taught individually by each PST in a different class. Instead of live observation, the PSTs watched an edited version of the video recording of one of the lessons. It was observed that the post-lesson mainly focused on evaluating the PST who taught the lesson. Although specific segments of the research lesson considered to be particularly relevant to spark discussion were included in the video, and despite the efforts of the facilitator, the discussion remained superficial.

Similarly, Bjuland and Mosvold (2015) found that PSTs had difficulties in planning the observation of the research lesson, and did not seem to make any focused observations. It was seen that discussion often drifted towards practical issues concerning the teacher's actions in the classroom. Baldry and Foster (2019) hence consider that clear prior direction is necessary to ensure that appropriate data are collected during the observation to enable productive post-lesson discussion. PSTs might require considerable coaching with regard to how to make such observations about students' behaviour to avoid delivering merely generalised judgements. The role of university instructors and/or school-based mentors as external experts is acknowledged to be crucial. Using observation and/or discussion protocols has also been found helpful to allow productive discussions.

This is what happened in Sims and Walsh's (2009) second LS cycle, in which PSTs observed the research lesson live. The authors prepared an "observation and evidence worksheet" for the PSTs, and developed the questions and prompts the facilitator would use in the post-lesson discussion in advance. The facilitator also adopted a more active role during the debriefing of the lesson. All these adaptations

led to more focused discussions, contributing to the development of PSTs' ability to observe and gather actual evidence about learning.

Likewise, Murata and Pothen (2011) asked PSTs to collect data specified in the lesson plan materials, centring discussion on those data. However, at times, discussion diverged from student learning to other issues. Ni Shuil-leabhain and Bjuland (2019) explicitly encouraged PSTs to carefully consider what evidence they would need to evaluate pupils' learning. As a result, the PSTs centred their reflections on the abovementioned observations instead of on analysing the lessons in terms of a smooth implementation of the activities. Nonetheless, the authors found the analysis of the research lesson during the collective reflection was superficial, and the PSTs did not recognise the complexities of analysing pupil learning.

All these studies show the crucial role of observation and reflection in PST learning within LS. They also highlight the complexity of meaningful observation and productive discussion. As shown by the studies described above, advances in this field have been made. Yet, studies rarely consider and discuss the role of PSTs' previously acquired knowledge within the observation and post-lesson discussion phases. Besides, research at a micro-level, aiming at a deeper understanding of the interactions that take place during observation and discussion, is scarce. This paper seeks to address these gaps by incorporating into the analysis of LS the knowledge that the PSTs had previously acquired and, building on this, trying to identify and explain some of the mechanisms that could justify why LS leads to teacher learning.

### 3 Theoretical framework and research questions

In this section, some notions of two theoretical frameworks will be presented. The first one, as a model of teacher knowledge. The second one, as the content of such knowledge.

#### 3.1 The praxeological equipment as a model of teacher knowledge

The Anthropological Theory of the Didactic (ATD) is built around the notion of praxeology as a unifying model of any human activity. Praxeologies consist of four components: the *praxis* part includes the *type of tasks* and the *techniques*, whereas the *logos* part includes *technologies* (describing, explaining, and justifying the technique(s) and *theories* (Bosch et al., 2020).

Praxeologies can model the mathematical activity in terms of the type of mathematical tasks and techniques, and the discourses that could explain and justify them. However,

they can also model the teaching activity, considering the type of tasks related to teaching, the didactic techniques (pedagogical gestures) the teacher mobilises, and the knowledge that explains and justifies teaching actions.

The ATD postulates a mutual dependence between what teachers teach (mathematical praxeologies) and how they teach it (didactic praxeologies). To be effective, teachers should possess and bring into play a complex, wide, and interconnected body of mathematical and didactic praxeologies, called praxeological equipment, that constitute their professional knowledge (Bosch & Gascón, 2009).

Considering LS can be broken down into different types of professional tasks for analytical purposes (García et al., 2019), the activity of PSTs can be studied in terms of the components of the praxeological equipment they mobilise. That is, from the mathematical and didactic techniques they activate to address such tasks, and the technological and theoretical elements that might justify both their mathematical and pedagogical work. PST learning can hence be followed up from the activation and evolution of the different components of their praxeological equipment.

#### 3.2 A teaching approach based on the Theory of Didactical Situations

García et al. (2019) and Lendínez et al. (2024) maintain that the way in which teachers cope with types of tasks specific to LS depends on the nature of the knowledge mobilised and shared within the LS group. In this paper, the teaching framework, which constitutes the PSTs' praxeological equipment, is based on the TDS. Some components of this equipment, called TDS-based praxeological equipment, are outlined in this section.

As a constructivist theory, in the TDS, student learning is interpreted as the result of their adaptation to a situation designed with a didactical purpose (Brousseau, 1997). In a broad sense, a situation is made up of the milieu, a problem or challenge within it, and the kind of interactions (strategies) students can establish with this milieu (Sadovsky, 2004). The knowledge to be learnt appears as the best approach students can use under the conditions of the situation, referred to as the optimal strategy.

