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Personalized individual semantics based on consistency in hesitant linguistic group decision making with comparative linguistic expressions

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ABSTRACT

In decision making problems, decision makers may prefer to use more flexible linguistic expressions instead of using only one linguistic term to express their preferences. The recent proposals of hesitant fuzzy linguistic terms sets (HFLTSs) are developed to support the elicitation of comparative linguistic expressions in hesitant decision situations. In group decision making (GDM), the statement that words mean different things for different people has been highlighted and it is natural that a word should be defined by individual semantics described by different numerical values. Considering this statement in hesitant linguistic decision making, the aim of this paper is to personalize individual semantics in the hesitant GDM with comparative linguistic expressions to show the individual difference in understanding the meaning of words. In our study, the personalized individual semantics are carried out by the fuzzy envelopes of HFLTSs based on the personalized numerical scales of linguistic term set.

1. Introduction

In real-world decision making, Computing with Words (CW) is often applied as a basis to solve the decision problems with linguistic information [14–16,28,29]. In recent years, different linguistic models are proposed for CW. Particularly, the 2-tuple linguistic representation model [8] provided a computation technique to deal with linguistic information without loss of information. Based on the 2-tuple linguistic representation model, the model based on a linguistic hierarchy [6] and the numerical scale model [2,3] are developed to provide good methods to deal with the linguistic decision making problems with single linguistic term.

However, the complexity and time pressure of decision making problems nowadays make decision makers need more elaborated expressions than a simple linguistic label [20]. Hence, to overcome this limitation, Rodríguez et al. [21] introduced the concept of hesitant fuzzy linguistic term set (HFLTS) to serve as the basis of increasing the flexibility of the elicitation of linguistic information by means of linguistic expressions.

To generate more elaborate linguistic expressions, Rodríguez et al. [21] provided a method to generate comparative linguistic expressions by using a context-free grammar and HFLTS. To deal with comparative linguistic expressions in group decision making (GDM), a decision model was proposed in [22] to facilitate the elicitation of linguistic information in hesitant situation. Besides, to represent the semantics of comparative linguistic expressions, Liu and Rodríguez [11] proposed a representation way by means of a fuzzy envelope to carry out the CW processes and discussed its application in multicriteria decision making. Some further developments about the hesitant linguistic decision making can be found in [19,23].

In GDM dealing with CW, there is a fact that words mean different things for different people [5,15,16]. For example, when evaluating the quality of a paper, three reviewers think the paper has "good" quality, but this term "good" has different semantics for these three reviewers. That makes the understanding and numerical meanings of "good" for different reviewers are different. The existing studies use the type-2 fuzzy sets [15] and multi-granular linguistic models [7,17] for managing this issue. Although both methods deal with multiple meanings of words are quite useful,

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they do not represent yet the specific semantics of each individual. To overcome this problem, Li et al. [10] proposed a personalized individual semantics approach to model and solve linguistic GDM by means of numerical scales [1–3] and the 2-tuple linguistic model [8] to improve the management of different meanings of words for different people. This approach shows the good features for managing linguistic information in CW processes and can reflect individual personalized differences in understanding the meaning of words.

In hesitant linguistic decision making, although there are many studies (e.g., [2,11,15,26]) to discuss the representations of HFLTSs, few studies consider the personalized individual semantics among decision makers when expressing the preferences using HFLTSs. Therefore, in this paper, we apply the idea of personalize individual semantics to reflect the different understanding of words for different decision makers in hesitant linguistic decision making. A new framework to personalize individual semantics in hesitant linguistic GDM with comparative linguistic expressions is proposed. This proposal consists of a two-step procedure:

- An average consistency-driven model is proposed to set personalized numerical scales for linguistic terms with comparative linguistic expressions. The proposed model is based on measuring the average consistency index (ACI) of hesitant fuzzy linguistic preference relations (HFLPRs) and provides a basis for developing the personalized individual semantics of HFLTSs.
- Based on the personalized numerical scales obtained from the average consistency-driven model, a process to personalize individual semantics with comparative linguistic expressions via the fuzzy envelope for HFLTSs represented by fuzzy membership function is proposed.

