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TWO DIMENSION 2-TUPLE LINGUISTIC APPROACH FOR MULTI-ATTRIBUTE GROUP DECISION MAKING METHOD UNDER UNCERTAINTY

THESIS MEMORY PRESENTED BY

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TO OBTAIN THE PHD DEGREE IN COMPUTER SCIENCE

SUPERVISORS

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The Thesis entitled *Two dimension 2-tuple linguistic approach for multi-attribute group decision making method under uncertainty*, presented by D. Zelin Wang to obtain the PhD degree in Computer Science, has been carried out in the Computer Science Department of the University of Jaén with the supervisors Dr. Luis Martínez López and Dra. Rosa María Rodríguez Domínguez. To be evaluated, this research memory is presented as a set of published articles, according to Article 23, point 3, Regulation of Doctoral Studies of the University of Jaén, approved in March 2022.

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Contents

Chapter 1

Introduction

1.1 Motivation

Group decision making (GDM) is a decision theory branch that has been widely applied in real world scenarios to solve important and complicated decision problems in a range of domains, such as public health [5], water supply engineering projects [127], foreign policy [8] and so forth. In GDM problems, decision makers (DMs) usually evaluate alternatives based on multiple attributes, leading to multiple attribute group decision making (MAGDM) problems [82]. However, because of the complexity of eliciting assessments and human beings bounded rationality, linguistic terms are easier elicited than crisp numbers for assessing attribute in MAGDM. Zadeh first introduced the concept of linguistic variable [206], it is a variable whose values are not numbers but artificial or natural language expressed as words or sentences. With the advantages in dealing with qualitative information for MAGDM problems, the number of researches on MAGDM approaches based on linguistic variables have been investigated increasingly [53, 108, 111, 117, 177].

For solve MAGDM problems under linguistic environment, it is necessary to carry out computing with words (CWW) processes [121, 208, 210] (see Figure 1.1), which is one of the most used methodologies.

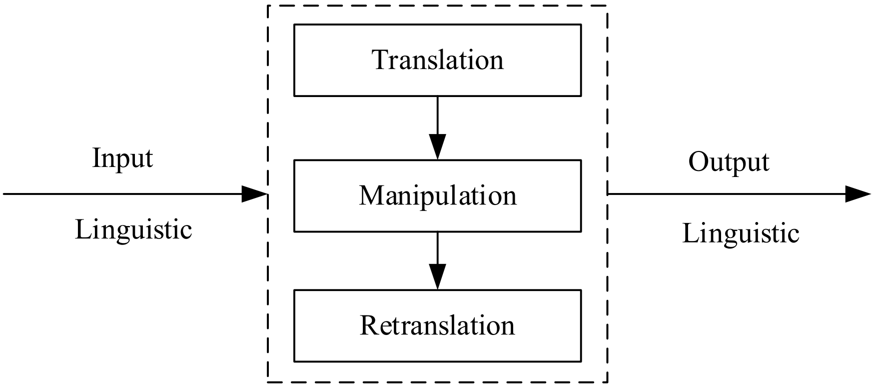

Figure 1.1 Computing with words process

In this process, linguistic outcomes are obtained from linguistic inputs, which are easily understandable and properly represented. Consequently, several linguistic computational models have been developed to accomplish the CWW processes [3, 60, 61, 118, 172, 197]. These models follow the computation scheme depicted by Yager [198, 199] that points out the importance of the translation and retranslation processes in CWW (see Fig. 1.1). However, there are some limitations when fusion processes are performed on linguistic variables. They performed the retranslation step as an approximation process to express the results in the original term set provoking a lack of accuracy [63]. In these approaches, the results usually do not exactly match with any of the initial linguistic terms, then an approximation process must be developed to express the results in the initial expression domain. This produces the consequent loss of information and hence the lack of precision.

To avoid such an inaccuracy in the retranslation step, a 2-tuple linguistic model [60] was developed, which is composed by a linguistic term and a numerical value called symbolic translation that represents the displacement of the linguistic term. Therefore, it avoids the loss of information and obtains more precise and interpretable results. For this reason, the 2-tuple linguistic model stands out as one of the most widely used in decision making [119, 142].

Furthermore, 2-tuple linguistic model has been extended to several different models, which have been proposed within MAGDM problems, such as, proportional 2-tuple linguistic model [172, 173], the 2-tuple semantic model [1, 163, 164], numerical scale model [34, 36, 37], multi-granular 2-tuple linguistic model [38, 62, 197], etc. Martínez and Herrera [120] provided an overview on these model based on the successful and extensive research of the 2-tuple linguistic models. The extant decision making models based on the 2-tuple linguistic information assume that all assessments have the same confidence level [112], which means the assessment is one hundred percent sure, while in practice, the reliability of the assessments needs to be measured. The same assessment with different reliability leads to different results. Hence, the reliability of the assessment is also important for MAGDM problems, Zhu et al. [225] considered the reliability information of the subjective assessments and proposed the concept of two-dimension linguistic information. Furthermore, two-dimension linguistic expression and 2-tuple linguistic information was combined and expressed as two dimension 2-tuple linguistic (TD2L) [224].

The assessment and the reliability of the assessment are provided at the same time and provide as TD2L, it was evident that the information expressed as TD2L is more reasonable and accurate. Considering the assistant of TD2L assessments in dealing with linguistic information, several researches on MAGDM problems based on TD2L [98, 99, 220] have been presented, such as:

- *Representation model of TD2L labels.* Generally, the TD2L labels are presented as a binary linguistic term form [223]. The two classes of linguistic information come from two different linguistic term sets respectively. The first term set represents the evaluation assessments provided by DMs. The second term set represents the reliability of the previous assessment, which is also the subjective information provided by DMs [202].

- *Operational and comparison rules of TD2L labels.* Different operators have been developed for different kinds of linguistic expression with two dimension, such as, two dimension uncertain linguistic aggregation operators [106, 110] used for aggregating the two-dimension linguistic labels under uncertainty, trapezoidal fuzzy two dimension linguistic power generalized aggregation operators [99] used to aggregate TD2L labels with the first class linguistic uncertain extended to trapezoidal fuzzy number, etc. Besides, the comparison rules between TD2Ls have been developed based on the traditional comparison rules of 2-tuple linguistic model [60], such as, two-dimension linguistic lattice implication algebra (2DL-LIA) [224] used for expressing and comparing the TD2Ls, the notation of expectation of TD2Ls [110] was proposed for comparing two-dimension uncertain linguistic variables, etc.

- *GDM methods based on TD2L expression.* Since TD2L has unique advantages in modelling information, its research and application combined with these classical GDM methods has attracted attention from scholars. Several GDM methods have been extended under the TD2L environment, such as, PROMETHEE [220], extended TODIM [105], extended VIKOR-QUALIFLEX [98], failure mode and effects analysis [104], extended prospect theory-VIKOR [33],etc.

- *Application of GDM methods based on TD2L labels in real life.* In some real situations, linguistic terms have been considered the most suitable modelling for assessing attributes, such as, emergency decision making [32, 33], quality evaluation of community question answering [97] power plant site selection [185] and risk assessment [186], etc.

Further research in GDM shows that consensus reaching processes (CRPs) are necessary to achieve the agreement on decision results in GDM problems. However, CRPs generally demand that the original assessments are adjusted if the expected consensus level is not satisfied. In such a situation, the reliability of the adjusted assessments is worth thinking. Obviously, original assessments' reliability could be given by experts in advance, however, the reliability of the adjusted assessments should be derived from an objective measure way.

Despite there are multiple models and approaches to deal with MAGDM and TD2L labels jointly, both theory and practice, it is remarkable that so far these models and approaches are not good enough when they are applied to real world MAGDM problems in which CRPs are applied to. Thus, the main motivations of this research are described with new difficulties and challenges as below:

- *The aggregation of the TD2Ls in MAGDM:* Aggregating the TD2Ls of DMs to rank or sort the alternatives, to select the best option is a necessary process. In MAGDM problems based on TD2L labels, individual DMs' preferences must be aggregated in a collective and well-structured way before the final decision has been made. The aggregation of the TD2Ls is of great importance in MAGDM because different aggregation operators could lead to different results. However, interpreting and analyzing these DMs' preferences is a complex task. And in the existing methods, no matter which aggregation operator is taken, the two-dimension information of the TD2L labels are taken separately for computing [99, 107, 110, 167, 200]. In fact, when the assessments are not completely reliable, they become random which means the assessment is uncertain. Therefore, an aggregation operator for dealing with the TD2L labels from a stochastic perspective is promising to research.

- *Measuring reliability of the adjusted TD2L assessment:* TD2L labels express the assessment and its reliability. With the advantage of the representation of the TD2L labels, they have been applied to many MAGDM problems [32, 185, 186]. However, by performing a CRP, the initial TD2L labels are modified and the reliability of the adjusted assessment should be recomputed. The reliability of the initial assessment is subjective. However, an objective measurement to improve the use of the TD2L labels in MAGDM is necessary.

- *Determining DMs' weights in MAGDM problems*: The calculation of DMs' weights in the literature can be divided into subjective methods, objective methods and methods combining the objective and subjective approaches [42, 178]. Subjective weight determination methods, such as the analytic hierarchy process (AHP) [146] and Delphi methods [73], assign weights to DMs based on subjective characteristics such as their background, professional levels and experience with the decision making problems. Objective weight determination methods [85], such as the entropy weight [46], technique for order preference by similarity to an ideal solution (TOPSIS) [68] and projection methods [204], etc. Mixed subjective and objective methods for computing comprehensive DMs' weights information [116, 147, 176] also very common in weights obtaining. When DMs' weights are not given in advance, the objective way to determine the reasonable weights information is important. Hence, to find out a more suitable and effective way to determine the DMs' weights it is a challenge for MAGDM problems.

- *The clustering of large scale number of DMs*: Clustering analysis can effectively simplify the CRP when a large scale number of DMs is involved in MAGDM. Therefore, the clustering analysis has become significant for solving MAGDM problems. Many scholars have focused their attention on clustering method, such as, k-means clustering algorithm [187], a fuzzy c-means based algorithm [151], a hierarchical clustering algorithm [21], etc. Using the clustering method, the DMs can be divided into several small clusters, then DMs' assessment information have higher consistency and a lower degree of conflict for each cluster. However, the existing clustering methods are complex for computing and they ignore the support degree on each alternative of different DMs. Thus, a new clustering method based on the support degree of each alternative of DMs need to be developed so that more information would be obtained during the CRP.

- *The consistency and consensus of DMs' opinions*: Consistency and consensus are other noteworthy challenges in the MAGDM process. Consistency is directly related to the credibility of the MAGDM results. Consensus, on the other hand, means that the agreement of DMs to accept the results of the process. During the CRP, some DMs do not modify at all their opinions, which could happen when there is not enough time to persuade these DMs. DMs agree to modify their preferences to a value that is within their tolerance degree at most. Therefore, the acceptance and tolerance degree of the adjusted opinion for stubborn DMs need to be considered and it might be a challenge to coordinate the stubborn DMs' assessments and the automatic feedback.

In real world MAGDM problems, current MAGDM approaches need to overcome previous challenges found in existing MAGDM problems in order to better satisfy the situations and demands in decision making. To deeply study the subjects regarding the challenges described above, this research memory conducts comprehensive and further researches to address those shortcomings.

1.2 Objectives

In accordance with the challenges pointed out previously in existing MAGDM methodologies based on TD2L labels, this research memory is focused on the improvements of current MAGDM techniques.

On the basis of such a design, the following three research purposes are considered:

1. *To develop a novel TD2L computation model.* It considers two dimensions' information of TD2L labels from stochastic perspective and then compare the computation models from the general and stochastic perspective by a case study. Additionally, some new aggregation operators and comparison rules will be introduced to improve previous studies.

2. *To consider the reliability degree of the adjusted assessment during CRPs in MAGDM.* Generally, original assessments provided by DMs are linguistic terms, and the adjusted assessments are still linguistic terms or the extension of a linguistic term, such as, 2-tuple linguistic value after the CRP. In such a case, the information of the reliability of the adjusted assessment is usually missing. Thus, another dimension for linguistic information will be obtained for representing the reliability of the adjusted agreed assessment. In this objective it will be considered the minimum adjustment during the CRP, a two-stage minimum adjustment consensus model based on linguistic assessment information will be proposed to show the obtained adjusted assessment and its reliability. Besides, the relations between the reliability of the adjusted assessment and the distance from the original assessment to the adjusted assessment will be discussed.

3. *To define a MAGDM framework.* It is used to solve the problems refer to a large number of DMs and consider the tolerance degree of DMs on changing their opinions. First, a support degree-based clustering method for classifying DMs into several subgroups is introduced, which make more manageable the large number of DMs. Besides, a minimum adjustment consensus model considers the tolerance degree of DMs to improve the reliability of the adjusted opinions will be presented to improve the consensus level gradually. Eventually, the adjusted assessment will be modelled as TD2L labels. Using the proposed method for comparing TD2Ls, the alternatives ranking could be obtained.

1.3 Structure

To carry out the objectives displayed in Section 1.2, and taking into account the article 23, point 3, of the current regulations for Doctoral Studies at the University of Jaén, according to the program established in the RD 99/2011, this research memory will be demonstrated as a summary of published articles by the PhD student throughout her PhD student period.

Two articles have been published in Q1 international journals indexed by JCR database, produced by ISI and one International conference paper (IEEE International Conference on Fuzzy Systems 2021) indexed in Engineering Village (Ranking in the Core Ranking list of conferences 2020 as CORE A). In a nutshell, the report consists of two papers which have been published in high quality international journals and one conference contribution.

The framework of this research memory is concisely explained below:

- Chapter 2: Various basic concepts that are used across the research memory to achieve our research goals are revised such as, related concepts of decision making, GDM, MAGDM, MAGDM under uncertainty, MAGDM based on linguistic information. And the methods and models that are used in our proposals, such as, fuzzy linguistic approach, 2-tuple linguistic model, two dimension 2-tuple linguistic label, consensus reaching process, minimum adjustment cost model and so on will be revised in short.

- Chapter 3: Research memory is briefly introduced through the published proposals with discussions of each result obtained in short, which elucidate the achievements arrived in our research.

- Chapter 4: The publications obtained as the research results are included in this chapter and thus it is the core of the doctoral thesis. The information about each journal in which the proposals have been published is further displayed.

- Chapter 5: Conclusions concerning this research memory and possible potential future works are indicated finally.

Chapter 2

Basic Concepts and Methods

This chapter builds the skeleton of concepts and tools connected to our research. In light of the fact that, the different articles that constitutes the research introduce and revise the fundamental setting for understanding the proposals, in this chapter we have presented a detailed and structured reviewing of the critical concepts related to the proposals including related decision making concepts, GDM, MAGDM, CRP and the handling of consensus under uncertainty in GDM by eliciting two dimension 2-tuple linguistic labels. Besides, the methods used for solving MAGDM problems under uncertainty, fuzzy sets, fuzzy linguistic approach, two dimension 2-tuple representation model, linear programming method, are revised. All these conceptions, approaches and instruments are further detailed in each specific article of the compendium displayed in this research (see Chapter 4 for further details).

2.1 Decision making

A concise introduction and a categorization of decision making are reviewed as the essential basic knowledge of this thesis in the section, which smooth the path of our approaching studies.