Moreover, a situation is considered a-didactical if the teacher aims to teach something hidden to the students, hence generating the students' "most independent and fruitful interaction possible" (Brousseau, 1997, p. 30) with the milieu. The TDS postulates that "each item of knowledge can be characterised by a (or some) a-didactical situation(s) which preserve(s) meaning" (Brousseau, 1997, p. 30), captured in the notion of a fundamental situation of given knowledge. The progressive adaptation of students to a learning milieu, on their way towards the learning of a mathematical concept, is mediated by the choices the teacher

makes regarding key features of the milieu itself and by the rules that regulate the students' possible interactions with it. These are called didactical variables (Brousseau, 1997).

To organise teaching in accordance with this model, the teacher's praxeological equipment should, at least, include knowledge about the type of situations and conditions of a learning milieu that could potentially encourage students to construct a certain mathematical topic. It should contain knowledge of the fundamental situation related to this topic, the didactical variables the teacher could modify to present different learning situations to the students, and the possible adaptations (in terms of mathematical approaches) the students could make to them. This praxeological equipment combines both mathematical and didactic praxeologies, since it is simultaneously related to the students' mathematical activity (type of mathematical tasks they face and mathematical techniques they use) and the design of situations that have a didactic purpose.

The teacher also needs to know how to manage these situations in the classroom in an efficient manner. The TDS highlights two challenges (didactic tasks) the teacher faces (Sadovsky, 2004): making the student accept responsibility for a learning situation (known as the devolution of a situation), and giving mathematical value to students' actions (called the institutionalisation of knowledge). In the former, the teacher organises the milieu so that the students accept the challenge posed by the situation, and engage with it in a productive and autonomous way under the conditions and rules set out by the teacher. In the latter, the teacher takes notes of the students' progression, of their questions and answers, distinguishing the wrong ones from the correct ones, and connecting the students' contextual knowledge to reference mathematical knowledge.

To do so, the teacher should be equipped with didactic praxeologies related to ways of organising successful devolutions of the situations and meaningful institutionalisations of the knowledge built by the students. This dimension of the teacher's praxeological equipment is closely related to the mathematical one, since devolution and institutionalisation processes are strongly linked to the knowledge to be taught and to the corresponding fundamental situation.

However, in practice, teaching TDS-based situations might challenge teachers to use other types of didactic techniques that are not explicitly described in the theoretical model. While trying to keep the essence of the model, based on student autonomy and self-regulation, teachers might feel the need to guide students whenever necessary to let them experience a successful adaptation to the situation.

### 3.3 The research questions

This study focuses on a university-based LS intervention with early childhood education PSTs in their fourth and

final year of their bachelor's degree. The TDS was explicitly used as the teaching framework within the LS experience. In Lendínez et al. (2024), an increase in their self-efficacy to plan and teach TDS-based lessons was already reported. The research here presented seeks to find evidence of PST learning that might have supported this increase. It also attempts to understand what makes LS produce such learning.

Given the extension and complexity of LS processes, this research mainly focuses on the post-lesson discussion phase, for which there is limited research (Shimizu & Kang, 2022). The following research questions are considered:

- RQ1: What elements of the TDS are activated during the post-lesson discussion phase of the LS intervention, and what role do they play in both noticing and commenting critical episodes?
- RQ2: What are the mechanisms in LS that could explain said activation and development during the post-lesson discussion?

## 4 Method

### 4.1 Context and participants

The LS intervention took place in the Bachelor's Degree in early childhood education at the University of Jaén (Spain) during the second semester of academic year 2017–2018 with 47 PSTs. A total of 8 groups of 5–6 PSTs per group were formed. The participants were not selected intentionally. The LS process was embedded in an optional course devoted to the teaching and learning of mathematics in early childhood education in the fourth year of the PSTs' training. The PSTs enrolled in the course voluntarily without knowing that an LS approach was going to be used and without any previous experience in LS.

In the third year of their degree, the PSTs took a compulsory methods course organised around the TDS as a framework for the teaching of mathematics in early childhood education (ages 3–6 years). The course included both learning about the TDS itself and its application to understand the teaching and learning of mathematical topics in the early years. The PSTs had access to examples of didactic situations, such as narratives, pupils' written productions, and short video clips of pupils in real classrooms. The PSTs engaged in didactic analysis practices from the perspective of the TDS. It is assumed that the course contributed to an initial development of PSTs' TDS-based praxeological equipment with a theoretical bias, since they did not have the opportunity to engage in actual teaching practices.

This paper considers the experience of one group of six PSTs that focused on 3-year-old pupils' learning of the cardinal value of numbers.