The proposed personalized individual semantics show the individual difference in understanding the meaning of comparative linguistic expressions. The use of the personalized individual semantics provides a new way to show decision makers' numerical meaning individually, and also provides a potential tool to obtain the optimal solution in hesitant linguistic GDM when dealing with the fact that words mean different things to different people.

The rest of this paper is arranged as follows. In Section 2, we present some basic knowledge. Then, in Section 3 the framework and models to personalize individual semantics with comparative linguistic expressions are proposed. Next, Section 4 provides numerical examples and analysis. Section 5 discusses the advantages and weakness of the proposed model. Section 6 concludes this paper with final remarks.

2. Preliminaries

In this section, we introduce the basic knowledge regarding the 2-tuple linguistic model, numerical scale, comparative linguistic expressions and HFLTSs.

2.1. The 2-tuple linguistic model and numerical scale

The 2-tuple linguistic representation model, presented by Herrera and Martínez [8], represents the linguistic information by a 2-tuple $(s_i, \alpha) \in \bar{S} = S \times [-0.5, 0.5)$, where $s_i \in S$ and $\alpha \in [-0.5, 0.5)$.

Definition 1 [8]**.** Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ be a value representing the result of a symbolic aggregation operation. The 2-tuple linguistic value that expresses the equivalent information to β is then obtained as:

$\Delta : [0, g] \rightarrow \bar{S}$,

being

$$\Delta(\beta) = (s_i, \alpha), \quad with \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases}$$

Function Δ, it is a one to one mapping whose inverse function $\Delta^{-1} : \bar{S} \rightarrow [0, g]$ is defined as $\Delta^{-1}(s_i, \alpha) = i + \alpha$. When $\alpha = 0$ in (s_i, α) is then called simple term.

A computational model for the 2-tuple linguistic model was defined in [8], in which different operations were introduced:

(1) A 2-tuple comparison operator: Let (s_k, α) and (s_l, γ) be two 2-tuples. Then:
   (i) if $k < l$, then (s_k, α) is smaller than (s_l, γ).
   (ii) if $k = l$, then
      (a) if $\alpha = \gamma$, then (s_k, α), (s_l, γ) represents the same information.
      (b) if $\alpha < \gamma$, then (s_k, α) is smaller than (s_l, γ).
(2) A 2-tuple negation operator: $Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$.
(3) Several 2-tuple aggregation operators have been developed (see [8,14]).

The concept of the numerical scale was defined to transform linguistic terms into real numbers:

Definition 2 [3]**.** Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and R be the set of real numbers. The function: NS: $S \rightarrow R$ is defined as a numerical scale of S, and $NS(s_i)$ is called the numerical index of s_i. If the function NS is strictly monotone increasing, then NS is called an ordered numerical scale.

Based on the concept of numerical scale, Dong et al. [2] proposed a connection of the numerical scale model with the 2-tuple linguistic model [8], the proportional 2-tuple linguistic model [25] and the model based on a linguistic hierarchy [6], respectively, by setting different certain values for $NS(s_i)$.

2.2. Comparative linguistic expressions and HFLTSs

To facilitate the elicitation of flexible and rich linguistic expressions, Rodríguez et al. [21] proposed an approach to generate comparative linguistic expressions by using a context-free grammar.

Definition 3 [21]**.** Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and G_H be a context-free grammar. The elements of $G_H = \{V_N, V_T, I, P\}$ are defined as follows,

$$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunctiond\rangle\}$$

$$V_T = \{lower\ than, greater\ than, between, and, s_0, s_1, \ldots, s_g\}$$

$$I \in V_N.$$

For the context-free grammar G_H, the production rules are as follows:

$$\begin{aligned} P = \{I &::= \langle primary\ term\rangle | \langle composite\ term\rangle\ \langle composite\ term\rangle \\ &::= \langle unary\ relation\rangle \\ &\langle primary\ term\rangle | \langle binary\ relation\rangle \langle primary\ term\rangle \\ &\langle conjuction\rangle \langle primary\ term\rangle \\ &\langle primary\ term\rangle ::= s_0 | s_1 | \ldots | s_g \\ &\langle unary\ relation\rangle ::= lower\ than | greater\ than \\ &\langle binary\ relation\rangle ::= between \\ &\langle conjuction\rangle ::= and\} \end{aligned}$$

By using the context-free grammar G_H, the comparative linguistic expressions are generated. Since they cannot be directly used for CW, Rodríguez et al. [21] provided a transformation function to transform them into HFLTSs.