2.1.1 Introduction

Decision making is a complex cognitive process proper of human beings. Within this process, individuals can decide actions based on either personal beliefs or the inference of various factors in various options, or decide the opinions that the individual wants to express. Every decision making process aims at producing the final decision and selecting the final choice [140]. Before making a decision, DMs are often faced with different plans and choices, as well as a certain degree of uncertainty about the consequences of their decisions; DMs need to weigh the pros, cons, and risks of various choices in order to achieve the best decision result.

The decision making process consists of an entire process from asking questions, determining goals, and going through program selection, decision making, and delivery to implementation. It emphasizes the practical significance of decision-making. It is clear that the purpose of decision making is execution, which in turn checks whether the decision is correct and whether the environmental conditions have undergone major changes [115].

In general, decision making is the process of making choices by identifying a decision, gathering information, and assessing alternative resolutions [83]. Seven steps could be considered to help DMs to execute the decision making process as follows [43]:

Step 1: To identify the decision problem: This step determines what the decision problem actually is.

Step 2: To gather relevant information: DMs' preference information is collected before decision making.

Step 3: To identify the alternatives: To list all possible and desirable alternatives.

Step 4: To weight the evidence: To place the alternatives in a priority order based on suitable decision methods.

Step 5: To choose the best possible option: To select the alternative that seems to be the best or even choose a combination of alternatives.

Step 6: To execute the action: The alternative derived from Step 5 is implemented.

Step 7: To review the decision result: The decision result is evaluated and then according to the performance of the alternative to improve next possible decision problems.

To better illustrate the decision making process, a flow chart is shown in Figure 2.1.

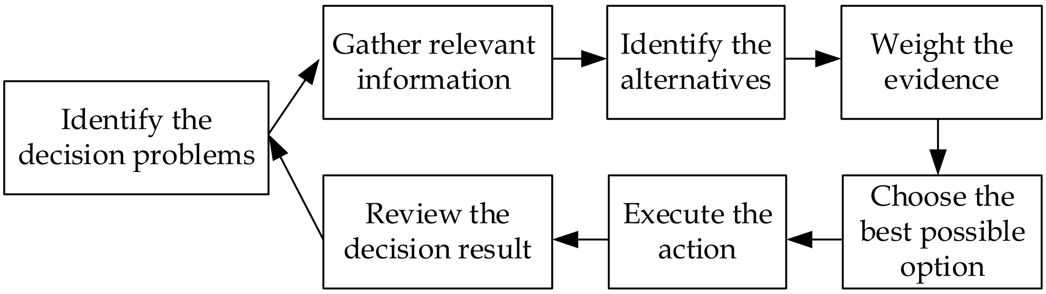

Figure 2.1: General decision making process

2.1.2 Classification

As a common behavior in politics, economy, science, technology, military, culture and other fields, decision-making is the core of modern management and runs through the whole management activities. The word decision-making now exists in the field of management science as a professional term. The continuous progress of society and the rapid development of economy are inseparable from the support of scientific decision-making. Such activities widely exist in various fields, such as environment, technology, economy, management, military, etc.

In decision-making problems, it is often necessary to choose the optimal alternative. Obviously, the methods used are different according to the decision-making situations or contexts. In this subsection, the main classification bases are introduced, such as *preference modelling*[113], *number of involved DMs* [126], *decision environment* [84] and so on.

(1) Preference modelling

In consideration of DMs may choose different types of assessments according to different decision situations, hence decision making could be divided into various types according to the way of modelling the preference assessment, such as: linguistic decision making [56, 129, 219], fuzzy decision making [6, 14, 144], decision making using numerical data [215, 221].

Some researchers deal with decision making problems based on fuzzy sets [207], hesitant fuzzy sets [162], 2-tuple linguistic term sets [60], type-2 fuzzy sets [206], etc. They are frequently conducted in qualitative circumstances because of cognitive limitations and the lack of sufficient information.

(2) Number of involved DMs

Decision making can be classified into two categories considering the involved number of individuals:

- Individual decision making, which means there is only one DM participating in the decision making process and the decision results are completely according to his/her judgment. Individual decision making saves time and cost and usually makes prompt decisions. Moreover, individuals are accountable for their acts by various people. The decision making would be high-quality if the individual has rich experience and excellent professionalism. However, individual is limited in all expertise to some extent and there may not be so many creative solutions generated.

- Group decision making. It is a type of decision making process in which multiple individuals acting collectively, analyze problems or situations, consider and evaluate alternative courses of action, and select from the different alternatives a solution. Group decisions take into account a wider scope of information because each group member may contribute distinct information and expertise. Organization decisions are much more technically and politically complex; hence they usually require GDM [31, 52]. Group members can identify more complete and robust solutions and recommendations through discussing, questioning and collaborative approaches. The classical solving scheme to solve GDM problems is a selection process that composes of two stages (see Figure 2.2) [128]: (1) an aggregation stage, in which each DM's information is collected according to aggregation operator, and (2) an exploitation stage, in which an optimal alternative or a subset of alternatives is derived as the solution to the decision making problem.

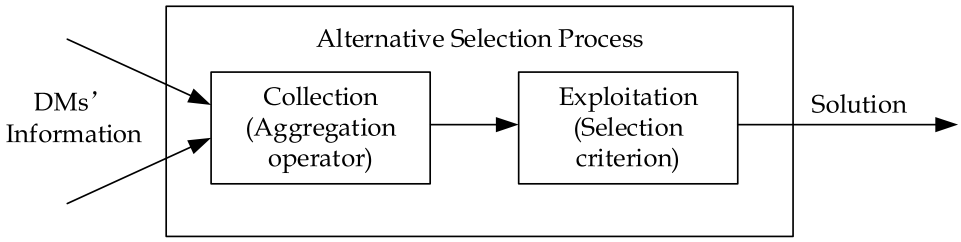

Figure 2.2: General group decision making chart

(3) Decision environment

In accordance with different decision situations in which the decision problem is conducted, it can be categorized as three kinds of decision problems [13]:

- Decision making under certain environment. It refers to DMs have a very definite comparison of what may happen in the future, such as the alternatives, the attributes, the weights information is definitely sure by DMs. In such a decision environment, the most commonly used decision making methods are linear programming decision making method [94], profit and loss sharing model [131], etc.

- Decision making under risk environment. It is a decision made by DMs based on the probabilities that various natural states may occur and the conditional benefit value of each alternative. The environment for risky decision making is not completely certain, but the probability of its occurrence is known. The commonly used methods for risky decision making are decision-making method based on expectations [182], decision-making method based on maximum probability [109], decision tree method [137], Markov decision process [2], etc.

- Decision making under uncertainty environment. The uncertainty handling has been one of the main concerns of DMs for many years [4]. It refers to a decision in which DMs cannot determine the probability of the occurrence of various natural states in the future. Uncertainty comes from many aspects, such as, incomplete information about the state of the world, practical and theoretical limitations of DMs [84], which means the future environment is unforeseeable. There are various uncertainty handing methods developed for dealing with the decision making under uncertainty environment [155], such as, fuzzy approach [205], information gap decision theory [50], robust optimization [156], interval analysis [124], etc.

2.2 Group Decision Making

In this section, it is revised the GDM problems and its classification according to the number of DMs involved, afterwards different processes and types of methods and models related to the GDM problem and its typology are briefly revised. Such a revision aims at introducing the necessary knowledge for understanding the proposals of this research memory, which prepare the way for our novel explorations in GDM.

2.2.1 Introduction

Decision making made by a single DM is a process in which only an individual is responsible for defining the problem, assessing the alternatives based on a set of attribute or preference relations and make a final decision [138]. In the context of economics, politics, military, and management, the decision making process is becoming increasingly complex, forcing stakeholders and DMs rely on group wisdom instead of individual judgements. Several DMs with the collective wisdom are more suitable for decision making.

GDM is a common phenomenon aims at choosing the optimal alternative from a set of possible alternatives based on the different opinions of DMs. Having more people involved in decision making is beneficial because each individual brings unique information or knowledge to the group, as well as different perspectives on the problem. However, with the increasing of the number of DMs, if the number is larger than 20, then the GDM problem could be large scale GDM [18]. According to the involved number of individuals, GDM can be classified into two categories [126]:

- Classical GDM: To obtain the most satisfactory alternative, a small group of DMs are invited to elicit their preferences. Hence, such decisions are usually taken by a few number of DMs, which can gather collective wisdom compared to individual decision making, which made decision making more reliable and credible. DMs are working together to find a solution for the specific problem. This turns GDM into a more effective and fast process. Groups can take advantage of the GDM to perform certain tasks, such as generating ideas and solutions through the group interaction. It is argued that DMs can enhance their ability to learn and stimulate their cognitive level with the GDM process. The classical GDM solving process is shown in Figure 2.2.

- Large scale group decision making (LSGDM): Unlike classical GDM, LSGDM refers to the selection of the best satisfactory alternative from a set of feasible alternatives, which is predicated on the preferences of a large number of DMs. Solving challenging problems can require a large group of DMs from different fields, the participating DMs are diverse and numerous [19], which has a wide range of applications in areas like earthquake shelter selection [193], urban resettlement [20], internet venture capital [45], financial inclusion [19], social networks [114], and emergency decision-making [95]. The evolution of GDM problems to LSGDM problems, has brought many new challenges, not only regarding the group size but also with regards to other problems such as knowledge distribution, the increase of cost and complexity in the decision making processes.

2.2.2 Consensus reaching process in GDM

In general, at the beginning of the GDM problem, DMs' opinions may differ substantially. The consensus reaching process (CRP) is often a necessity in GDM to achieve a general consensus regarding the selected alternatives [57, 58, 133]. Usually, consensus is defined as the full and unanimous agreement of all the DMs regarding all the feasible alternatives. However, a complete agreement is difficult to achieve in practice, thus "soft" consensus is a common phenomenon in real decision making problems [24, 59, 77]. Reaching consensus implies that DMs should modify their initial opinions throughout different discussion rounds in order to bring them closer to the opinions of the rest of the group.

Consensus can be achieved with or without feedback. The CRPs without feedback achieve consensus by modifying the initial assessments without considering DMs, while CRPs with feedback involve discussions among DMs and they should modify their initial assessments to reach a consensus. Particularly, the feedback process is often guided by a moderator, then the moderator suggests to modify the original assessments far from the collective agreement according to the identification and direction rules [54, 59]. Figure 2.3 shows the general process of consensus reaching.

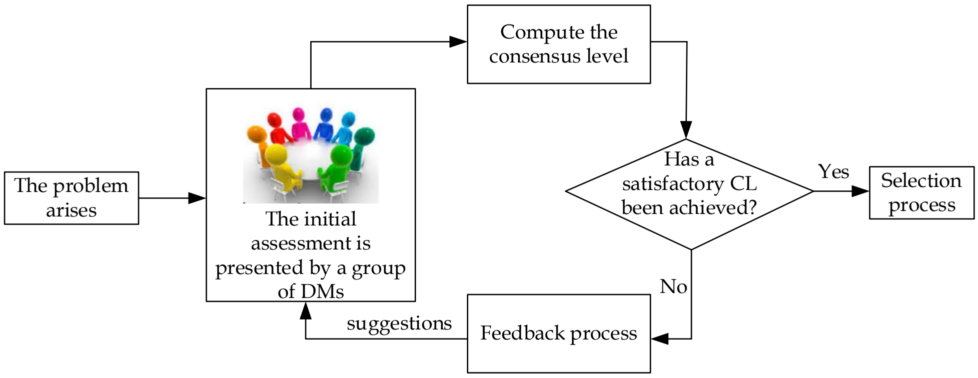

Figure 2.3: Consensus reaching process

However, the feedback mechanism has some limitations [217], such as, it is time consuming, it will result in a huge cost consumption and even in deadlocks.

Furthermore, in GDM problems, due to the existence of polarized opinions, the group consensus process is becoming more and more important and worthy of attention. Therefore, the core problems are the assessment adjustment and the consensus cost in CRP. Based on the consideration of consensus cost, the cost of reaching a non-strict consensus is smaller, more effective, and more feasible than strict consensus that is time-consuming and costly. Therefore, the acceptable level of consensus and the coordination cost of reaching a consensus are two very important factors in GDM. Obviously, DMs will prefer a low-cost group consensus process, and the minimum adjustment cost consensus model to solve this problem well.

Since the existing resources are limited, it is expected to spend the least adjustment cost to reach a consensus. The two most common minimum cost consensus models used in the specialized literature to deal with linguistic information are introduced below [9, 35].

(1) Minimum Adjustment Consensus Model (MACM)

The minimum adjustment of this type of model [35] has two core points: one is based on the distance, which aims to minimize the distance between the initial assessment of the DM and the adjusted assessment. The second is based on the number of assessments that need to be adjusted, that is, to minimize the number of changes in the process of reaching a consensus.

Suppose that $E=\{e_k \mid k=1,2,...,m\}$ is a set of DMs, $w=\{w_1,w_2,...,w_m\}$ are the DMs' weights with $\sum_{k=1}^{m} w_k=1$ and $w_k \in [0,1]$. $S=\{s_0,s_1,...,s_g\}$ is the linguistic term set used for expressing the initial assessment. $O = \{o_1, o_2, ..., o_m\}$ and $\overline{O} = \{\overline{o}_1, \overline{o}_2, ..., \overline{o}_m\}$ are the initial preferences and adjusted preferences of the DMs, respectively. Usually, o_k is a linguistic term belong to set S, $\overline{o}_k$ is a 2-tuple linguistic value. According to Dong et al. [35], the minimum adjustment cost consensus model in the group consensus process based on linguistic assessment is as follows

$$\min \sum_{k=1}^{m} d(o_k, \overline{o}_k)$$
$$s.t. \begin{cases} d(\overline{o}_k, \overline{o}^c) \le \varepsilon, k = 1,2,...,m \\ f(\overline{o}^c) = F_w(f(\overline{o}_1), f(\overline{o}_2), ..., f(\overline{o}_m)) \end{cases} \tag{2.1}$$

where f represents the linguistic information conversion function, $d(o_k, \overline{o}_k)$ represents the distance between o_k and $\overline{o}_k$, ε is the given distance threshold and $0 \le \varepsilon \le 1$, $F_w(\cdot)$ is the aggregation function used to obtain the collective preferences of the DMs.

For GDM when the assessments are expressed, both by numerical or linguistic information, the DMs' opinions can be not only elicited in the form of evaluation values in utility vectors, but also in the form of preference relations [37, 54, 55]. Let $R_k = (r_k^{ij})_{n\times n}$ be the assessment matrix given by DM e_k presented as preference relations and the preference relation r_k^{ij} belong to set S, then the MACM is as follows.

$$\min \sum_{k=1}^{m} \sum_{j=i+1}^{n} \sum_{i=1}^{n-1} \left| f(r_k^{ij}) - f(\overline{r}_k^{ij}) \right|$$
$$s.t. CL \ge \sigma \tag{2.2}$$

where $\overline{r}_k^{ij}$ represents the adjusted preference relation, CL represents the overall consensus level obtained, σ is the CL threshold given in advance, $0 \le \sigma \le 1$.