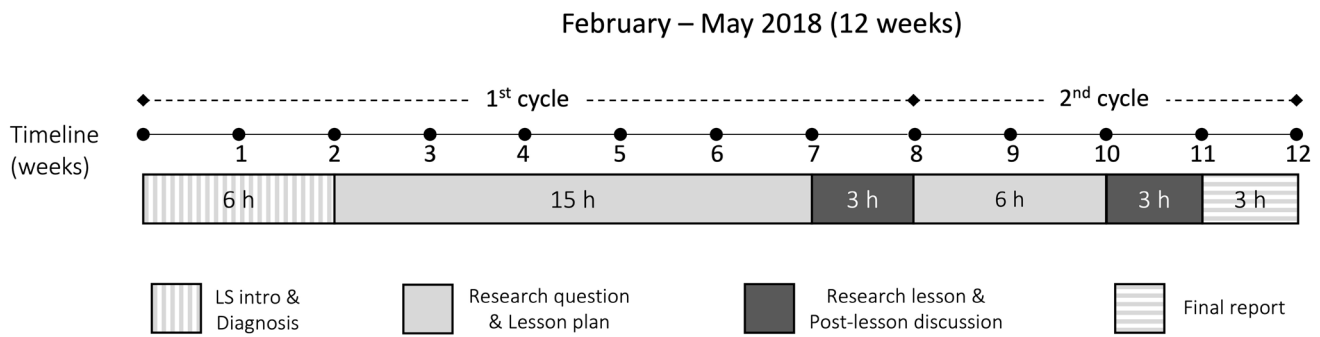


Fig. 1 Structure of the LS intervention with early childhood education PSTs (Lendínez et al. 2024)

## 4.2 The lesson study intervention

The LS process lasted 12 weeks and the PSTs worked for 36 h (see Fig. 1) to complete two whole LS cycles. The authors attended the sessions in turns, collaborating as university tutors, external experts, and also as observers. The second author was the instructor of the course. The research lesson and post-lesson discussion sessions took place in early childhood schools, while the rest of the sessions took place at the university.

The course instructor made it clear that the PSTs' work had to be based on the contents of the methods course about the teaching and learning of mathematics from the perspective of the TDS. The LS, which is widely described in Lendínez et al. (2024), included the seven components that Seleznyov (2018) identified as crucial to the success of Japanese LS.

## 4.3 Data collection and analysis

The progress reports of the different groups' planning sessions, templates of their lesson plans, final reports after each cycle, and observation notes of the research lesson were collected throughout the LS intervention as evidence. Besides, the research lesson was video-recorded and the post-lesson discussion audio-recorded. The authors of the paper observed the research lesson in situ and participated, as experts, in the post-lesson discussion.

Building on the theoretical framework explained above, a qualitative analysis restricted to the post-lesson discussion phase was performed. However, since the discussion in this phase was related to what happened in the research lesson, it had to be considered as well.

To structure the analysis, it is considered that, amongst all the events that take place during the research lesson, some of them could be labelled as critical, in the sense that they could be powerful drivers for teacher learning. Following Mynott's (2019) LS outcome model, critical events are understood as any kind of event that happens throughout an

LS process, and that might be relevant as long as they create some kind of dissonance within the group, which might challenge participants' knowledge. Specific components of PSTs' TDS-based praxeological equipment are used as an analytical tool.

The next section starts by briefly describing the background of the group to explain the research question and the lesson designed by the group. The data of the research lesson are then analysed to try to identify possible critical events. Their selection is mainly based on the potential we, as experts, anticipated in some of the episodes during the research lesson to create dissonance in the PSTs' praxeological equipment. However, as Mynott (2019) points out, this does not necessarily mean it leads to PST learning. Three critical events were therefore analysed in detail, following these steps: (1) providing a brief description of each event, (2) discussing, as experts, the possibilities foreseen in each of them to promote the development of the PSTs' TDS-based praxeological equipment, (3) reviewing the data from the post-lesson discussion to see if the event was considered in the post-lesson discussion, who encouraged it, and the kind of debates it generated, and (4) based on the evidence gathered from the post-lesson debate, analysing if it is possible to conclude that the PSTs' praxeological equipment has evolved.

## 5 Results

### 5.1 Brief background of the group: research question and lesson plan

#### 5.1.1 LS cycle 1

To formulate the research question, the PSTs revisited the fundamental situation for the learning of cardinal numbers. They decided to focus on the relationship between the notion of cardinality and numerals. To gain an initial understanding, they designed some activities to enquire into

**Fig. 2** Materials created by the PSTs for the learning milieu of the different situations they proposed in their lesson plan



the connections that their pupils would be able to establish between small collections of objects and numerals. They found that the pupils experienced difficulties when they had to express the quantity of a small collection (between 4 and 6 objects) in written form. According to the fundamental situation, the PSTs already knew that the cardinal value of numbers emerges from situations in which pupils observe a first collection of objects, and have to create/select a second one that is equal in quantity to the first one. This needs to happen in a situation in which they cannot see the first collection when they are creating/choosing the second one, which is a key didactic variable to the learning of numbers in this stage. The rationale and meaning of numerals emerge when pupils need to ask someone for the second collection, instead of creating it themselves (Sierra et al., 2012).