Definition 4 [21]**.** Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. A HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S.

Definition 5 [21]**.** Let H_S be a HFLTS of S. Let $H_S^- = \min_{s_i \in H_S}(s_i)$, $H_S^+ = \max_{s_i \in H_S}(s_i)$ and $env(H_S) = [H_S^-, H_S^+]$. Then, H_S^-, H_S^+ and $env(H_S)$ are called the lower bound, the upper bound and the envelope of H_S.

Definition 6 [21]**.** Let S_{ll} be the expressions generated by G_H, and let E_{G_H} be a function that transforms linguistic expressions, $ll \in S_{ll}$, obtained by using G_H, into HFLTS, H_S. S is the linguistic term set used by G_H and S_{ll} is the expressions domain generated by G_H:

$E_{G_H} : S_{ll} \to H_S$

The comparative linguistic expressions generated by G_H using the production rules are converted into HFLTS by means of the following transformations:

$$E_{G_H}(s_i) = \{s_i\};$$

$$E_{G_H}(less\ than\ s_i) = \{s_j \,|\, s_j \in S\ and\ s_j \le s_i\};$$

$$E_{G_H}(greater\ than\ s_i) = \{s_j \,|\, s_j \in S\ and\ s_j > s_i\};$$

$$E_{G_H}(between\ s_i\ and\ s_j) = \{s_k | s_k \in S\ and\ s_i \le s_k \le s_j\}.$$

Based on the use of HFLTSs, Rodríguez et al. [22] proposed the concept of the HFLPR as Definition 7.

Definition 7 [22]**.** Let M_S be a set of HFLTSs based on S. A HFLPR based on S is presented by a matrix $H = (H_{ij})_{n \times n}$, where $H_{ij} \in M_S$ and $Neg(H_{ij}) = H_{ji}$.

3. Personalizing individual semantics with comparative linguistic expressions in hesitant linguistic GDM

As aforementioned, there is a fact that words mean different things for different people. To represent the specifically personalized individual semantics of each decision maker in decision making, this section proposes the process to personalize individual semantics with comparative linguistic expressions in hesitant linguistic GDM.

3.1. Framework

GDM is defined as a decision situation where two or more experts, who have their own knowledge and preferences regarding the decision problem, take part and provide their preferences to reach a collective decision. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives and $E = \{e^1, e^2, \ldots, e^m\}$ be a set of decision makers. Each decision maker provides his/her preferences with comparative linguistic expressions over X by a preference relation $P^k = (p_{ij}^k)_{n \times n} (k = 1, 2, \ldots, m)$.

In order to carry out CW processes with comparative linguistic expressions p_{ij}^k, it is necessary to transform them into HFLTS H_{ij}^k by means of the transformation function E_{G_H}, i.e.,

$$E_{G_H}(p_{ij}^k) = H_{ij}^k \tag{1}$$

Therefore, in decision making, by using the transformation function E_{G_H}, the preference relation with comparative linguistic expressions P^k can be transformed into the HFLPR H^k [21,22],

$$H^k = \left(H_{ij}^k\right)_{n \times n} = \left(E_{G_H}(p_{ij}^k)\right)_{n \times n} \tag{2}$$

Following the existing semantics definitions of linguistic terms and HFLTSs, in this paper it is assumed that:

(1) The semantics of linguistic terms $s_i \in S$ are represented by the trapezoidal (triangular) membership functions $A(s_i) = T(a_L^i, a_M^i, a_M^i, a_R^i)$. For simplicity we note $A(s_i) = T(a_L^i, a_M^i, a_R^i)$.
(2) The semantics of HFLTSs H_{ij}^k are defined by fuzzy envelopes using trapezoidal fuzzy membership functions, $env_F(H_{ij}^k) = T(a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k)$.