The consensus level can be considered from the following three aspects [189]:

- The consensus level on each pair of alternatives (x_i, x_j): CL_{ij}, where CL_{ij} is measured by the similarity between the alternative x_i and x_j.
- The consensus level on each alternative x_i : CL_i , where $CL_i = \sum_{j=1, j\ne i}^{n} CL_{ij} \Big/ (n-1)$.
- The overall consensus level: CL, where $CL = \sum_{i=1}^{n} CL_i \Big/ n$, $0 \le CL \le 1$, the closer the CL to 1, the closer the opinions between DMs.

(2) Minimum Cost Consensus Model (MCCM)

Compared with the previous model MACM, this type of model takes into account the cost of persuading each DM to change a unit's point of view, that is, the unit adjustment cost, which was proposed by Ben-Arieh and Easton [9] and Ben-Arieh et al. [10]. In general, the adjustment cost is the unit adjustment cost multiplied by the adjustment distance.

Suppose that $E=\{e_k \mid k=1,2,...,m\}$ is a set of DMs, $w=\{w_1,w_2,...,w_m\}$ is the DMs' weights with $\sum_{k=1}^{m} w_k = 1$ and $w_k \in [0,1]$. The symbols involved have the same meaning as above. The adjustment cost of adjusting a unit opinion of the DM e_k is recorded as c_k, the MCCM based on linguistic assessment is as follows.

$$\min \sum_{k=1}^{m} c_k \left| f(o_k) - f(\overline{o}_k) \right|$$
$$s.t. \left| f(\overline{o}_k) - f(\overline{o}^c) \right| \le \varepsilon, k=1,2,...,m \tag{2.3}$$

where f represents the linguistic information conversion function, $\varepsilon \in [0,1]$ is the distance threshold, $\overline{o}^c$ is the collective opinion of the optimal adjusted opinions.

The solution of the previous model is the optimal adjusted opinion, and then the collective opinion of the optimal adjusted opinion can be obtained. However, there is no explanation in the collective opinion model (2.3) of how to obtain the collective opinion of the optimal adjusted opinions. Therefore, Zhang et al. [213] proposed an extended version of the model (2.3) by considering the operator that aggregates DMs opinions as follows:

$$\min \sum_{k=1}^{m} c_k \left| \phi(o_k) - \phi(\overline{o}_k) \right|$$
$$s.t. \begin{cases} \left| \phi(\overline{o}_k) - \phi(\overline{o}^c) \right| \le \varepsilon, k=1,2,...,m \\ \phi(\overline{o}^c) = F_w(\phi(\overline{o}_1), \phi(\overline{o}_2), ..., \phi(\overline{o}_m)) \end{cases} \tag{2.4}$$

where ϕ represents the linguistic information conversion function, $F_w(\cdot)$ is the aggregation function that obtains the collective opinion of DMs.

The MACM and the MCCM models obtain the adjusted opinions automatically. After achieving the consensus, the selection process is presented to obtain an optimal alternative under agreement. Therefore, a GDM process should including a CRP and a selection process [57, 76, 145].

2.3 Multiple attribute group decision making

To better evaluate a decision making problem, DMs tend to perform the evaluation process from different aspects, which is called multi-attribute group decision making (MAGDM). With the advancement of society and the improvement of technology, more and more real world group decision-making problems are actually modelled as MAGDM problems. Moving from GDM setting to MAGDM setting introduces a great deal of new problems into the analysis, for example, the assessment of the attribute can be provided as different forms.

According to the different expressions of information given by DMs, the GDM problem can be sorted into two different points of view:

- According to the opinions assessment, where DMs considering multiple attributes and give their assessment values on each attribute on different alternatives.

  MAGDM refers to selecting the best alternative or ranking the possible alternatives according to several attributes from different DMs' opinions. For a MAGDM problem, let $A=\{A_1, A_2, ..., A_n\}(n \geq 2)$ be a finite set of alternatives, $C=\{c_1, c_2, ..., c_m\}(m \geq 2)$ be a set of attributes and $E=\{e_1, e_2, ..., e_g\}(g \geq 2)$ be a set of DMs. Let $W=\{w_1, w_2, ..., w_m\}$ be the associated weighting vector of DMs, where $w_k \geq 0(k=1,2,...,g)$ and $\sum_{k=1}^{g} w_k = 1$. Let $X^k = (x_{ij}^k)_{m\times n}(k=1,2,...,g)$ be the evaluation matrix given by DM e_k. The decision problem consists of ranking the alternatives and choosing the best one based on the evaluation matrices X^k, where the assessments provided by DMs are presented as evaluation matrices as follows.

$$\begin{array}{c} \\ \textit{alternative } A_1 \\ \textit{alternative } A_2 \\ \vdots \\ \textit{alternative } A_n \end{array} \begin{array}{c} \begin{array}{cccc} \textit{attribute } C_1 & \textit{attribute } C_2 & \cdots & \textit{attribute } C_m \end{array} \\ \begin{pmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1m}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2m}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1}^k & x_{n2}^k & \cdots & x_{nm}^k \end{pmatrix}_{n\times m} \end{array}$$

  A MAGDM process refers to different opinions provided by several DMs. Owing to the complexity of the decision making problem, quantitative or qualitative information are both used to represent the DMs' opinions on different attributes, such as, 2-tuple linguistic values [60], hesitant fuzzy linguistic term sets [141], interval data [69], grey number

[102], real number [152], etc. Usually, multi-attribute evaluation requires DMs to provide the relative importance of the attribute with respect to the overall objective of the problem [30].

- According to the preference structure used to provide the assessments [65]. As each DM has their own ideas, attitudes, motivations and expertise, it is common to see that the different DMs will give their preferences in a different way. Usually, it can be presented in one of the following three ways.

   1. *A preference ordering of the alternatives*. In this case, DM e_k gives his preferences on an alternative set A as an individual preference ordering, $O^k = \{o^k(1), o^k(2), ..., o^k(n)\}$, where $o^k(\cdot)$ is a permutation function over the index set $\{1, 2, ..., n\}$ [23, 149]. Therefore, an ordered vector of alternatives from best to worst is given.

   2. *A utility function*. In this situation, DM e_k gives his preference on the alternative set A as a set of n utility values, $U^k = \{u_i^k, i = 1, 2, ..., n\}$, where u_i^k represents the utility evaluation given by the DM e_k in terms of alternative x_i [160].

   3. *A multiplicative/additive preference relation*. In this condition, DM e_k gives his preferences on the alternative set A on the pair of alternatives. Let $R^k = (r_{ij}^k)_{n\times n} (k = 1, 2, ..., g)$ be the preference relations matrix given by DMs e_k, where r_{ij}^k represents the preference relation of alternative x_i in terms of x_j. The decision problem is how to rank the alternatives and choose the best one based on the preference relations matrices R^k, where the preference relations provided by DMs are presented as preference relations matrices as follows.

$$\begin{array}{l} \\ \textit{alternative } A_1 \\ \textit{alternative } A_2 \\ \vdots \\ \textit{alternative } A_n \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \end{array} \\ \begin{pmatrix} r_{11}^k & r_{12}^k & \cdots & r_{1n}^k \\ r_{21}^k & r_{22}^k & \cdots & r_{2n}^k \\ \vdots & \vdots & r_{ij}^k & \vdots \\ r_{n1}^k & r_{n2}^k & \cdots & r_{nn}^k \end{pmatrix}_{n\times n} \end{array}$$

Preference relations are frequently-used structures to reflect DMs' opinions by pairwise comparisons of alternatives. Many kinds of preference relations have been proposed, including fuzzy preference relations [55], intuitionistic fuzzy preference relations [158], hesitant fuzzy preference relations [222], linguistic preference relations [57] and hesitant fuzzy linguistic preference relations (HFLPRs) [168]. For MAGDM problems based on the expression form of preference relations, the consistency checking is the first priority before the selection process.

Despite there are different kinds of MAGDM, they have the common features as following [68]:

- Multiple attributes: for each MAGDM problem, it has multiple attributes evaluated by DMs;
- Assessment values: they are provided by DMs, which could be presented as various expressions and be expressed either as utility vectors or preference relations;
- Inconsistent units: attributes may have incommensurable units of measurement;
- Selection: an alternative or subset of alternatives is obtained as the solution to the problem.

2.4 Multiple attribute group decision making under uncertainty

Most of real-world GDM problems are defined under uncertain contexts, this is particularly interesting for MAGDM problems in which fuzzy information expressions have been commonly used for modelling preferences. Therefore, this subsection presents a previous knowledge about GDM in case of uncertainty and afterwards the methods for dealing with MAGDM problems under uncertainty are presented.

Owing to the fact that in real-world it is often hard to describe something precisely or completely, uncertainty is very common in reality. The uncertainties in decision problems mainly come from three different aspects, including the uncertainty of assessment value, the uncertainty of weights information and the uncertainty of reliability of assessment.

- *The uncertainty of assessment value*. An important phenomenon is that most of decision-making processes are dealing with uncertain and imprecise data. If the vagueness of the mankind process of decision making is ignored, the outcomes could be misleading. Fuzzy set theory [226] can model ambiguity and vagueness, additionally, it provides formalized tools that deal with the imprecision of many problems.

- *The uncertainty of weights information*. The weights information for MAGDM problems refer to the DM weights information and the attribute weights information. The increasing complexity of the decision circumstances makes it hard for DMs to provide the attribute weights or the appropriate DM weights in advance. The weights information just based on the DMs' knowledge and capabilities is not usually enough, many factors should be considered when determine the weights information, such as the similarity of preferences among DMs [78], the incompatibility of attributes [25], the credibility of the evaluations [135], etc.

- *The uncertainty of reliability of assessment*. In view of the complex decision environment and insufficient information that appears in them, DMs tend to trust in the reliability on the original assessment [225], which are often provided as linguistic information given by DMs in advance [224]. However, after the CRP, the original assessments have been usually changed if the desired consensus level has not been achieved. In this situation, the reliability of the adjusted assessment is less than the reliability of the original assessment. Hence, the adjusted assessment implies a greater uncertainty, which is worth to be studied and measured.

Since the process of MAGDM involves human intervention, uncertainty and vagueness are implicit factors. Take the different decision situations into consideration in which the GDM is conducted, MAGDM problem can be sorted into three kinds [84]: MAGDM in certain conditions [94, 131], MAGDM under risk environment [2, 109, 137, 182] and MAGDM under uncertainty environment [28, 40, 90].

Uncertainty includes randomness, fuzziness, random fuzzy mixing, interval, etc. Uncertain theory is the foundation and tools for studying uncertainty. The existing uncertain theories can be roughly divided into the following categories: random mathematical methods [26], interval mathematical methods [123], fuzzy mathematical methods [79], rough set theory [130], grey system theory [74], etc.

MAGDM under uncertainty is the central theme in this study, common methods for dealing with MAGDM under uncertainty are:

1. *Random mathematical method*. Random mathematical method is one of the earliest methods to deal with uncertainty in real life. It uses probability theory, mathematical statistics, random process and other models and methods in mathematical research to operate on data that follows a probability distribution.

2. *Interval mathematical method*. Accurate values sometimes cannot fully summarize certain data characteristics. Therefore, some scholars use interval numbers to describe certain uncertainties. A variable is represented by an interval number, and the variable can take any value within the value interval [148]. In most cases, the value of a variable satisfies a certain probability distribution in the interval [183]. Probability distributions commonly used could be uniform distributions that include uniform distribution and normal distribution.

3. *Fuzzy mathematical method*. It was developed on the basis of the fuzzy set theory introduced and developed by Prof. Zadeh [205]. According to the ambiguity of the type of set division and the extension of the boundary of the set, Zadeh uses fuzzy sets to expand the classic set. The ambiguity and uncertainty of data are described using the membership function. Fuzzy mathematics method has become one of the most effective methods to deal with uncertain information.

In real life, we will encounter some difficulties in choices inevitably, such as choosing a career, buying a property, choosing a partner, choosing a university, etc. Most of these choices are decision making situations under uncertainty in which multiple attributes describe the different actions of the problem. MAGDM under uncertainty includes five factors [82]: DMs, attributes set, alternatives set, attributes weight and decision making method. The general solving process is shown as Figure 2.4 as follows.

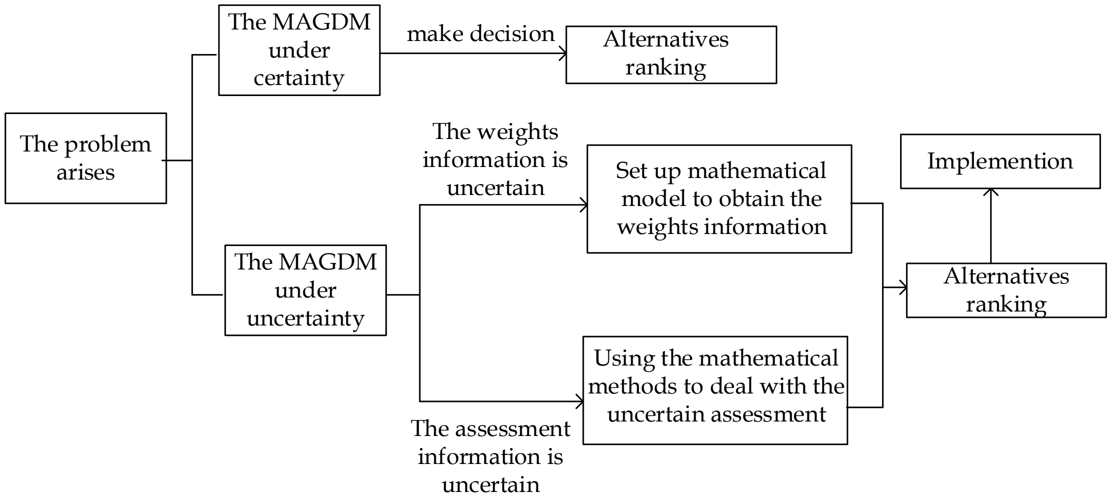

Figure 2.4: The general scheme of MAGDM under uncertainty

Besides, DMs can only predict the possible nature states of each alternative and their corresponding profit and loss values. Due to the lack of decision making information and experience, the probability of each natural state is unknown, therefore the attitude of DMs towards risk should be considered. There are five types of criteria to deal with the decision making problems under uncertainty [66]:

1. Maximum maximum criterion. Making decisions based on the best objective state, then find out the optimal alternative with the best expected effect. This criterion is actually based on the most optimistic estimation of alternative chosen, which is also the riskiest criterion. Maximum minimum criterion. Looking for the best expected effect alternative based on the worst objective state. The criterion is based on the most pessimistic estimation, thus the criterion is the most conservative criterion.

2. Minimum maximum regret value criterion. Assume that any action taken is a state with the largest regret value, then find the optimal alternative with the smallest regret value. This criterion is based on the worst objective state, which is similar to the maximum and minimum criterion.

3. Equal probability criterion. Assuming that the probability of the occurrence of each natural state is the same, then use a simple arithmetic average method to calculate the average return of each alternative in various natural states, and find the optimal alternative with the largest average return.

4. Hurwice criterion. This is a kind between the maximum maximum criterion and the maximum minimum criterion. When applying this criterion, we must first determine an optimism coefficient indicating the optimism of the DM, then calculate the weighted average of the maximum and minimum benefits of each alternative.