The PSTs hence came up with the following research question: *what situations should we generate in the classroom for children to understand the functionality of written numerals?*

The PSTs then reviewed the materials and developed the lesson plan. They decided that the learning milieu would consist of puppies, as a collection the pupils would observe, and bones, as a collection the pupils would have to create (Fig. 2). The pupils' mathematical activity would revolve around the problem of getting one bone for each puppy (from a nearby box containing a lot of bones). The future teachers revisited their materials from the methods course to enquire into how the milieu could be structured (management of the didactic variables in terms of the TDS) and possible pupil strategies. This led to the writing of the lesson plan, which included three situations depending on different arrangements of the milieu:

- situation 1: the box with the bones is placed close to the collection of six puppies.
- situation 2: the box with the bones is placed at a distance from the collection of six puppies (visual control was blocked). The pupils can only look at the collection of puppies once. They are not allowed to go back and forth between the two collections.
- situation 3: the pupils ask for the bones they need for a collection of six puppies, using pencil and paper. They have to observe the collection of puppies, and then write down their written message at a certain distance from the objects (visual control remained blocked).

Both the PSTs' creation of the learning milieu and the choices they made to propose different learning situations clearly show certain expertise in the purposeful design of teaching situations in accordance with the TDS teaching model. This means important elements of said model had already been incorporated into their praxeological equipment.

In the lesson plan, the PSTs also wrote down possible techniques the pupils could use to get the bones in different situations (one-to-one correspondence, counting, subitising, guessing, randomly). The future teachers relied on their previous knowledge and review of the materials used in the methods course. This proves that their praxeological equipment also includes knowledge about the pupils' approaches to adapt to a learning milieu designed in accordance with the fundamental situation, and that may vary because of changes in certain didactic variables.

Given the relevance within the TDS framework of pupils' self-evaluation of their approaches (a-didactical nature of the situation), the PSTs anticipated how the milieu would inform the pupils, and how the teacher could act to direct the pupils' attention to the feedback from the milieu. However, the PSTs did not include any recommendations as to how the teacher should react to certain approaches used by the pupils, although they knew unusual ones could emerge.

From the statements the PSTs wrote in their progress reports, it is observed they activated components of their TDS-based praxeological equipment concerning how an a-didactical situation should work, as well as possible teacher actions consistent with the TDS model so as to avoid weakening the a-didactical nature of the situation.

However, other decisions were not explicitly supported. Some were even implicit. In general, it seems the PSTs' TDS-based praxeological equipment is still under construction and revision. Their work showed they were at times strongly dependent on the materials used in the methods course. Occasionally, they even required our support, as university instructors, to feel confident about some of their choices.

The PSTs then went to a school to teach the research lesson to early childhood education pupils. The PST that taught the lesson followed the lesson plan, although she introduced some changes that will be discussed later. The other PSTs and the authors of this paper observed the lesson, which lasted for approximately 60 min. A detailed description of



**Fig. 3** Pupils writing down how many syringes they need (LS cycle 2, situation 3)

what happened in the research lesson is beyond the scope of this paper. Only some critical events that took place during the lesson will be discussed.

In the final reflection that closed this first LS cycle, the PSTs concluded they did not have enough evidence to answer their research question. Therefore, they decided to keep it for the second cycle, and discussed possible adaptations to the lesson plan. This linked LS cycle 1 to LS cycle 2.

### 5.1.2 LS cycle 2

Throughout the first cycle, the PSTs realised the pupils could not handle the size of the collection of puppies, which hindered the prospective teachers' enquiry into the kind of written representations the pupils were capable of using. The PSTs proposed adjusting some didactic variables by reducing the collection to 4 puppies in situations 2 and 3, and by making the pupils write their messages only a short distance away from the puppies (Fig. 3). They also decided that the new situation would resemble situation 1, but changing the bones for syringes to vaccinate the puppies.

Again, they activated components of their TDS-based praxeological equipment to make decisions in context.

Their progress reports showed that having the pupils write down their messages in close proximity to the puppies would allow even those pupils experiencing difficulties in quantifying the whole collection to use an alternative approach. Said approach is based on one-to-one correspondence between the puppies and iconic representations. These changes are not only an expression of their TDS-based praxeological equipment, but are also consistent with the research question guiding the LS process.

However, as in the first cycle, the PSTs did not anticipate the possible reactions of the teacher to the pupils' approaches. Still, they knew from the previous cycle that unusual and challenging approaches could arise. As before, this is interpreted as a sign their praxeological equipment is still being developed, and that it is not extensive enough to address all the issues that might arise when creating a TDS-based teaching situation.

The research lesson was taught by a different PST, and observed by the other PSTs and the authors of the paper, who also participated in the post-lesson discussion.

## 5.2 Critical event 1 in LS cycle 1: teacher-pupil interactions to determine the validity of the pupils' approaches

*Brief description:* following the lesson plan, after each pupil provided a solution to the situation (in this case, they observed a collection of puppies and picked some bones), they had the opportunity to check if their answer was right (by giving each puppy a bone). The teacher asked them questions like: “do all the puppies have a bone?”, “how did you do it?”, “what did you do when you went to the box to pick the bones?”, “can you tell your classmates?”

*Potential of the event to foster PST learning:* it was observed that the PST who acted as a teacher<sup>1</sup> employed a didactic technique asking the pupils questions to increase the feedback regarding the milieu, and the pupils' awareness of the information they were getting from it. Discussing possible teacher actions in this situation, or their impact on the pupils' learning, could contribute to broadening the PST's TDS-based praxeological equipment with regard to certain components of their didactic praxeologies. It particularly concerns the features of an a-didactical situation, the student-milieu interaction, and how the teacher could intervene to ensure this interaction was meaningful to the pupils, while keeping it as autonomous as possible. The PSTs would be given the opportunity to reflect on the type of didactic techniques a teacher could use to engage pupils in the most fruitful interaction possible within an a-didactical situation.