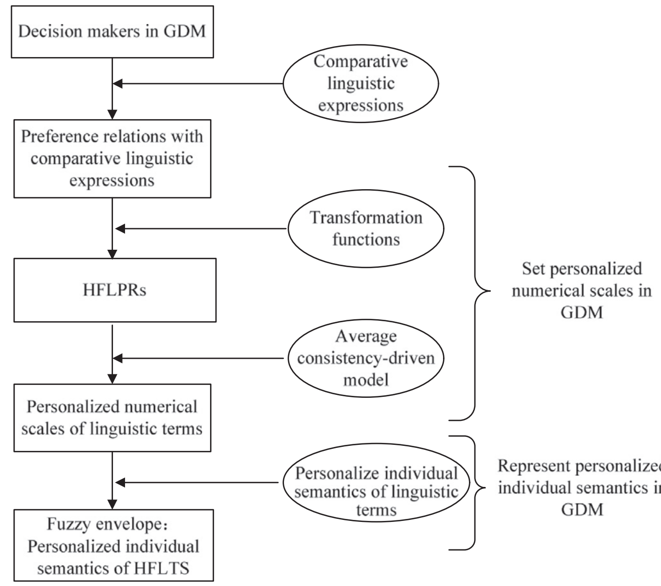

Fig. 1. Framework to personalize individual semantics with comparative linguistic expressions.

Considering the fact that words mean different things for different people in CW processes, in linguistic GDM, the representation way for the HFLTS should reflect the individual differences to understand the meaning of words. Keeping the previous fact in mind, an approach to express the personalized individual semantics of HFLTS in GDM is presented, which reflects the different meanings of HFLTS for different decision makers. It is implemented by a two-step procedure:

(1) The process to set the personalized numerical scales of linguistic terms over S. To achieve this process, we propose a consistency-driven approach based on the ACI of HFLPR in Section 3.2. The ACI is determined as the average consistency degree of all linguistic preference relations associated to a HFLPR.
(2) The process to represent the personalized individual semantics of HFLTSs. Based on the personalized numerical scales, we propose an approach to represent the personalized individual semantics by means of constructing the fuzzy envelope for HFLTSs in Section 3.3.

The framework to personalize individual semantics with comparative linguistic expressions in hesitant GDM is provided below (see Fig. 1).

3.2. Setting personalized numerical scales of linguistic terms in GDM

According to Eqs. (1) and (2), the preference relations with comparative linguistic expressions are transformed into HFLPRs using the transformation function E_{G_H} to facilitate the CW processes in linguistic decision making. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and $H = (H_{ij})_{n \times n}$ be a HFLPR based on S, where $H_{ij} = \{H_{ij}^t | t = 1, \ldots, \#H_{ij}\}$ and $\#H_{ij}$ is the number of linguistic terms in H_{ij}.

Definition 8. Let $H = (H_{ij})_{n \times n}$ be a HFLPR defined as before. $L = (l_{ij})_{n \times n}$ is a linguistic preference relation associated to H, if $l_{ij} = H_{ij}^t$, $t \in \{1, \ldots, \#H_{ij}\}$, and $l_{ij} = Neg(l_{ji})$.

We denote N_H as the set of the linguistic preference relations associated to H.

Example 1. Let $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$ be a linguistic term set. The HFLPR H is given as follows [26],

$$H = \begin{pmatrix} \{s_4\} & \{s_5\} & \{s_6, s_7\} & \{s_6, s_7\} \\ \{s_3\} & \{s_4\} & \{s_4, s_5\} & \{s_5, s_6\} \\ \{s_1, s_2\} & \{s_3, s_4\} & \{s_4\} & \{s_5\} \\ \{s_1, s_2\} & \{s_2, s_3\} & \{s_3\} & \{s_4\} \end{pmatrix}$$