5. Minimize regret criterion: It is a decision-making method based on opportunity cost. It judges the advantages and disadvantages of each alternative according to the opportunity loss of each alternative. First, calculate the regret value of each alternative in natural state, find out the maximum regret value of each alternative, then compare it, and then select the alternative with the minimum maximum regret value as the optimal alternative.

In the environmental *uncertainty* conditions, the information about the problem is vague and imprecise, and can be modelled by fuzzy information. In this situation we talk about MAGDM problems under a fuzzy environment [96]. For MAGDM in which preferences are elicited as linguistic assessments, the classical mathematics cannot handle such uncertainty, then fuzzy linguistic approach has been successfully applied but, there are still situations that cannot be properly managed [206]. Especially for TD2L information, there are few literatures study on the CRP with TD2L approach, besides, the analysis of the reliability of assessment for TD2L information is neglected. Therefore, the further studies on TD2L representation and computation model are necessary for MAGDM under uncertainty.

In this memory, the MAGDM problems are studied based on TD2L information, which are MAGDM under uncertainty, the evaluation value and the reliability of the evaluation value are both expressed by linguistic information. Different extensions of the 2-tuple linguistic model and TD2L model will be proposed to handle the difficulties and respond to challenges stated in Section 1.1.

2.5 Multiple attribute group decision making based on linguistic information: State of art and limitations

In this section, fuzzy linguistic approach and its use to model the uncertainty in MAGDM problems are briefly revised, besides the limitations in current MAGDM approaches dealing with linguistic information are described to illustrate the necessity and importance of the projects.

2.5.1 Fuzzy linguistic approach

The fuzzy linguistic approach models the uncertainty by linguistic variables using words or sentences [206]. Most DMs cannot give exact numerical values to express their opinions, more appropriately, measurements are stated as linguistic assessments rather than numerical values. Linguistic MAGDM problems have provided very good results in many fields and applications [27, 117, 129, 188]. The use of linguistic information implies computing with words (CWW) processes [209]. There are different linguistic models for accomplishing such computing processes, one of the most widely-used is the 2-tuple linguistic model [60]. The 2-tuple linguistic model was inspired by the symbolic models used in decision making. Its main application field has been decision analysis and decision making. 2-tuple linguistic model has been widely used as basis for different models. For example, multi-attribute decision making based on 2-tuple linguistic model [134, 170, 175], consensus reaching process based on 2-tuple linguistic model [37, 218], 2-tuple linguistic aggregation operators [169, 171, 174], etc.

Suppose that $S=\{s_0,s_1,...,s_g\}$ is a pre-defined linguistic term set, and the cardinality of S is $g+1$. For any $s_i,s_j\in S$, the following properties should satisfy [62, 64]:

(1) The set is ordered: if $i>j$, then $s_i>s_j$;

(2) Maximum operator: if $s_i>s_j$, then $\max(s_i,s_j)=s_i$;

(3) Minimum operator: if $s_i>s_j$, then $\min(s_i,s_j)=s_j$;

(4) Negation operator: $neg(s_i)=s_{g-i}$.

Usually, the cardinality is odd number, more than 5 and less than 9 in general, for describing linguistic term set. Take linguistic term set S with 5 cardinalities as an example could be:

$$S=\{s_0=very\ poor, s_1=poor, s_2=medium, s_3=good, s_4=very\ good\}$$

With the purpose of obtaining more correct results in CWW processes, 2-tuple linguistic model (s_i,α) was proposed by Herrera and Martínez, where s_i is a linguistic term belong to the set S and α is a numerical value symbolizing the translation from s_i.

Definition 1 [60] Let $S=\{s_0,s_1,...,s_g\}$ and $\overline{S}=S\times[-0.5,0.5)$ be a linguistic term set and a 2-tuple set associated with S, respectively. The 2-tuple linguistic value (s_i,α) is equivalent to β through the function Δ as follows.

$$\Delta:[0,g]\rightarrow S\times[-0.5,0.5) \tag{2.5}$$

$$\Delta(\beta)=(s_i,\alpha),\ with\begin{cases}s_i, i=round(\beta)\\ \alpha=\beta-i,\alpha\in[-0.5,0.5)\end{cases} \tag{2.6}$$

where $round(\cdot)$ is the round operation in general, it allocates the closet integer number $i\in\{0,1,...,g\}$ to β.

Definition 2 [60] Let $S=\{s_0,s_1,...,s_g\}$ be a linguistic term set and $(s_i,\alpha)\in\overline{S}$ be a 2-tuple linguistic value. $\beta\in[0,g]$ is a numerical number transformed from (s_i,α) by the function Δ^{-1} as follows.

$$\Delta^{-1}:S\times[-0.5,0.5)\rightarrow[0,g] \tag{2.7}$$

$$\Delta^{-1}(s_i,\alpha)=\alpha+i=\beta \tag{2.8}$$

Remark 1 Given any two 2-tuple linguistic values (s_i,α_i) and (s_j,α_j) with $s_i,s_j\in S$, $\alpha_i,\alpha_j\in[-0.5,0.5)$, the relations between the two 2-tuple linguistic values can be given as follows.

(1) $(s_i,\alpha_i)>(s_j,\alpha_j)$ for $i>j$;

(2) $(s_i,\alpha_i)>(s_j,\alpha_j)$ for $i=j$ and $\alpha_i>\alpha_j$;

(3) $(s_i,\alpha_i)<(s_j,\alpha_j)$ for $i=j$ and $\alpha_i<\alpha_j$;

(4) $(s_i,\alpha_i)=(s_j,\alpha_j)$ for $i=j$ and $\alpha_i=\alpha_j$.

The 2-tuple linguistic information can describe the assessments in MAGDM. However, the real decision making problems may be more complex and uncertain, and it could happen that DMs have to provide not only their evaluations on alternatives, but also elicit the self assessments on the given evaluation results. In this situation, another dimension linguistic information expressed as linguistic term, which is needed to provide subjective evaluation or self-confidence on reliability of the given attribute preferences. Thus, Zhu et al. [225] first proposed 2-dimension linguistic information as follows.

Definition 3 [225] Let $S=\{s_0,s_1,...,s_g\}$ and $\dot{S}=\{\dot{s}_0,\dot{s}_1,...,\dot{s}_h\}$ be two linguistic label sets, where $g+1$ and $h+1$ are the cardinality of the sets S and $\dot{S}$, respectively. Then a two-dimension linguistic expression is denoted as $(s_u,\dot{s}_v)$, where $s_u\in S$ represents the evaluation about the alternative given by the DM, $\dot{s}_v\in\dot{S}$ represents the self-assessment of DM.

Inspired by the 2-tuple linguistic model [60], Zhu et al. [223] extended the two-dimension linguistic expression to two-dimension 2-tuple linguistic label. It can be seen as an extension of the 2-tuple linguistic model from one dimension to two dimensions.

Definition 4 [223] Let $S=\{s_0,s_1,...,s_g\}$ and $\dot{S}=\{\dot{s}_0,\dot{s}_1,...,\dot{s}_h\}$ be two linguistic term sets. Let $\alpha,\dot{\alpha}\in[-0.5,0.5)$ be two numerical values. Then $\hat{S}=\big((s_u,\alpha),(\dot{s}_v,\dot{\alpha})\big)$ is a TD2L expression, where $s_u\in S$, $\dot{s}_v\in\dot{S}$, (s_u,α) shows the evaluation value about the alternative provided by DMs, $(\dot{s}_v,\dot{\alpha})$ depicts the self judgment of the DM on reliability of the given evaluation value.

Remark 2 If $\alpha=\dot{\alpha}=0$, then $\hat{S}=\big((s_u,\alpha),(\dot{s}_v,\dot{\alpha})\big)$ is simplified as $\hat{S}=(s_u,\dot{s}_v)$. The more details about the original two dimension linguistic information expressed as $\hat{S}=(s_u,\dot{s}_v)$ are described in Zhu et al. [225].

Let $S=\{s_0,s_1,...,s_g\}$ and $\dot{S}=\{\dot{s}_0,\dot{s}_1,...,\dot{s}_h\}$ be two linguistic term sets, $\beta\in[0,g]$ be a numerical value representing the aggregation result of the indexes of the linguistic labels in S, and $\dot{\beta}\in[0,h]$ be the numerical value representing the aggregation result of the indexes of the linguistic labels in $\dot{S}$. According to Definition 2, there exist two functions Δ_1 and Δ_2 such that

$$\Delta_1:[0,g]\to S\times[-0.5,0.5),\beta\to\Delta_1(\beta)=(s_u,\alpha) \tag{2.9}$$

$$\Delta_2:[0,h]\to \dot{S}\times[-0.5,0.5),\dot{\beta}\to\Delta_1(\dot{\beta})=(s_v,\dot{\alpha}) \tag{2.10}$$

where $\alpha=round(\beta)$, $\dot{\alpha}=round(\dot{\beta})$, $\alpha=\beta-u$, $\dot{\alpha}=\dot{\beta}-v$, $\alpha,\dot{\alpha}\in[-0.5,0.5)$, $round(\cdot)$ is the usual round operation.

Definition 5 [223] Let $S=\{s_0,s_1,...,s_g\}$ and $\dot{S}=\{\dot{s}_0,\dot{s}_1,...,\dot{s}_h\}$ be two linguistic term sets. Let $\alpha,\dot{\alpha}\in[-0.5,0.5)$ be two numerical values, β and $\dot{\beta}$ be two numerical values representing the aggregation result of the indexes of the linguistic labels respectively in S and $\dot{S}$. The function Δ is equivalent to a binary numerical array $(\beta,\dot{\beta})$, which used to obtain a TD2L defined as

$$\Delta:[0,g]\times[0,h]\to(S\times[-0.5,0.5),\dot{S}\times[-0.5,0.5)) \tag{2.11}$$

$$(\beta,\dot{\beta})\to\Delta(\beta,\dot{\beta})=(\Delta_1(\beta),\Delta_2(\dot{\beta}))=((s_u,\alpha),(\dot{s}_v,\dot{\alpha})) \tag{2.12}$$

Definition 6 [223] Let $S=\{s_0,s_1,...,s_g\}$ and $\dot{S}=\{\dot{s}_0,\dot{s}_1,...,\dot{s}_h\}$ be two linguistic label sets. Let $\alpha,\dot{\alpha}\in[-0.5,0.5)$ be two numerical values, there is a function Δ^{-1}, that maps a TD2L to its equivalent binary numerical number $(\beta,\dot{\beta})$, which is defined as follows.

$$\Delta^{-1}:(S\times[-0.5,0.5),\dot{S}\times[-0.5,0.5))\rightarrow[0,g]\times[0,h] \tag{2.13}$$

$$\Delta^{-1}((s_u,\alpha),(\dot{s}_v,\dot{\alpha}))=(\Delta_1^{-1}(s_u,\alpha),\Delta_2^{-1}(\dot{s}_v,\dot{\alpha}))=(u+\alpha,v+\dot{\alpha})=(\beta,\dot{\beta}) \tag{2.14}$$

Remark 3 The general two dimension linguistic label can be represented by two dimension 2-tuple linguistic expression by combining 0 with each linguistic label, which means $(s_u,\dot{s}_v)=((s_u,0),(\dot{s}_v,0))$.

The linguistic term set [60], 2-tuple linguistic representation model [60], and TD2L approach [223] are introduced because they are the main assessment expression way throughout the study.

2.5.2 Multiple attribute group decision making dealing under linguistic environment

MAGDM problems coping with linguistic information are quite common, because of the advantage of expressing preferences as linguistic information. Through a plenty collection of studies, discuss and an extensive analysis, the core themes corresponding to MAGDM based on linguistic assessment have been discussed in current MAGDM studies.

Searching "linguistic" and "multiple attribute group decision making" as the subject words from the beginning of the year 2005 to the end of the year 2021 through Web of Science Core Collection, all the research and publication fruits of each year are depicted in Figure 2.5.

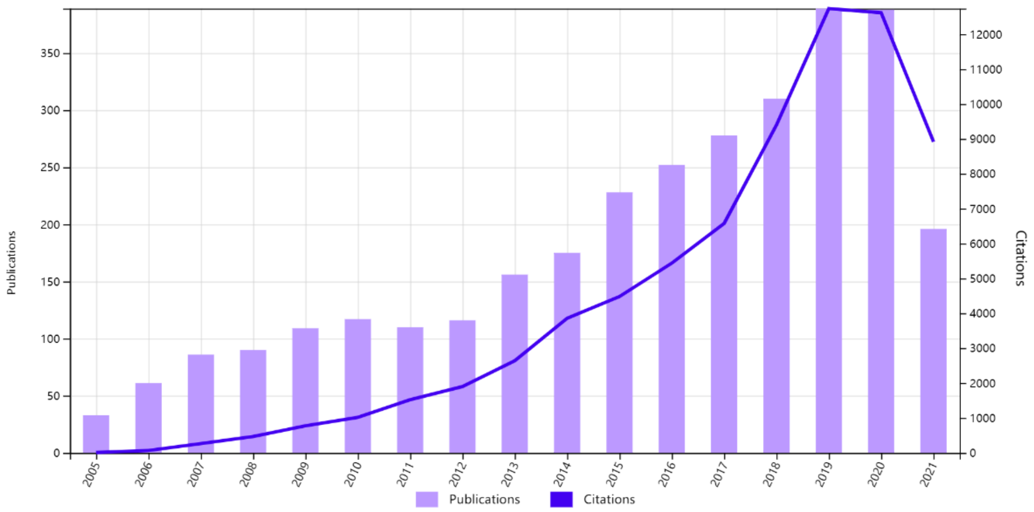

Figure 2.5: Publications of each year on MAGDM based on linguistic assessment

It has been evident that the related researches on MAGDM problems appears increasing tendency during recent years, and has been turned out to be one of the popular exploration topics in MAGDM. As the mainstream field in the research of MAGDM methods, the existing research has achieved relatively fruitful results.

Face to multiple alternatives, the joint participation of group DMs is required to evaluate the attribute values under different alternatives. The evaluation presented as linguistic information is a common phenomenon. The solution to such problems is divided into at least two processes: the acquisition of decision making data and the ranking of alternatives.

The acquisition of decision data also includes the acquisition of evaluation values and the acquisition of attribute weights information. evaluation values on attribute are the directly given by the DMs in the initial stage. In view of different decision making needs and decision making situations, DMs have their own preferences when giving linguistic evaluation values. According to the different manifestations of linguistic information provided by DMs, MAGDM based on fuzzy linguistic assessment is divided into the following main categories:

- **Multi-attribute group decision making based on linguistic terms.**
  Due to the fact that linguistic expression is the direct ways to express the opinions used by DMs for information transfer and owing to simply the MAGDM with linguistic information, some certain linguistic terms belong to a set given in advance, then DMs will choose one linguistic term from the certain set to express their preferences. Commonly MAGDM problems based on linguistic information assess the attributes by linguistic terms [67].

- **Multi-attribute group decision making based on linguistic 2-tuple model.**
  Some authors pointed out that the use of single linguistic terms is not enough to represent the linguistic information because during the CWW processes there is loss of precision [142]. Hence the linguistic 2-tuple model includes a parameter to improve the accuracy of the linguistic computations and the interpretability of the results [60]. The 2-tuple linguistic information is able to represent the linguistic results that do not match with the initial terms of the linguistic term set.