*Event in the post-lesson discussion:* the event seemed to have caused some dissonance on the teacher's side, since she decided to take it to the post-lesson discussion. She was aware she had used a didactic technique, but seemed not to be fully satisfied with the results obtained. The following excerpt, from the post-lesson discussion, summarises her concerns:

Teacher: I asked them “how did you do it?” However, they did not answer: “counting”, but “I picked [the bones] with my hands.”

She stressed how difficult it was for her to make the children reflect on what they had done and on the results of their actions, even if she asked them directly. However, the discussion did not continue, as none of the other PSTs reacted to her comment. An external expert then pointed out that in this kind of interactions with pupils, teachers should not only try to make pupils express what they did (“I picked them”), but also to make them think about what they did, talk about their thoughts, and about the success of their actions. The teacher should consider using an alternative question like “what did you think when you took only those bones?” instead of “how

did you do it?” He added that creating meaningful interactions between pupils and the learning milieu is challenging, and sometimes needs to be encouraged.

*Evidence of PST learning:* although it was brief, the discussion about the event addressed an important issue of PSTs' praxeological equipment, as described earlier. The fact that the teacher considered this event worth discussing shows connections between her actions in the context of the lesson and her TDS-based equipment. Particularly interesting is how she started noticing that, despite the fact that the TDS theoretical model states that the teacher should not intervene in the student-milieu interaction within an a-didactical situation, in reality she might need to strengthen this interaction. However, neither the teacher nor the other PSTs were able to provide deeper reflections on how the teacher could mediate in the pupil-milieu interactions. This could be perceived as a sign of certain limitations of their praxeological equipment. This prompted the intervention of the experts in an attempt to introduce new insights that could make the PSTs' praxeological equipment develop. In our interpretation, the way the event was recalled in the post-lesson discussion and the debate around it were not enough to maximise PST learning.

## 5.3 Critical event 2 in LS cycle 1: modification of the learning milieu of situation 3 during the research lesson

*Brief description:* according to the lesson plan, the box containing the bones was placed at a certain distance from the collection of puppies, so that the pupils could not see the puppies when they had to produce their written representations. However, during the research lesson, the teacher decided to place the box with the bones close to the puppies. The pupils could thus see them when they were writing down their messages.

*Potential of the event to foster PST learning:* the event shows how the teacher decided to change the value of a didactic variable relevant to the situation, resulting in a different situation from the one described in the lesson plan. According to the fundamental situation for teaching cardinal numbers (Sierra et al., 2012), changes like this might be necessary when pupils continue to have difficulties to use counting to quantify a collection and, as a result, struggle to write down a message that might work. The teacher observed these difficulties in the situations she had just implemented. According to her TDS-based praxeological equipment, she knew that introducing changes like this in the milieu of the situation would create new conditions that would enable the pupils to use one-to-one correspondence to produce iconic representations of the collection. The PSTs' discussion of this event might have an impact, on the one hand, on the mathematical components of their praxeological equipment in aspects like the symbols

<sup>1</sup> Henceforth, she will be referred to as the teacher.



the pupils can use to communicate a quantity (drawings, tally marks, numerals), the relationship between the symbols they use and their ability to count, or how this depends on the size of the collection. On the other hand, it might have an impact on the didactic components related to how the teacher, in accordance with the TDS model, should intervene by modifying a didactic variable. In doing so, she would ensure the pupils stayed engaged in the task, thus avoiding failure, or ending up telling them what to do.

*Event in the post-lesson discussion:* the experts observing the research lesson considered the event might be relevant, and therefore decided to generate dissonance in the group. They asked the teacher why she decided to change the conditions of the situation with respect to what was written in the lesson plan. She argued the following:

Teacher: placing the box at a distance [from the pupils] when the pupils produce their written representations adds more complexity to the situation. I therefore opted for rectifying the situation this way [putting the box near the puppies], promoting the use of one-to-one correspondence as a first step.

She added that she had noticed the difficulties the pupils experienced in situation 2 to quantify the collection of puppies, and she realised that situation 3 would not work unless she changed something. The rest of the PSTs nodded in agreement with this decision.

The experts supported the teachers' arguments, highlighting how the pupils failed when they had to quantify the collection in situation 2, but it seemed they were confident about using one-to-one correspondence. The teacher said this had been her intention when she decided to adapt the situation.

*Evidence of PST learning:* throughout her LS intervention, the teacher demonstrated having TDS-based knowledge enabling her to make decisions in context to adapt the situation to the pupils' needs, presenting arguments that supported her actions. She is aware of the mathematical praxeologies involved in the situations, and of the didactic technique she used (changing the learning milieu). The fact that the other PSTs agreed can also be interpreted as evidence of having developed those connections. Therefore, we decided the event generated a discussion that can be considered valuable in terms of recalling certain components of the PSTs' TDS-based praxeological equipment. This opens up the opportunity to activate and test said components in the context of a real teaching situation with pupils.

#### 5.4 Critical event 3 in LS cycles 1 and 2: interpreting pupils' written messages in situation 3

*Brief description:* during situation 3, in both cycles, the pupils produced different kinds of written representations



Fig. 4 Sergio's written representations in cycle 1 and 2

using drawings, tally marks, letters, and even numerals. One boy in particular, Sergio, used some lines that resemble digits in cycle 1 to ask for 6 bones, and then some digits in cycle 2 to ask for 4 syringes (Fig. 4).

In cycle 1 (Fig. 4, on the left), the teacher asked Sergio to explain his solution:

Teacher: Can you tell me about this?

Sergio: One, two, four, seven, six, and nine [while he recites the number-words, he points at the signs in his written representation, from left to right].

Teacher: Are these all the bones that you want?

Sergio: Yes, I want this for this [he points at the first sign and at the first puppy], I want this for this [he points at the second sign and at the second puppy, and he continues until he points at the last sign and the last puppy].

Finally, the teacher decided to give Sergio a bone for each symbol on the sheet of paper:

Teacher: I am giving you this bone for this [she points at the first symbol], this bone for this [she points at the second symbol, and so forth].

In cycle 2 (Fig. 4, on the right), the teacher asked Sergio again:

Teacher: How many syringes are you ordering?

Sergio: one, two, three, four [he points at the puppies, one by one, as he is reciting].

Teacher: [do you want] four?

Sergio: Yes, I counted and I have written the number. I want four.

The teacher decided accept Sergio's request, giving him 4 syringes. She ignored another possible interpretation of Sergio's solution (for instance, giving him one, two, three, and four syringes), which could have challenged Sergio's thinking.

*Potential of the event to foster PST learning:* the event clearly shows the diversity of approaches pupils can use. It also confronts the teacher with two challenges: understanding what the pupils are trying to communicate with their written messages, and deciding how to react to certain messages.

In the case of Sergio's first representation, the teacher did not know if he simply used lines, or if he drew digits. Therefore, she asked him, and his answer was surprising. Sergio listed a series of number-words in an invented order. The teacher asked him again and, according to his answer, it seemed that Sergio used one-to-one correspondence between the puppies and the lines he drew, accompanied by an invented recitation of number-words. The teacher ignored Sergio's wrong enumeration, and reproduced, gesticulating, the one-to-one correspondence to give him one bone for each symbol written on the sheet of paper.

The same was observed in Sergio's second representation: he used one-to-one correspondence between the symbols and the collection. This time, however, he recited the number-words correctly and used them as intermediaries between the collection and the message. He came close to making proper use of the cardinal principle ("I want four"), although he still used digits as icons (each digit does not really have a cardinal value for him). He employed one-to-one correspondence between each digit and each puppy.

Discussing this event might challenge key components of PSTs' TDS-based praxeological equipment related to the fundamental situation for the teaching of cardinal numbers (Sierra et al., 2012). On the one hand, it might challenge the future teachers' understanding of the type of symbols pupils can initially use in situations involving cardinal numbers when they are not yet able to give a cardinal value to digits. It is particularly interesting to see how children may start using digits as icons before they give them a proper cardinal value. On the other hand, during the discussion, the PSTs could reflect on how different ways of interpreting pupils' solutions could make the feedback they get from the milieu successful or not. Since the TDS postulates that pupil learning takes place from successive adaptations to the milieu, the teacher needs to be aware of the possible reactions she can have to Sergio's written representations, depending on whether she wants to temporarily reinforce his approaches, or challenge them.

*Event in the post-lesson discussion:* the event around Sergio's solutions and the interaction with the teacher created dissonance in the LS group. The PSTs addressed Sergio's case in the post-lesson discussion both in cycle 1 and 2. With respect to Sergio's representation in cycle 1, the PSTs agreed that Sergio was the only child that evaluated the situation as being mathematical and, consequently, tried to use mathematical objects. Actually, they mentioned that Sergio said several times that his classmates had to use numbers. For instance, a PST mentioned a comment she had written down in her observation sheet:

PST: Sergio said, "Juan [another child] is not doing it correctly because he is not writing numbers, but letters."

The PSTs also commented they realised that Sergio was not using the numerals in accordance with their meaning. The teacher who taught the lesson stated:

Teacher: He [Sergio] told me "this 'O' [first symbol in his message, see Fig. 4] is for this puppy." I wanted to give him zero bones in accordance with the cardinal value of 0, then 7, 4... but he wasn't asking for that... He is only 3 years old. He couldn't explain it to me, but he stared at me like saying "don't you get it? It is not this [I'm not asking for so many], but this for this, and this for this" [again, Sergio gesticulated the one-to-one correspondence between the symbols he wrote and the puppies]. He knew that every puppy needed a symbol, although he didn't know how to use the numerals.

With respect to Sergio's representation in cycle 2, the PSTs again highlighted that he was the only child using numerals, but, as in cycle 1, he was not giving them a proper cardinal sense. However, they argued that they observed some progress in Sergio's knowledge, since he used the number-words sequence in the right order. They realised that Sergio used a bijection between the puppies and the digits.