- **Multi-attribute group decision making based on hesitant fuzzy**

linguistic term sets.

When DMs hesitate among different linguistic terms to elicit their opinions, the use of just one linguistic term is not enough to represent such opinions. In these situations, DMs can provide their opinions by using comparative linguistic expressions which are based on hesitant fuzzy linguistic term sets [141].

- **Multi-attribute group decision making based on interval linguistic information.**

  When DMs cannot give specific linguistic evaluation information, but the evaluation value is given in the form of interval linguistic form or the weight information cannot be completely determined, the research on multi-attribute group decision-making methods is necessary for such uncertain linguistic information.

- **Multi-attribute group decision making based on linguistic distribution evaluation information.**

  When faced with group DMs expressing their opinions alone and unwilling to present them in the collective form, they often choose the expression form of linguistic distribution evaluation information, which can not only present the linguistic term for evaluation, but also reflect the probability information of the evaluation value.

The previous review shows that various studies have examined the characteristic of MAGDM based on linguistic assessment from different perspectives, and have achieved successful results, which has made a significant contribution to the progression of MAGDM. On the basis of this review, however, there are still some unresolved problems in the current research, and there are also some limitations. For sake of clarity, the following subsections describe such restrictions in further detail.

2.5.3 Limitations in current MAGDM under linguistic environment

As mentioned before, the current MAGDM research based on linguistic assessment has some limitations, as listed below:

1. It was previously mentioned that in MAGDM problems could be necessary CRPs for smoothing out conflicts. In such situations, the reliability of the adjusted linguistic preferences after the CRP has not been either studied or evaluated. The reliability of the initial linguistic preferences given by DMs presented as a second term in the TD2L information as a whole [32, 185, 186] is clear because represent DMs' preferences. However, after the CRP, the initial linguistic preferences may be changed [159, 214]. In this situation, the adjusted linguistic preferences are not so reliable, because some automatic adjustments either might not represent or might not be accepted by the DMs [45, 92, 136], thus the study of reliability is necessary for automatic CRP to assure the adjusted linguistic preferences are reliable.

2. In terms of the aggregation of the TD2L labels, the correlation between the two dimensions information has not been considered yet. The existing methods failed to consider the degree of relative importance of the two dimensions although they provided more importance to the assessment than the degree of reliability [202, 220, 223]. Besides, previous studies for handling the TD2L information have taken two dimensions into consideration as independent information without keeping in mind that the relations between the degree of reliability and the uncertainty of the assessment. Thus the general aggregation operators of TD2L labels do not reflect the reliability degree of the overall assessment, which may result in the misrepresenting of information.

3. For automatic CRP during decision making process, the minimum of the adjustment and the minimum of the consensus cost are also considered during CRP [22, 100, 181, 212]. However, how to make sure the number of the DMs keeping the original assessment as much as possible is an important problem, especially for large scale MAGDM. Besides, face to large scale number of DMs, the suitable way for clustering is the key for better obtaining the collective opinion of the DMs and searching for the deviant opinions. The existing clustering methods [80, 165, 192] are mostly the expansion of fuzzy c-means [11]. These methods usually need to preset several subjective clustering coefficients, which may reduce the objectiveness of the clustering results.

In view of the previous limitations, this research memory will conduct in-depth research on these deficiencies and associated issues to address these shortcomings and improve the theoretical methods and basis of existing MAGDM based on linguistic assessment.

2.6 Methods and models

Different models and methods applied across this research memory are concisely introduced in this subsection, including linear programming, probability theory, stochastic approach and so on. These are related to the different proposals that will be further explored in this study to complete the research objectives.

2.6.1 Linear programming

Linear programming is an important branch of operations research that has been studied, developed rapidly and widely used in economic activities such as water supply system development [154], production scheduling [12], social networks [47], nurse rostering problem [157] and so on. It is an indispensable requirement for GDM, and improving economic effects generally taking two ways [44]:

1. The improvement in technology, such as improving the production process, using new equipment and new raw materials.

2. The improvement of production organization and plan, that is, reasonable arrangement of human and material resources.

Generally, the linear programming consists of three elements: variables, objective function and constraints. The problem of finding the maximum or minimum value of a linear objective function under linear constraints is collectively called a linear programming problem. Decision variables, constraints, and objective functions are the three elements of linear programming.

In the process of GDM based on linguistic assessment, linear programming is a common method. Specific applications are reflected in the following aspects.

- Computing weights information. For MAGDM, the weights can be associated to DMs or attribute and they could be provided in advance or unknown. If the weights information is not given in advance, the objective method is needed to obtain the weights information. To construct an optimal model is a common way for obtaining the weights information [7, 29, 203]. The objective function is usually the minimum of the distance among the DMs' opinions or the balance of each alternative from the best or worst alternative.

- Obtaining the adjusted opinions during CRP. The consensus of the DMs' assessment is the prerequisite of further decision making. The adjustment of initial opinions is inevitable if the consensus level is not satisfied. The acquisition of the adjusted opinions is often through the building of a linear programming model [72, 93, 184, 216]. The objective function is usually the minimum adjustment between the original and adjusted opinions or minimum adjustment cost from the original opinion to the adjusted opinion.

2.6.2 Stochastic Approach

Probability theory [39], as the basis of the stochastic approach, is a branch of mathematics that studies the quantitative laws of random phenomena. Since probability theory involves extensive knowledge, here we only introduce the common knowledge often used in MAGDM. According to the category of the stochastic variable, the attribute value could be divided into three parts as:

1) Attribute value is discrete [190, 201]. It is the general distribution of attribute values. Usually, DM gives the assessment of the attribute in advance, then the attribute value is the possible value with possibility value equal to 1.

2) Attribute value is continuous [86]. Usually, the value range will be given in advance. In this situation, the attribute value is presented with probability density function. For the uncertain attribute value, its value is usually views as a stochastic variable that belong to normal distribution or uniform distribution.
   - Normal stochastic variable

     Suppose that the probability density function of continuous stochastic variable X is $f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-u)^2}{2\sigma^2}}$, where μ and σ^2 represent the expectation and variance of X, then X is normal stochastic variable, denoted as $X \sim N(\mu,\sigma^2)$. If $Y = aX + b$ with $X \sim N(\mu,\sigma^2)$, a,b are real numbers and $a \neq 0$, according to the knowledge of probability theory and mathematical statistics, Y is still a normal stochastic variable, its probability density function is as

$$g(y)=\frac{1}{|a|\sigma\sqrt{2\pi}}e^{-\frac{[y-(a\mu+b)]^2}{2(a\sigma)^2}} \tag{2.15}$$

where $Y \sim N(a\mu+b,(a\sigma)^2)$

Suppose that $X_1,X_2,...,X_n$ are n mutually independent normal random variables, denoted as $X_i \sim N(\mu_i,\sigma_i^2)$. If these stochastic variables are linearly added, which is $Z=c_1X_1+c_2X_2+\cdots+c_nX_n$, where $c_1,c_2,...,c_n$ are real numbers that exist at least one $c_i \neq 0$, then according to the knowledge of probability theory and mathematical statistics [39], Z is still a normal stochastic variable, its probability density function is as

$$h(z)=\frac{1}{\sqrt{2d\pi}}e^{-\frac{(x-c)^2}{2d}} \tag{2.16}$$

where $c=\sum_{i=1}^{n}c_i\mu_i$, $d=\sum_{i=1}^{n}c_i^2\sigma_i^2$, $Z \sim N(\sum_{i=1}^{n}c_i\mu_i,\sum_{i=1}^{n}c_i^2\sigma_i^2)$.

Generally, the larger the expectation μ and the smaller the variance σ^2 of a normal stochastic variable X, the greater the X. If $\sigma^2=0$, then X is a real number μ. The comparison rules between any two normal stochastic variables $X_1 \sim N(\mu_1,\sigma_1^2)$ and $X_2 \sim N(\mu_2,\sigma_2^2)$ are as [39]:

a) If $\mu_1>\mu_2$ and $\sigma_1^2<\sigma_2^2$, then $X_1>X_2$;
b) If $\mu_1=\mu_2$ and $\sigma_1^2<\sigma_2^2$, then $X_1>X_2$;
c) If $\mu_1>\mu_2$ and $\sigma_1^2=\sigma_2^2$, then $X_1>X_2$.

- Uniform stochastic variable

Suppose that the probability density function of continuous stochastic variable X is $f(x)=\frac{1}{b-a}, a<x<b$ and $f(x)=0, else$, where a and b are the boundary values of x, then X is a uniform stochastic variable, denoted as $X \sim U(a,b)$.

3) Attribute value is non-discrete discontinuous [153]. DM can only make sure the attribute value under certain circumstances, however, in some cases, the attribute value is uncertain [49, 101, 195]. In this situation, the attribute values are the combination of continuous and discrete

then they could be analyzed based on the above two cases.

Stochastic perspective is a common way to deal with uncertainty [41, 89, 91]. When the attribute values are not deterministic, the process of arriving at the weights of objects becomes more complicated [139]. As Honert [166] pointed out that the attributes allows to interpreted as a stochastic variable when there is a need to manage quite a few evaluation values for the same assessment. Thus, stochastic approach can be defined as an approach to handle uncertainty that defines probability distribution for each input value or parameter in the MAGDM process [125]. For example, Tervonen et al. [161] proposed a stochastic method based on stochastic multi-criteria acceptability analysis for assessing the stability of the parameters in sorting problems. Celik et al. [15] gave a comprehensive review on stochastic MAGDM applications and approaches. Therefore, the stochastic approach is very useful for the condition when a MAGDM is based on the stochastic initial information.

Chapter 3

Research Results

A summary of the main proposals will be presented in this chapter. Research outputs and results will be mentioned for each proposal in brief. Different objectives are implemented through three proposals are related with them presented in the Introduction section as follows:

- A new representation and computation model of TD2L from stochastic perspective.
- The calculation of the objective reliability degree of the adjusted assessment modeled by TD2L information.
- A CRP based on minimum adjustment in consideration of the tolerance of DMs for changing their opinions in GDM.

3.1 A MAGDM method based on TD2L information from a stochastic perspective

For the purpose of reaching the first proposal mentioned in Section 1.2, we spotlight the operation rules between TD2L labels from the stochastic perspective, and then analyze the limitations in current computation model of TD2L. Afterwards, a MAGDM method with the TD2Ls assessment from the stochastic perspective is proposed and tested on a real life decision making problem.

3.1.1 A new representation and computation model of TD2L

As mentioned previously, TD2L label represents the assessment and its reliability of the assessment are both given by DMs simultaneously. Therefore, all preferences provided as linguistic term without reliability degree can be viewed as uncertain ones. In general, due to the boundness in perceptions and the complication of objects things, the degree of reliability is a subjective judgement on reliability of the provided preferences. The existing computation model of TD2L considers the two dimensions information as dependent information, in fact, the two dimensions are related to each other, thus the relations should be considered in the process of information transformation.

To address such limitations about the representation and computation of TD2L labels, a new proposal that designs to extend a new representation and computation model from a stochastic perspective is introduced, and then to propose the new rules for similarity measure and comparison rules for TD2L labels according to the importance proportional of the two dimensions linguistic information.

3.1.2 MAGDM method based on the new TD2L representation model

This new MAGDM method is mainly based on the new aggregation function of TD2L labels, the new MAGDM method based on the proposed TD2L computation model is then developed accordingly, all of the contributions are enumerated and briefly explained below:

1. This proposal aims at developing a corresponding rule from TD2L label to a stochastic variable and its inverse. Hence, the comparison and similarity measurement between two TD2L labels have been defined with the consideration of the relative importance degree of the two dimensions of information.
2. To deal with the uncertainty of the initial assessment, another dimension linguistic information is added to ensure the reliability of the initial assessment. To further deal with the TD2L information, a TD2L label is

viewed as a stochastic variable with corresponding expectation and variance.

3. To reflect the reliability of the overall assessments accurately, a new aggregation function of TD2L labels is developed. When all DMs provide the identical preferences about the decision objects, then the collective preferences derived from a general aggregation function is the same with the preferences given by all DMs, Obviously, the degree of reliability of the collective preferences is improved by a new aggregation function of TD2Ls compared to initial preferences provided by each DM, which is more acceptable and explainable in real MAGDM problems.

Additionally, an experimental process on a real life MAGDM problem has been described about a business group with a great wealth of investment experience, which determines to choose an appropriate investment plan from several small companies, then implement comparisons with other studies regarding the optimal alternative.

The article associated to this proposal is the following one:

Z. L. Wang, Y. M. Wang, L. Martínez. A stochastic perspective on a group decision making method based on two-dimension 2-tuple linguistic information. International Journal of Fuzzy Systems, 2022, https://doi.org/10.1007/s40815-021-01199-3.

3.2 A GDM method based on two-stage MACM with the TD2L labels for reliability measure

The DMs' adjusted preferences are different from the initial ones after carrying out a CRP based on minimum adjustment cost rule. The initial preferences were reliable, however, the reliability of the adjusted preferences cannot be ensured. Obviously, the reliability of the adjusted preferences is critical in the decision making process. Adjusted preferences with high agreement degree but low reliability degree would be worthless. Accordingly, the adjusted preferences and its reliability degree should both be discussed for solving GDM problems. However, it has been disregarded in existing studies when DMs' preferences are automatically adjusted without moderator's suggestion.

3.2.1 The features and related limitations of MACM

CRPs can be classified according to their feedback process into two types in accordance with the categorization provided in [88]: CRP with feedback or CRP without feedback. In general, CRP with feedback improves the level of agreement among DMs, which also leads to the reliability increase on adjusted preferences. However, consider the high-quality for some decision making problems, like emergency decision making [33, 191, 194] within the limited time, it is not impossible to take too much time on feedback process and wait the DMs change their preferences. Because less time on decision making means more effective and successful in this situation. We try to develop an automatic CRP to balance the low cost of CRP without feedback and the increased reliability of CRP with feedback. It is an automatic CRP with minimum adjustment cost rule with the consideration of the reliability of the adjusted preferences. The critical points of this subsection and some involved outputs gained are presented in brief:

1. The use of the existing MACMs leads to accepted preferences by changing DMs' initial ones. However, the reliability of the adjusted preferences received by these models with minimum adjustment cost rule is often neglected, which results in the uncertain reliability of the collective preference. Accordingly, an objective detection approach on reliability of adjusted opinions is necessary for GDM.

2. Regarding the adjustment cost, the more DMs' opinions needed to change, the higher the cost of the adjustment. Therefore, the number of the adjusted preferences should be considered. With the number of the involved DMs increased, GDM turned into large scale GDM problems, if too many DMs need to change their initial opinions, then the CRP would be with low execution. A two-stage MACM with the minimum adjustment rule and the minimum number of adjusted preferences rule is considered. It includes the following two stages:

   Stage one: To improved the consensus degree maximized at the alternatives level with the minimum preferences adjustment rule.

   Stage two: To acquire the adjusted preferences with a certain consensus degree according to the first stage adjustment solution.

3. The connection between the reliability of the adjusted preferences and the total preference adjustment are discussed. Not only the adjustment distance and the number of the adjusted opinions are considered, but also the reliability of the adjusted opinions is derived from the measure of the distance between the original and adjusted opinions.