PST: Previously [cycle 1], Sergio said [wrote] "1, 2, 6, 0, 9", or something like that, but now he has said: "one, two, three, four". He enumerates the numbers, and he also writes them down. He looked at each puppy when he wrote down each digit.

The experts pointed out the existence of connections with the PSTs' TDS praxeological equipment, adding that Sergio's written representation corresponds to a difficulty typical of children that age, when they do not yet understand that saying "four" and writing "4" is enough. They also mentioned that the teacher could have acted differently, depending on what she meant to achieve with regard to Sergio's learning. For instance, if Sergio had remained stuck in a one-to-one approach, it would have been interesting to try out a new situation in which the teacher challenged Sergio's thinking by interpreting his solution in accordance with the cardinal value of the digits. However, the experts expressed their support to the way the teacher decided to interpret Sergio's written message at that point.

*Evidence of PST learning:* The PSTs' interventions indicate the activation of important components of their TDS-based praxeological equipment. Mathematically speaking, the discussion reveals that it contributed to deepen their understanding of the relationships between numbers as the cardinality of a set, and numerals as the symbols used to express the numbers. Besides, it shows that the PSTs understand and are able to notice how pupils go through transitional phases in their adaptation to situations. The PSTs are able to establish connections with their TDS-based equipment about the teaching of cardinal numbers in early

childhood education. They realised that pupils do not give numerals and number-words a cardinal value at once, even if this is explicitly stated by the teacher. Sergio's responses showed how pupils start grasping the notion of quantity while they struggle to make sense of certain cultural symbols (numerals), number-words, and routines like counting.

The PSTs also addressed important aspects that could be related both to the mathematical and didactic component of their TDS-based praxeological equipment. They discussed the didactic techniques the teacher could use to react to certain kinds of messages, depending on whether she wanted the pupils to make successful adaptations to the milieu or not. The fact that this appeared in the discussion, triggered by the events in the research lesson, contributed to broaden the PSTs' equipment insofar as they realised how the TDS model worked in practice, and how important teacher actions are to enhance meaningful and productive interactions of pupils with the learning milieu. We consider that, as the PSTs themselves brought this event into discussion, and given the depth of the debate and the participation of the experts, it allowed activating and developing important components of the PSTs' TDS-based praxeological equipment related to the teaching of cardinal numbers in early childhood education.

## 6 Discussion and conclusions

This paper describes an LS process with early childhood education PSTs, related to a methods course on the teaching and learning of mathematics in the early years from the perspective of the TDS. With a focus on the observation and post-lesson discussion, two research questions were formulated: the first one about the activation of PSTs' TDS-based praxeological equipment during the post-lesson discussion, and the second one about the possible mechanisms that could explain said activation and the potential development of PSTs' praxeological equipment.

Regarding RQ1, the three critical events selected to be presented in this paper show the opportunities provided to activate and challenge the PSTs' TDS-based praxeological equipment. In connection with the mathematical dimension, the PSTs had to mobilise their knowledge about the fundamental situation for the learning of cardinal numbers, including how the learning milieu could be structured and which different approaches the pupils could use. In critical event 2, the teacher modified a didactic variable in cycle 1 because she noticed that the pupils were not able to adapt to the learning milieu as it had been conceived in the lesson plan. The discussion showed how the teacher justified this change, and how the rest of the group agreed with that decision. In critical event 3, the PSTs had to interpret a student's peculiar solution, which challenged

their knowledge of the kind of solutions pupils could use (one-to-correspondence and counting). The debate shows how the PSTs established connections between the way in which this pupil used counting (reciting number-words in an invented order in cycle 1, and reciting them in the right order in cycle 2), the rudimentary numerals he wrote in his message, and how he used them deprived of their cardinal value to establish one-to-one correspondence.

As for the didactic dimension of their praxeological equipment, in the three events the PSTs observed a lack of congruence between the ideal character of a theoretical model and the eventuality of a teaching situation. Thus, in critical event 1, the notion of a didactical situation, which stresses a possible independent interaction of the pupil with the learning milieu, was challenged when they realised the teacher might need to intervene in the situation to strengthen the pupil-milieu interaction. In critical event 2, they experienced how, in accordance with the TDS model, the teacher managed the learning situations through the choices she made concerning the didactic variables. Finally, in critical event 3, they faced the challenge of deciding how to respond to the tricky answer of a pupil, depending on the kind of interaction with the milieu they wanted to promote or block at that moment.