3.2.2 A large scale GDM method considering the two-stage MACM with the TD2L labels for reliability measure

As previously mentioned, the decision method for dealing with large number of DMs and the reliability measure of the adjusted opinions after CRP are challenges for large scale GDM, aiming at improve the existing methods, a new large scale GDM method is proposed that considers a large number of DMs and a reliability measure of the adjusted opinions after CRP during the decision making process. At the same time, our proposal presents the alternatives ranking with reliability, which illustrates the reliability of one alternative is better or worse than another alternative. The initial assessments provided by DMs are linguistic terms, 2-tuple linguistic information will appear during calculation, while the final decision result is made based on TD2L information. In the process of linguistic transformation from linguistic term to TD2L information, a large scale GDM method based on a two-stage MACM plays a key role. This proposed method has the following novelties.

1. A new support degree (SD)-based clustering method is proposed to classify the large number of DMs into several subgroups for large scale GDM.

2. A new two-stage automatic consensus model with minimum adjustment is proposed.

3. The connections between the reliability of the adjusted preferences and the adjustment from the original preferences to the agreed ones are considered to attain a reliable collective decision solution by using TD2L approach.

4. A novel ranking rule for obtaining the optimal alternative is developed, the new ranking rule not only takes the optimal alternative into consideration but

also refers to the alternative with greatest reliability.

To illustrate the validity and feasibility of our proposal, we have carried out numbers of comparisons with the classical existing CRP methods that are conducted from different perspective.

The contribution associated to this proposal is the following one:

Zelin Wang, Rosa M. Rodríguez, Ying-Ming Wang, Luis Martínez. A two-stage minimum adjustment consensus model for large scale decision making based on reliability modeled by two-dimension 2-tuple linguistic information. Computers & Industrial Engineering, 2021, 151(3): 106973.

3.3 A CRP with MACM in GDM considering the tolerance of DMs for changing their opinions

During our research related to CRP with MACM, it was detected that there are still multiple difficulties that have not been effectively dealt with, including the following ones:

1. Classically many CRPs consider that the minimum distance between original preferences and the adjusted preferences is the key rule for achieving the agreement, but in classical MACM the number of adjusted preferences should be also considered. Zhang et al. [211] proposed a MACM with these two consensus rules, however, they are separately used in the consensus mechanism, which complicates the consensus process.

2. To reach an agreement among DMs, there will be a lot of consensus rules, like, minimize adjustment between the original and adjusted opinions, minimize the number of the original opinions need to be changed, maximize the number of DMs that could stay their original opinions, etc. However, how to balance these consensus rules is also an important factor, which will influence the decision results of GDM.

3. There must be exist an upper and lower limit that DMs could accept or reject the adjusted opinions during the CRP. If the tolerance of DMs for changing their opinions is neglected, then the feedback mechanism is needed, which is contradict with the automatic CRP. Therefore, the tolerance of DMs for

changing their opinions is necessary for CRP in GDM.

In order to address previous issues, a new consensus model based on the consideration of tolerance degree of DMs, and two consensus rules are considered:

(i) minimum distance between the original and adjusted preferences, and (ii) minimum number of adjusted preferences. Furthermore, the detection of the degree of the reliability of adjusted preference is presented. Therefore, the third proposal stated in Section 1.2 can be achieved.

3.3.1 Dealing with the tolerance of DMs on the adjusted opinions

The proposed CRP considers the following two consensus rules: (1) minimize the distance between the original and adjusted preferences. (2) minimize the number of adjusted preferences. In order to balance these two consensus rules, a DM tolerance degree that defines how much is willing the DM to change his original opinion will play an important role, which means DMs only accept the adjusted preferences within tolerance interval.

The adjustment for DMs' preferences is necessary if the overall consensus level is less than the consensus threshold. Hence, DMs' tolerance degree is proposed as the maximum change that DM willing to accept for the adjusted preferences, which need to be considered. The adjusted preferences to be accepted must satisfy the normalized distance between the original and the adjusted preferences less than the tolerance degree of DMs. The tolerance degree ranges from 0 to 1.

If DM does not accept any change of the original preferences, then he/she is a stubborn DM, which means the tolerance degree is 0. If tolerance degree is 1, then DM could accept any change of the original preferences, where he/she is a benevolent DM.

In fact, the consideration of tolerance degree of DMs is a strict view for minimizing the number of the adjusted preferences. If the minimum number of adjusted preferences is the only condition to be considered, then the distance between the original and adjusted preferences may out of the tolerance interval of DMs. In such situation, the minimum number of adjusted preferences is meaningless.

Thus, it is important to consider both DMs' tolerance degree of DMs and the minimum number of adjusted preferences. To simplify the CRP, a consensus mechanism with priority adjustment rule is designed, then a linear programming model with the minimum number of adjusted preferences is developed.

3.3.2 A CRP in GDM based on the reliability measurement considering the tolerance of DMs

It has been already pointed out the lack of considering of the tolerance of DMs for changing their opinions will lead to the unreliability of the adjusted opinions during CRP. For better understanding, Algorithm Ⅰ is designed to obtain the optimal adjusted preference with minimum number of adjusted preferences considering the tolerance interval of DMs.

Algorithm Ⅰ
Input: The original preference matrices provided by DMs, the tolerance degree of DMs, the consensus threshed.
Output: The final adjusted preference.
Step 1: Check the overall CL of DMs' preferences based on three consensus levels as described in Section 2.4.2, if whole CL is larger than or equal to the consensus threshed, then the CRP is done, otherwise continue to the next step.
Step 2: Set up consensus model with the first round, if it can be solved by software LINGO 11 and obtain the optimal preference relations. Then the output preferences are as the adjusted preference relations. If the model is unsolved, then go to the next step.
Step 3: Set up consensus model with second round and repeat the process as described in Step 2, if it can be solved, then output the adjusted preferences as the obtained results. If the model is unsolved, then repeat the above steps until the consensus model can be solved.

After using the Algorithm Ⅰ, the adjusted opinions are derived, however, the reliability of the adjusted preferences are not assured. Here we give a reliability model to compute the degree of reliability of the adjusted preferences by using the proposed consensus model, where the reliability degree comes from the concept of stability degree of the original preferences. In this subsection, we introduce a concept: stability degree of original preferences. Then, a comparison measure for TD2L is provided in order to facilitate the selection of the best alternative of the GDM problem.

The reliability degree of the adjusted preferences derives from the stability degree of original preferences, which describes the similarity between the original and adjusted preferences after CRP. The more similar the original preference to adjusted preference is, the higher the stability degree of adjusted preferences is. The higher the stability degree is, the more stable the initial preferences are, then the reliability degree of adjusted preference is more likely higher.

With the introduction of the consensus model with new adjustment rule and the description of the relations between the adjustment and the reliability of the adjusted preferences, the steps to execute the decision making process are as follows.

Step 1: To use Algorithm Ⅰ to obtain the optimal adjusted opinions.

Step 2: To compute the reliability of the adjusted preferences.

Step 3: To obtain the final assessment information expressed as TD2L labels.

Step 4: To compare the TD2L labels, then obtain the alternative ranking.

Finally, an illustrative example is shown to certificate the effectiveness of the proposed method.

The contribution associated to this proposal is the following one:

Z. L. Wang, R. M. Rodríguez, Y. M. Wang, L. Martínez. A Consensus Reaching Process with Minimum Adjustment in Group Decision Making with Two-dimensional 2-tuple Linguistic Information based on Reliability Measurement. 2021 IEEE International Conference on Fuzzy Systems, Luxembourg, 11th-14th July.

Chapter 4

Publications

By virtue of the provisions of article 25, point 2, of the current regulations for Doctoral Studies at the University of Jaén, corresponding to the RD program. 99/2011, this chapter presents the publications that make up the core of this doctoral thesis.

These publications correspond to two scientific articles published in International Journals indexed by the JCR (Journal Citation Reports) database, produced by Clarivate Analytics and a conference paper indexed in Engineering Village.

4.1 A new presentation and computation model of TD2L from stochastic perspective

- State: Published.
- Title: A stochastic perspective on a group decision making method based on two-dimension 2-tuple linguistic information.
- Authors: Zelin Wang, Ying-Ming Wang, Luis Martínez.
- Journal: International Journal of Fuzzy Systems.
- Abstract: The two-dimension 2-tuple linguistic (TD2L) label, based on the traditional 2-tuple linguistic representation model, adds another 2-tuple linguistic term to express the reliability degree of the assessments. However, the reliability degree is a subjective evaluation on reliability of the given assessments and variables due to the limitations in cognitions and the complexity of decision objects. All assessments without total reliability degree are viewed as uncertain ones. Based on this idea, this paper proposes a new TD2L representation model from a stochastic perspective. The assessment expressed by TD2L is a variable that fluctuates around the given linguistic term, and the fluctuation range is decided by the reliability of the assessment. Therefore, the assessments are regarded as stochastic variables, where the expectancy and deviation of the stochastic variable are corresponding to the first dimension and the second dimension information of TD2Ls, respectively. Consequently, two new aggregation functions (AFs) for aggregating TD2L labels based on the algorithms among stochastic variables are proposed. In addition, the comparison and similarity measure between TD2L labels are developed, which considers the relative importance of the two dimensions of TD2L labels. Finally, the proposed method is applied to an investment decision of medium sized enterprise (SME) and alternatives ranking is provided with the probability of superiority. A comparison analysis conducted from three aspects illustrates the effectiveness of the proposed method.
- DOI: https://doi.org/10.1007/s40815-021-01199-3

- ISSN: 1562-2479.

- Impact Factor (JCR 2020): 4.673.

  - Quartiles:

  - * Quartile 1 in Computer Science, Information Systems. Ranking 35/162

4.2 The measurement of the reliability of the adjusted preferences modeled by TD2L information

- State: Published.

- Title: A two-stage minimum adjustment consensus model for large scale decision making based on reliability modeled by two-dimension 2-tuple linguistic information

- Authors: Zelin Wang, Rosa M. Rodríguez, Ying-Ming Wang, Luis Martínez.

- Journal: Computers & Industrial Engineering.

- Abstract: Consensus reaching processes (CRPs) have been required to assure the consensus in large scale group decision making (LSGDM). Opinion reliability detection has been demanded to ensure the reliable of the original information. The reliability degree of original opinions is often given in advance as subjective evaluation, and after adjustment during CRP, the reliability of the adjusted opinions is often neglected especially for automatic CRP, which may lead to unreliable decisions. In real decision making, considering the interest of decision makers (DMs) themselves, the self-assessment of the DMs on the reliability of the given opinions could be manipulated by DMs. To reduce the subjective of decision making, we propose a method to obtain the objective detection on the reliability of adjusted opinions through a two-stage minimum cost consensus model based on 2-tuple linguistic additive preference relations. Firstly, a support degree (SD)-based clustering method will be developed for classifying DMs into several subgroups to make more manageable the large number of DMs. Subsequently, a two-stage minimum adjustment consensus model will be presented to improve the consensus level (CL) gradually. Eventually, the adjusted opinions will be presented as two-dimension 2-tuple linguistic (TD2L) information. A comparative performance analysis of this CRP based LSGDM approach will be provided to show its effectiveness.

- Volume: 151. Page: 106973. Year: 2021.

- DOI: 10.1016/j.cie.2020.106973

- ISSN: 0360-8352.

- Impact Factor (JCR 2020): 5.431.

  - Quartiles:

  - * Quartile 1 in Computer Science, Interdisciplinary. Ranking 21/112.

4.3 A CRP with minimum adjustment in GDM considering the tolerance of DMs for changing their opinions

- State: Published.
- Title: A consensus reaching process with minimum adjustment in group decision making with two-dimensional 2-tuple linguistic information based on reliability measurement.
- Authors: Zelin Wang, Rosa M. Rodríguez, Ying-Ming Wang, Luis Martínez.
- Journal: 2021 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE) Luxembourg, 2021, 11th-14th, July.
- Abstract: This paper designs a consensus reaching process (CRP) with minimum adjustment considering that decision makers (DMs) have a tolerance for changing their preferences, which means DMs only accept the adjustment if such adjustment fall within their tolerance interval. In addition, two consensus rules are considered: minimum distance between original and adjusted preferences, and minimum number of adjusted preferences. Unlike to classical CRP, our proposal includes a reliability detection of adjusted preferences, in order to avoid biased solutions because of the inherent subjectivity of group decision-making problems. To obtain the agreed adjusted opinions and their reliability, the two-dimensional 2-tuple linguistic (TD2L) information is used in which, the first dimension of a TD2L is 2-tuple linguistic value that assess the adjusted preference and, the second dimension information is another 2-tuple linguistic value assessing the reliability degree of the adjusted preferences, which is derived according to similarity between adjusted and original preferences. Then a novel comparison method for TD2Ls is developed. Finally, an illustrative example is given to verify the proposed method and the results show that the approach is feasible and effective.
- DOI:10.1109/FUZZ45933.2021.9494397
- Quality indicator:

Ranking Core: Core A

Chapter 5

Conclusions and Future Works

The research memory is concluded in this chapter by reviewing the conclusions about the main proposals and obtained results, and indicating achievable future works.

5.1 Conclusions

GDM is widely applied in real life to solve important and complicated problems in a range of domains, such as emergency decision making [51, 71, 180], medical service assessment [150, 179], the selection of supplier [16, 17, 48] etc. Since the importance of GDM in selecting and evaluating the management and economic issues, several models and approaches for GDM problems have been developed [81, 103, 122, 152].

MAGDM involves that DMs provide evaluations regarding the performance of the alternatives under multiple attributes [75]. With the increasing complexity of the decision making environment and the limited DMs' expressiveness, the MAGDM based on linguistic assessment has attracted more attention [129, 132, 143, 196]. Considering the complexity and uniqueness of the linguistic expression, the general MAGDM methods are not always suitable for linguistic MAGDM. Even though, the existing research has achieved numerous and successful achievements

[27, 101, 188], but there are still many methods and theoretical systems need to be improved. Besides, in some situations, the use of only one dimension to represent information by linguistic information is not enough to ensure the accuracy of such information, the reliability of the assessment is also an important factor to be considered. Therefore, the study of TD2L is necessary and meaningful. Furthermore, several new methods need to be developed aiming at solving MAGDM based on TD2L information.

Throughout the research project we have obtained novel, remarkable and relevant results. Several new views in dealing with processes of MAGDM by TD2L approach are displayed and new research opportunities for the future works are pointed out. As a consequence, the following results is concluded from our research:

1. In spite of the successful application of the two-dimension linguistic information to deal with the representation and computation of two-dimension linguistic labels, the analysis of the uncertainty of assessments according to the second dimension information has not been explored. Thus, a new representation model of TD2L from stochastic perspective has been proposed. A corresponding rule from TD2L label to a stochastic variable and its inverse have been presented, which is more suitable for the computation of large scale GDM. And the similarity measurement and comparison rule between any two TD2L labels have been proposed refers to the computation of the relative importance degree of the two dimensions of TD2L labels from the stochastic perspective, which has made that the decision analysis provides more useful information.