Overall, the critical events show how the PSTs had to rely on formal components of their TDS-based equipment, which came from an out-of-school methods course. However, at the same time, they had to go beyond them to adapt to what happened during the research lesson. Leavy and Hourigan (2016) found that LS triggered the development of knowledge acquired by PSTs in university-based pedagogical courses, but that, to a certain extent, had remained inert. Murata and Pothen (2011) add that LS may facilitate connections between aspects of teaching that may seem discrete from one another for PSTs. The results are consistent with the ones reported in Lendínez et al. (2024), which showed an increase in the self-efficacy beliefs of the PSTs to plan and teach TDS-based lessons. In fact, our study is one of the few that have explicitly taken into account the activation, use and adaptation of PSTs' previously acquired knowledge in the context of LS, considering it in a functional way. The results are also innovative, having carried out a detailed analysis of PST learning linked to a specific framework, the TDS, for the teaching and learning of mathematics. We have shown how the PSTs made use of theoretical tools and notions that they had gained in a course taken before their participation in LS. We have also analysed the role that these tools played in PSTs' observations of the research lesson and in the identification and discussion of critical events. Besides, we have interpreted PST learning as closely connected to, and depending on, these theoretical notions. This is important since, as Larssen et al. (2018) suggested, research on LS

should be more explicit in how it describes the learning of PSTs.

Concerning RQ2, the analysis of the case here presented, and of other cases not included in this paper, reveals an existing mechanism in LS that might, to some extent, explain teacher learning throughout the post-lesson discussion. Thus, in LS, the implementation of the research lesson provides a complex and, to some extent, unpredictable empirical milieu in which the praxis block of the PSTs' praxeological equipment is activated. While the research lesson may contain numerous noteworthy teaching and learning events (Leavy & Hourigan, 2016), some of them might be particularly relevant (critical) because they can create dissonance in the LS participants. However, these events do not reach their full potential until they are taken up in the post-lesson phase, and only if the teachers engage in meaningful discussions about them, mobilising and challenging the logos block of their praxeological equipment.

According to this mechanism, unsatisfactory results of LS processes may, amongst other reasons, be due to the participants' difficulties to identify critical events. This leads to an absence and/or dysfunctional dissonance (Mynott, 2019), as was experienced in some of the LS groups in this study. Even if critical events are identified and reconsidered in the post-lesson discussion, superficial and/or unfocussed discussions can also limit PST learning. This is what occurred in critical event 1. The PSTs' learning potential was not completely developed, which provoked limited dissonance (Mynott, 2019). In contrast, the discussion that took place around critical events 2 and 3 showed they were better used to challenge and develop certain components of the PSTs' praxeological equipment.

This study shows it is worth paying attention to the people who identify the events that occur during the research lesson, and bring them up for discussion. The results reveal that some of the events were taken to the discussion by the PSTs themselves, but others by the external experts. This also happened with other LS groups not reported here. It may be because the PSTs have difficulties in identifying those events (Baldry & Foster, 2019; Bjuland & Mosvold, 2015; Hourigan & Leavy, 2019), as they do not consider them relevant enough to be discussed. It could even be because of the power relationships established between PSTs and their tutors in LS in initial teacher education (Corcoran, 2011). Although it is desirable for PSTs to develop the ability to identify events (which is connected to the development of their noticing skills), to consider them as valuable and, therefore, to include them in the discussion, it cannot be expected to be a straightforward process. Difficulties may be linked to the lack of knowledge and/or experience of PSTs (Bjuland & Mosvold, 2015; Hourigan & Leavy, 2019; Lewis, 2019), or to not knowing how to connect their course work and field experiences (Murata & Pothen, 2011). Mynott (2019) considers participants' lack of expertise

as a potential limiting factor of dissonance within an LS experience. However, it could also be related to the insufficient development of PSTs' noticing skills to identify what is important in a learning situation (Bjuland & Mosvold, 2015; Leavy & Hourigan, 2016; Sims & Walsh, 2009; van Es & Sherin, 2002). This deserves greater attention in future research. Planning observation in advance, as suggested in other research (Murata & Pothen, 2011; Ni Shuilleabhain & Bjuland, 2019; Sims & Walsh, 2009), might enable PSTs to develop the ability to identify critical events within the research lesson, something that will be explored in future research.

The results of this study also suggest the crucial role that teacher educators could play in LS with PSTs, as has been acknowledged by others (Baldry & Foster, 2019; Leavy & Hourigan, 2016; Ni Shuilleabhain & Bjuland, 2019). Thanks to their expertise, they might contribute to the identification of critical events unnoticed by PSTs, or noticed but underestimated. Besides, they can also help to amplify the dissonance these events might provoke within the group. Last, but not least, they could play an important role as activators of the knowledge learnt by PSTs in previous courses. Nonetheless, from our experience, we can say that this can also be challenging for them, being an aspect that deserves more attention in future research.

To sum up, our research documents PST learning, and identifies a mechanism in LS that could explain why and how teacher learning happens. This may have major implications for the planning and implementation of future LS processes with PSTs. This research also stresses the need to be more explicit about the nature of the professional knowledge teachers use and develop in LS, as shown here using the TDS.

It goes without saying that this is not the only internal mechanism in LS that might be responsible for teacher learning. However, our research helps to make it visible. Further research at a micro-level could be relevant in the process of unravelling how LS works and how it leads to teacher learning.

**Funding** Funding for open access publishing: Universidad de Jaén/CBUA. This work was supported by the Ministry of Education, Culture, and Sports (Spain) under the predoctoral contract for University Teacher Education FPU14/06496 and by the Ministry of Science and Innovation under the research Grant PID2021-126717NB-C32.

## Declarations

**Conflict of interest** The authors have no competing interests to declare that are relevant to the content of this article.

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