2. The reliability of the initial assessment provided by DMs is usually provided as linguistic information with two dimensions for MAGDM problems, however, during the CRP, especially for CRP with automatic feedback, the reliability of the adjusted preferences cannot be guaranteed. For this reason, a novel two-stage consensus mode with minimum adjustment rule for large scale GDM problems based on TD2L expressions has been proposed, which refers to the two consensus rules: minimize the total adjustment and the number of adjusted preferences. Besides, the connections between the the reliability degree of the adjusted preferences and total preference adjustment have been discussed. The proposed

method has completed the two dimension 2-tuple linguistic approach for large scale GDM.

3. The measurement of the degree of reliability of the adjusted preferences can increase the accuracy of the decision results with automatic CRPs. However, the reliability is based on the reasonable tolerance of DMs on changing their opinions. Therefore, the tolerance degree of the DMs is a crucial factor should be considered in advance during the CRP. The proposal of the new consensus model based on the consideration of tolerance degree of DMs has improved the MAGDM method with TD2L information based on reliability measurement.

5.2 Future Works

There are still some challenges for MAGDM problems based on linguistic assessment and the TD2L approach even though several approaches and tools have been put forward in this research memory. In the period ahead, we will focus on the development of solutions for new MAGDM problems and the extension of the presented proposals:

1. Usually, it is considered that DMs are completely rational in most existing researches, however, in real situation, DMs are bounded rational, and they may feel uncomfortable when they are suggested to adjust their opinions within a minor adjustment range, thus non-cooperative behavior could appear in GDM. Therefore, the psychological behavior of DMs would be considered in the future work.

2. For CRP in GDM, the minimum adjustment is often considered to make the DMs' opinions changed, however, make DMs to change their opinions have different levels of difficulty, the unit adjustment cost is almost given in advance with the default condition that they are symmetric, in fact, the unit adjustment cost is not always identical in upward and download adjustment tendencies [70]. Therefore, the determination of the unit adjustment cost is also needed to be considered, especially for the symmetric unit cost, which is a puzzle for minimum cost consensus model and is the next step worth thing

about it.

3. As the linguistic term set for expressing the initial assessment is sometimes unbalanced, the linguistic term for expressing the reliability of the adjusted opinions could be also unbalanced, how to design the representation and computation model with the unbalanced reliability information will be studied in the future.

5.3 Additional Publications

When it comes to the diffusion of our research results, except for the publications involved in this research memory, the following contributions are highlighted:

- International Journals
  - Z. L. Wang, Y. M. Wang. Prospect theory-based group decision-making with stochastic uncertainty and 2-tuple aspirations under linguistic assessments. *Information Fusion*, vol. 56, issue 1, pp. 81-92, 2020.
  - Z. L. Wang, Y. M. Wang, L. Wang. Tri-level multi-attribute group decision making based on regret theory in multi-granular linguistic contexts. *Journal of Intelligent & Fuzzy Systems*, vol 35, issue 3, pp. 793-806, 2018.
- International Conferences
  - Z. L. Wang, R. M. Rodríguez, Y. M. Wang, L. Martínez. A Novel Method for Group Decision Making based on Two-dimensional 2-tuple Linguistic from a Stochastic Perspective.
    International Virtual Workshop on Business Analytics Eureka 2021 held in Ciudad Juárez (México), 2-4 June 2021.

Appendix A

Resumen escrito en Español

Título de la tesis: *Enfoque lingüístico de dos dimensiones y 2 tupla para toma de decisión en grupo con múltiples atributos bajo incertidumbre*

El presente apéndice incluye el título, el índice, la introducción, la motivación, los objetivos, la estructura, el resumen, las conclusiones y los documentos futuros como parte de las condiciones necesarias para obtener un doctorado de conformidad con el párrafo 2 del artículo 23 de la Ordenanza sobre estudios de doctorado de la Universidad of Jaén.

El índice se proporciona primero. A continuación se presenta brevemente el estudio y se detallan la motivación, los objetivos y la estructura de los capítulos. Por último, se presentan las conclusiones y el trabajo futuro.

Contenido

A.1 Motivación

La toma de decisiones en grupo (TDG) es una rama de la teoría de decisiones que se ha aplicado ampliamente en escenarios del mundo real para resolver problemas de decisión importantes y complicados en una variedad de dominios, como salud pública [5], proyectos de ingeniería civil [127] y política exterior [8]. En los problemas de TDG, los decisores suelen evaluar alternativas basadas en múltiples atributos, lo que se conoce como un problema TDG con múltiples atributos (TDGMA) [82]. Sin embargo, debido a la complejidad de proporcionar las opiniones y la racionalidad limitada de los seres humanos, el uso de términos lingüísticos es más intuitivo, flexible y cercano al lenguaje utilizado por los seres humanos para evaluar los criterios en TDGMA que el uso de valores numéricos. El concepto de variable lingüística fue introducido por Zadeh [206], para modelar la incertidumbre de la información. Una variable lingüística es una variable cuyos valores no son números sino palabras u oraciones en lenguaje natural o artificial. Es una herramienta muy utilizada para resolver problemas de TDGMA con información cualitativa. Por tanto, existen muchos enfoques de TDGMA que utilizan variables lingüísticas para modelar la incertidumbre [54, 109, 111, 117, 177].

Para resolver un problema de TDGMA con información lingüística, es necesario realizar procesos de computación con palabras (CWW) [121, 208, 210] (ver Figure A.1), que es una de las metodologías más utilizadas en toma de decisión lingüística.

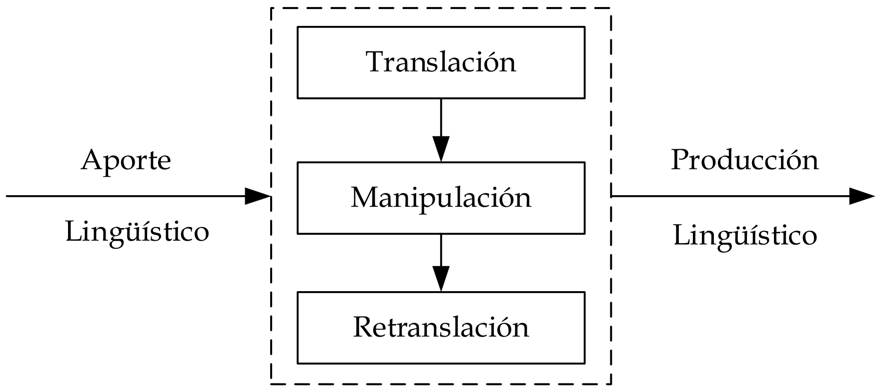

Figure A.1: Proceso de computación con palabras

En los procesos de CWW en TDGMA, los resultados lingüísticos se obtienen a partir de entradas lingüísticas, que son fácilmente comprensibles y se representan adecuadamente. En consecuencia, se han desarrollado varios modelos computacionales lingüísticos para llevar a cabo los procesos de CWW [3, 60, 61, 118, 172, 197]. Estos modelos siguen el esquema de computación descrito por Yager [198, 199] que señala la importancia de los procesos de translación y retranslación en CWW. Sin embargo, existen algunas limitaciones cuando se realizan procesos de fusión sobre variables lingüísticas, según estos modelos originales. En estos enfoques, los resultados no suelen coincidir exactamente con ninguno de los términos lingüísticos iniciales, por lo que se debe desarrollar un proceso de aproximación para expresar el resultado en el dominio de la expresión inicial [63]. Esto produce la consiguiente pérdida de información y por ende la falta de precisión.

Para evitar tal inexactitud en el paso de retranslación, se propuso el modelo lingüístico 2-tupla [60]. Una representación lingüística de 2-tupla está compuesta por un término lingüístico y un valor numérico llamado translación simbólica que representa el desplazamiento del término lingüístico. Por tanto, evita la pérdida de información y obtiene resultados más precisos e interpretables. Por ello, el modelo lingüístico 2-tupla destaca como uno de los más utilizados en toma de decisiones [119, 142].

Además, se han propuesto varias extensiones del modelo lingüístico 2-tupla para resolver problemas de TDGMA, como el modelo semántico de 2-tupla [1, 163, 164], el modelo lingüístico multigranular de 2 tupla [38, 62, 197], el modelo lingüístico proporcional de 2-tupla [172, 173], modelo numérico escalar [34, 36, 37], etc. Teniendo en cuenta la extensa y exitosa investigación de modelos lingüísticos basados en 2-tupla, Martínez y Herrera [120] realizaron una revisión del estado del arte de estos modelos. Los modelos lingüísticos de 2-tupla se han utilizado con éxito para obtener resultados precisos e interpretables, pero la fiabilidad de las evaluaciones también es un tema importante para los decisores. Los modelos de toma de decisiones existentes basados en información lingüística 2-tupla asumen que todas las evaluaciones tienen el mismo nivel de confianza [112], lo que no es realista en la práctica. Por tanto, Zhu et al. [225] propusieron el concepto de información lingüística bidimensional, que incluye la información de fiabilidad de las evaluaciones subjetivas. Posteriormente, se propuso el concepto de información lingüística bidimensional de 2-tupla (LB2T) [224] combinando la expresión lingüística de dos dimensiones y la información lingüística 2-tupla.

Evidentemente, la información expresada como LB2T es más precisa y razonable, porque la valoración y la fiabilidad de la valoración se proporcionan al mismo tiempo. Debido a las ventajas de utilizar evaluaciones LB2T, se han desarrollado diferentes enfoquespara resolver problemas de TDGMA con evaluación lingüística bidimensional [98, 99, 220], tales como:

- *Modelo de representación de etiquetas LB2T.* Generalmente, las etiquetas LB2T se representan como un término lingüístico binario [223]. Las dos clases de información lingüística proceden de dos conjuntos de términos lingüísticos diferentes. El primer conjunto de términos lingüísticos representa las evaluaciones proporcionadas por los decisores y el segundo conjunto de términos lingüísticos representa la fiabilidad de la evaluación anterior, que también es información subjetiva proporcionada por los decisores [202].

- *Operaciones y comparación de etiquetas LB2T.* Se han desarrollado diferentes operadores para diferentes tipos de expresión lingüística bidimensional, como los operadores de agregación de información lingüística incierta bidimensional [106, 110] utilizados para agregar las etiquetas lingüísticas bidimensionales bajo incertidumbre, operadores de agregación generalizados de potencia lingüística trapezoidal de dos dimensiones. Los operadores de agregación [99] se utilizan para agregar las etiquetas LB2T. Además, las operaciones de comparación entre LB2T se han desarrollado sobre la base de las operaciones de comparación tradicionales del modelo lingüístico 2-tupla [60], como el álgebra de implicación lingüística bidimensional [224] que se utiliza para expresar y comparar las LB2Ts, la notación de expectativa de las LB2Ts [110] propuesta para comparar variables lingüísticas inciertas de dos dimensiones, etc.

- *Métodos TDG basados en expresión LB2T.* Dado que LB2T tiene ventajas importantes en la expresión de información, su investigación y aplicación combinadas con estos métodos clásicos de TDG han atraído la atención de los investigadores y se han extendido a varios métodos de TDG en el entorno LB2T, como PROMETHEE [220], TODIM extendido [105], VIKOR-QUALIFLEX extendido [98], modo de fallo y análisis de efectos [104], teoría prospectiva extendida-VIKOR [33] , etc.

- *Aplicación de métodos TDG basados en etiquetas LB2T en la vida real.* En algunas situaciones reales, los términos lingüísticos se han considerado el modelo más adecuado para evaluar atributos, como la toma de decisiones de emergencia [32, 33], la evaluación de la calidad [97], la selección del lugar para la construcción de una central eléctrica [185], la evaluación de riesgos [186], etc.

La investigación en TDG muestra que es necesario un proceso de alcance de consenso (PAC) para asegurar el acuerdo sobre los resultados de las decisiones en los problemas de TDG basados en información lingüística. Sin embargo, los PAC generalmente exigen que las preferencias u opiniones iniciales se modifiquen si no se satisface el nivel de consenso esperado durante el PAC. En esta situación, vale la pena pensar en la fiabilidad de las preferencias u opiniones modificadas. Obviamente, los decisores podrían dar por adelantado la fiabilidad de sus preferencias; sin embargo, esta fiabilidad debería obtenerse de una forma de medición objetiva.

A pesar de que existen múltiples modelos y enfoques que tratan los problemas de TDGMA y la información LB2T de manera conjunta, tanto en la teoría como en la práctica, estos modelos y enfoques no son lo suficientemente buenos cuando se aplican a problemas de TDGMA del mundo real en los que se aplican PAC. Así, los nuevos desafíos que se describen a continuación son las principales motivaciones de esta memoria de investigación:

- *La agregación de los LB2T en TDGMA:* Agregar las opiniones modeladas mediante LB2T de los decisores para clasificar u ordenar las alternativas, y seleccionar la mejor opción, es un proceso necesario. En los problemas de TDGMA basados en etiquetas LB2T, las preferencias de los decisores individuales deben agregarse de forma colectiva y bien estructurada para tomar la decisión final. La agregación de los LB2T es de gran importancia en TDGMA porque diferentes operadores de agregación pueden conducir a resultados diferentes. Sin embargo, interpretar y analizar las preferencias de estos decisores es una tarea compleja. Y en los métodos existentes, independientemente del operador de agregación utilizado, la información bidimensional de las etiquetas LB2T se toma por separado para su cálculo [99, 107, 110, 167, 200]. De hecho, cuando las evaluaciones no son completamente fiables, se vuelven aleatorias, lo que significa que el valor de la preferencia u opinión es muy incierto. Por lo tanto, un operador de agregación para agregar las etiquetas LB2T de una perspectiva estocástica parece adecuado.

- *Medición de la fiabilidad de la evaluación LB2T modificada:* Las etiquetas LB2T expresan la valoración y su fiabilidad y se han aplicado a muchos problemas de TDGMA [32, 185, 186]. En un PAC, las etiquetas LB2T iniciales se modifican y es necesario volver a calcular la fiabilidad de la evaluación modificada. La fiabilidad de la evaluación inicial es subjetiva, sin embargo, es necesaria una medición objetiva para mejorar el uso de las etiquetas LB2T en TDGMA.

- *Cálculo de los pesos de los decisores en problemas TDGMA:* El cálculo de la importancia de los decisores se puede dividir en métodos subjetivos, métodos objetivos y métodos que combinan los enfoques objetivo y subjetivo [42, 178]. Los métodos que obtienen el peso subjetivo, como el proceso de jerarquía analítica (analytic hierarchy process, AHP) [146] y los métodos Delphi [73], asignan pesos a los decisores en función de características subjetivas como su formación, nivel profesional y experiencia con los problemas de toma de decisiones. Los métodos de cálculo del peso objetivo [85], como el peso obtenido de la entropía [46], la técnica de orden de preferencia por similitud a una solución ideal (the order preference technique for similarity to an ideal solution, TOPSIS) [68] y los métodos de proyección [204], etc., son algunos de los más usados. Los métodos mixtos (subjetivos y objetivos) para calcular los pesos de los decisores combinan los pesos subjetivos y objetivos para obtener los pesos de los decisores [116, 147, 176]. Cuando los pesos de los decisores no se dan por adelantado, el método objetivo de cálculo de las ponderaciones es importante. Por lo tanto, es un desafío encontrar una manera más adecuada y eficaz de obtener el peso de los responsables políticos TDGMA con evaluaciones lingüísticas.

- *Clustering para manejar grandes grupos:* Los métodos de clustering pueden simplificar eficazmente el PAC cuando hay una gran cantidad de decisores involucrados en el problema. Por tanto, es importante aplicar clustering para resolver problemas de TDGMA. Muchos investigadores se han centrado en el método de clustering utilizado, como el algoritmo de clustering k-means [187], fuzzy c-means [151], clustering jerárquico [21], etc. Usando un método de clustering, los decisores se pueden dividir en varios grupos pequeños, por lo que las preferencias de los decisores tienen mayor consistencia y menor grado de conflicto para cada grupo. Sin embargo, los métodos de clustering existentes son complejos de aplicare ignoran el grado de soporte en cada alternativa de diferentes decisores.

  Por lo tanto, es necesario desarrollar un nuevo método de clustering basado en el grado de soporte de cada alternativa de los decisores para obtener más información durante el PAC.

- *La consistencia y el consenso de las opiniones de los decisores:* La consistencia y el consenso son otros retos dignos de mención en el proceso TDGMA. La consistencia está directamente relacionada con la credibilidad de los resultados. El consenso, por otro lado, significa el acuerdo de los decisores para aceptar los resultados del proceso. Durante el PAC, algunos decisores no modifican sus opiniones, lo que podría suceder cuando no hay tiempo suficiente para persuadirlos o cuando mantienen su grado de tolerancia. Los decisores podrían aceptar modificar sus preferencias como máximo un valor que esté dentro de su grado de tolerancia. Por lo tanto, es un reto coordinar las preferencias de los decisores que no quieren modificar sus preferencias y el proceso de feedback automático con el grado de aceptación y tolerancia de la opinión modificada de los decisores.

En los problemas de TDGMA del mundo real, los retos encontrados si son superados pueden hacer que los enfoques de TDGMA satisfagan mejor las situaciones y necesidades en la toma de decisiones. Esta memoria de investigación se centra en estudiar en profundidad dichos retos y como superarlos.

A.2 Objetivos

Según los retos señalados anteriormente en los enfoques de TDGMA basados en etiquetas LB2T, esta memoria de investigación se centra en proponer nuevos modelos que permitan hacer frente a los retos indicados.

En base a tal propósito, se consideran los siguientes tres objetivos de investigación:

1. Desarrollar un modelo computacional LB2T que considere la información de dos dimensiones con etiquetas LB2T desde una perspectiva estocástica y permita comparar los modelos computacionales mediante un caso de estudio. Además, se introducirán algunos nuevos operadores de agregación y reglas de comparación para mejorar los estudios anteriores.

2. Considerar el grado de fiabilidad de las preferencias modificadas durante el

PAC. En general, las preferencias iniciales las proporcionan los decisores mediante términos lingüísticos, y las preferencias modificadas se presentan mediante un término lingüístico o la extensión de un término lingüístico, como el valor lingüístico 2-tupla. En cualquier caso, falta la información de fiabilidad de las preferencias modificadas. Por tanto, es necesario otra dimensión de información lingüística para representar la fiabilidad de las preferencias modificadas. Considerando el ajuste mínimo durante el PAC, se propone un modelo de consenso de ajuste mínimo de dos etapas basado en información lingüística para mostrar la preferencia modificada obtenida y su fiabilidad. Además, se discuten las relaciones entre la fiabilidad de las preferencias modificadas y la distancia entre la preferencia original y la modificada.

3. Definir un modelo de TDGMA en el que se manejen grandes grupos de decisores y se considere el grado de tolerancia de los decisores al cambiar de opinión. Además, se desarrollará un método de clustering basado en el grado de soporte para clasificar a los decisores en varios subgrupos haciendo más manejable situaciones con gran cantidad de decisores. Se considerará el grado de tolerancia de los decisores para mejorar la fiabilidad de las opiniones modificadas, y se presentará un modelo de consenso de ajuste mínimo con dos reglas de consenso para mejorar gradualmente el nivel de consenso. Eventualmente, las preferencias modificadas se modelarán con etiquetas LB2T. Usando la forma de comparación propuesta entre LB2T, se puede obtener un ranking de alternativas.

A.3 Estructura

Teniendo en cuenta el párrafo 3 del artículo 23 de las actuales normas de doctorado de la Universidad Jaén y con el fin de alcanzar los objetivos establecidos en el párrafo 1.2, el presente estudio se basa en el plan establecido en el documento RD 99 / 2011, en el que se resumen los artículos publicados por los estudiantes de doctorado durante su doctorado.

Se han publicado dos artículos en revistas internacionales indexadas por la base de datos JCR, producida por ISI y una contribución en el congreso IEEE CIS International Conference on Fuzzy Systems 2020 (Clasificación en la lista de conferencias Core Ranking 2020 como CORE A). En resumen, el informe dos artículos publicados en revistas internacionales de alta calidad (Q1) y una contribución a la conferencia CORE A.

La estructura de esta memoria de investigación se describe brevemente a continuación:

- Capítulo 2: Se revisan algunos conceptos básicos que se utilizan en la memoria de investigación para lograr nuestros objetivos, tales como conceptos relacionados con la toma de decisiones, TDG, TDGMA, TDGMA bajo incertidumbre, TDGMA basado en información lingüística. Y se introducen brevemente los métodos y modelos que se utilizan en nuestras propuestas, tales como, el enfoque lingüístico difuso, el modelo lingüístico 2-tupla, la etiqueta lingüística 2-tupla de dos dimensiones, proceso de consenso, el modelo de coste de ajuste mínimo etc.

- Capítulo 3: Se introducen brevemente las propuestas publicadas que componen la memoria de investigación, además, se examinan brevemente los resultados para esclarecer los logros alcanzados en nuestra investigación.

- Capítulo 4: Este capítulo es la parte principal de este estudio, ya que muestra la literatura obtenida a través de la propuesta. Para cada publicación se indica la revista en la que se ha publicado, así como su factor de impacto y cuartil.

- Capítulo 5: Por último, se presentan las conclusiones de este estudio y el trabajo futuro realizable.

A.4 Resumen

Los términos lingüísticos son más intuitivos y cercanos al lenguaje usado por los seres humanos para representar las preferencias de los decisores que participan en los problemas de TDGMA. Por tanto, se han investigado ampliamente los enfoques de TDGMA que utilizan información lingüística. Las metodologías y modelos existentes que manejan información lingüística no hubieran sido posibles sin metodologías para llevar a cabo los procesos de computación con palabras [87, 209]. El modelo lingüístico 2-tupla [60] fue introducido para evitar la pérdida de información y obtener resultados más interpretables y precisos durante los procesos de computación con palabras. Como consecuencia es uno de los modelos computacionales lingüísticos más utilizados en TDGMA. Por lo tanto, una revisión profunda en la literatura especializada muestra el rápido crecimiento y aplicabilidad del modelo de representación lingüística de 2-tupla, que se ha aplicado a distintos problemas del mundo real. Sin embargo, con el aumento de la complejidad de los problemas de toma de decisiones, la información lingüística de una dimensión no siempre es suficiente para que los decisores tomen decisiones con precisión. Por lo tanto, se proponen etiquetas LB2T, que son una extensión del modelo lingüístico 2-tupla [60]. Consecuentemente, en los últimos años se han estudiado los enfoques correspondientes para resolver problemas TDGMA basados en información LB2T. Sin embargo, todavía existen algunas limitaciones en los estudios existentes, tales como, la precisión del modelo computacional y la agregación de las etiquetas LB2T, el PAC durante la TDGMA teniendo en cuenta la fiabilidad de las preferencias modificadas, el grado de tolerancia de los decisores cuando se sugiere modificar las preferencias originales, etc. Para superar estos retos, esta investigación ha realizado las siguientes propuestas.

1. Se han presentado unas funciones para transformar una etiqueta LB2T en una variable estocástica y su inversa. Además, para la comparación y la medición de la similitud entre dos etiquetas LB2T se han desarrollado operadores teniendo en cuenta el grado de importancia relativa de las dos dimensiones de la información. Es evidente que el nuevo modelo de representación y computación de LB2T puede reflejar la influencia de la información de la segunda dimensión en los resultados de la decisión final, y también se han discutido el impacto de los diferentes métodos en los resultados de la decisión final.

2. Se propone un modelo de consistencia en dos etapas con un ajuste mínimo, que no solo considera el ajuste mínimo, sino que también minimiza el número de preferencias modificadas. La primera etapa es maximizar la mejora del nivel de consenso para cada par de alternativas con un ajuste mínimo. La segunda etapa es obtener las preferencias ajustadas con un determinado nivel de consenso en la primera etapa dentro de un ajuste mínimo. La información de la segunda dimensión se puede obtener a

través de un modelo matemático como la fiabilidad de las preferencias modificadas, que evitan la subjetividad de la información lingüística. Se construyen las relaciones entre el ajuste de preferencia total y el grado de fiabilidad de las preferencias ajustadas, lo que mejora la precisión del ranking de alternativas.

3. Se ha propuesto un modelo de consenso basado en la consideración del grado de tolerancia de los decisores, siguiendo dos reglas de consenso: ajuste mínimo entre preferencias originales y modificadas, y el número mínimo de preferencias modificadas. Mediante el uso de la función de comparación para LB2Ts, las expresiones LB2T utilizadas para describir la evaluación general brindan más información para la toma de decisiones y mejoran la fiabilidad del ranking de alternativas.

A.5 Conclusiones y Trabajos Futuros

El capítulo 5 concluye nuestra memoria de investigación revisando las conclusiones sobre las principales propuestas y resultados obtenidos, y señalando posibles trabajos futuros.

A.5.1 Conclusiones

La TDG se utiliza ampliamente en la vida real para resolver problemas importantes y complicados en una variedad de dominios, como la toma de decisiones de emergencia [51, 71, 180], la evaluación de servicios médicos [150, 179], la selección de proveedores [16, 17, 48], etc. Dada la importancia de TDG en la selección y evaluación de la gestión y los problemas económicos, se han propuesto muchos modelos y enfoques para los problemas de TDG [81, 103, 122, 152].

La TDGMA implica que los decisores proporcionan evaluaciones con respecto al desempeño de las alternativas bajo múltiples criterios [75]. Con el aumento de la complejidad de los problemas de toma de decisiones y la limitación de la expresión de los decisores, la TDGMA basada en la evaluación lingüística ha atraído más atención [129, 132, 143, 196]. Teniendo en cuenta la complejidad y singularidad de la expresión lingüística, los métodos TDGMA generales no siempre son adecuados para resolver problemas de TDGMA con información lingüística. A pesar de que la investigación existente ha obtenido numerosos logros [27, 101, 188], todavía hay muchos métodos que deben mejorarse. Además, en algunas situaciones, el uso de solo una dimensión de la información lingüística no es suficiente para garantizar la precisión de la información inicial, la fiabilidad de la evaluación también es un factor importante a considerar. Por tanto, el estudio de LB2T es necesario y significativo. Además, es necesario desarrollar varios métodos nuevos con el objetivo de resolver TDGMA basados en información LB2T.

En este estudio, obtenemos algunos nuevos resultados respecto a aquellos retos que no solo cumplen con los objetivos señalados en el apartado 1.2, sino que también aportan nuevas visiones en los procesos de resolución de TDGMA basados en etiquetas LB2T y la nueva tendencia de la investigación futura.

Por lo tanto, nuestros resultados han llegado a algunas conclusiones:

1. A pesar de la aplicación exitosa de la información lingüística bidimensional mediante el modelo de representación y computación de etiquetas lingüísticas bidimensionales, no se había explorado el análisis de la incertidumbre de las evaluaciones según la información de la segunda dimensión. Así, se ha propuesto un nuevo modelo de representación de LB2T desde una perspectiva estocástica. Se han presentado funciones de transformación entreuna etiqueta LB2T y una variable estocástica y su inversa, que es más adecuada para el cálculo de TDG a gran escala. La comparación y medida de similitud entre dos etiquetas LB2T se ha desarrollado teniendo en cuenta el grado de importancia relativa de las dos dimensiones de información desde la perspectiva estocástica, lo que ha hecho que el análisis de decisiones brinde información más útil.

2. La fiabilidad de la preferencia inicial proporcionada por los decisores generalmente se presenta como información lingüística de dos dimensiones para problemas TDGMA basados en información lingüística, sin embargo, durante el PAC, especialmente para los PAC automáticos, la fiabilidad de las preferencias modificadas a menudo se obvia. Con base a esta observación, Sobre la base de la expresión LB2T, se propone un modelo de ajuste de consistencia mínima en dos etapas para resolver el problem a TDG en decisores a gran escala con la consideración del grado de fiabilidad de las preferencias ajustadas, que no solo considera el ajuste

mínimo, sino que también minimiza el número de preferencias ajustadas. Y se han discutido las relaciones entre el ajuste de preferencia total y el grado de fiabilidad de las preferencias modificadas. El método propuesto ha completado el enfoque lingüístico de dos dimensiones y 2-tupla para la TDG a gran escala.

3. La medición de la fiabilidad de las opiniones modificadas puede mejorar la precisión de la toma de decisiones con los PAC automáticos. Sin embargo, la fiabilidad se basa en la tolerancia de los decisores al cambiar sus opiniones. Por tanto, el grado de tolerancia de los decisores es un factor importante a considerar de antemano durante el PAC. La propuesta del nuevo modelo de consenso basado en el grado de tolerancia de los decisores ha mejorado el método de TDGMA con información LB2T basada en la medición de fiabilidad.

A.5.2 Trabajos Futuros

Aunque se han propuesto varios métodos, herramientas y enfoques en esta investigación, todavía existen retos dentro de la TDG basados en la evaluación lingüística y el enfoque LB2T que deben estudiarse más a fondo. En un futuro próximo, nos centraremos en ampliar el alcance de la orientación y desarrollar soluciones a los problemas de TDG:

1. En la mayoría de los estudios existentes, los decisores se consideran totalmente racionales, pero en la realidad, los no lo son ya que presentan ciertas limitaciones respecto a su racionalidad a la hora de tomar decisiones. Debido a esta racionalidad limitada algunos decisores pueden no estar cómodos cuando se les sugiere que ajusten sus opiniones en un rango menor y limitado, por lo que se pueden producir comportamientos no cooperativos en la TDG. . Esto nos lleva a la necesidad de estudiar y considerar el comportamiento psicológico de los decisores en nuestros trabajos futuros.

2. Para PAC en TDG, a menudo se considera el ajuste mínimo para hacer que las opiniones de los decisores cambien, sin embargo, hacer que los decisores cambien sus opiniones tiene diferentes niveles de dificultad, el

coste de ajuste unitario está definido bajo el supuesto de que no esdireccional, de hecho, el coste de ajuste unitario no siempre es igual en las direcciones de ajuste ascendente y descendente [71]. Por lo tanto, también se debe considerar la determinación del coste de ajuste unitario, especialmente para el coste unitario simétrico, que es un reto a considerar para el modelo de consenso de coste mínimo.

3. Como el conjunto de términos lingüísticos establecido para expresar la evaluación inicial a veces no está balanceado, el conjunto de términos lingüísticos para expresar la fiabilidad de las opiniones modificadas también podría ser no balanceado, por tanto, cómo diseñar el modelo de representación y computación con la información de fiabilidad no balanceada se estudiará en el futuro.

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