For any $l_{ij} \in H_{ij}$ and $l_{ij} + l_{ji} = 1$, we have the linguistic preference relation $L = (l_{ij})_{4\times4} \in N_H$, such as,

$$L = \begin{pmatrix} \{s_4\} & \{s_5\} & \{s_6\} & \{s_7\} \\ \{s_3\} & \{s_4\} & \{s_5\} & \{s_5\} \\ \{s_2\} & \{s_3\} & \{s_4\} & \{s_5\} \\ \{s_1\} & \{s_3\} & \{s_3\} & \{s_4\} \end{pmatrix}$$

Let NS be an ordered numerical scale on S, and in this paper we set the range of NS in the interval [0,1]. Additive transitivity is often used to character the consistency of linguistic preference relations [9,27]. Following the additive transitivity, the consistency index (CI) of a linguistic preference relation L based on the numerical scales NS is defined as,

$$CI(L) = 1 - \frac{2}{3n(n-1)(n-2)} \sum_{i,j,z=1}^{n} \left| NS(l_{ij}) + NS(l_{jz}) - NS(l_{iz}) - 0.5 \right| \tag{3}$$

with $NS(l_{ij}) \in [0, 1]$.

To measure the consistency of HFLPRs, we propose the ACI based on Eq. (3) as follows.

Definition 9. Let H be a HFLPR. The value of $ACI(H)$ is determined by the average consistency degree of all linguistic preference relations associated to the HFLPR, i.e.,

$$ACI(H) = \frac{1}{\#N_H} \times \sum_{L \in N_H} CI(L) \tag{4}$$

where $\#N_H$ is the number of linguistic preference relations in H, i.e., $\#N_H = \prod_{i=1}^{n} \prod_{j=i+1}^{n} \#H_{ij}$.

Example 2. Let S and H be as in Example 1, we have $\#N_H = \prod_{i=1}^{4} \prod_{j=i+1}^{4} \#H_{ij} = 16$. Using Eq. (3) to compute the consistency of the linguistic preference relation associated to H, such as, the consistency of the linguistic preference relation L provided in Example 1 is $CI(L) = 0.9583$. Then, by computing the average consistency of all the linguistic preference relations $L \in N_H$, the ACI of H are obtained, $ACI(H) = 0.9375$.

As mentioned before, it is possible to transform linguistic terms into the numerical scales, and both linguistic terms and numerical scales represent the same preference of decision maker. Considering this statement, we provide the following premise.

Premise 1: If HFLPRs are consistent, then the transformed preference relation based on the established numerical scale should be as much consistent as possible.

In the following, we construct an optimization-based model to set personalized numerical scales for linguistic terms with HFLPRs based on the average consistency measure.

Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and $E = \{e^1, e^2, \ldots, e^m\}$ be a set of decision makers. Let $H^k = (H_{ij}^k)_{n\times n}$ be a HFLPR based on S associated to decision maker e^k, where $H_{ij}^k = \{H_{ij}^{t,k} | t = 1, \ldots, \#H_{ij}^k\}$, and let $L^{h,k} = (l_{ij}^{h,k})_{n\times n}$ $(h = 1, 2, \ldots, \#N_{H^k})$ be the linguistic preference relations associated to H^k, i.e., $L^{h,k} \in N_{H^k}$. Let NS^k be the numerical scale associated with e^k.

Based on Premise 1, in order to guarantee that the HFLPR H^k is as consistent as possible, the objective function to maximize the ACI of H^k is as follows,

$$max \quad ACI(H^k) \tag{5}$$

where $ACI(H^k) = \frac{1}{\#N_{H^k}} \sum_{h=1}^{\#N_{H^k}} \left(1 - \frac{2\sum_{i,j,z=1}^{n} \left| NS(l_{ij}^{h,k}) + NS(l_{jz}^{h,k}) - NS(l_{iz}^{h,k}) - 0.5 \right|}{3n(n-1)(n-2)}\right)$ with $l_{ij}^{h,k} \in H_{ij}^k$ and $l_{ij}^{h,k} = Neg(l_{ji}^{h,k})$.

In this paper, without loss of generality, we set the range of numerical scales for linguistic terms $NS^k(s_i)$ as follows,

$$NS^k(s_i) \begin{cases} = 0 & i = 0 \\ = 0.5 & i = \frac{g}{2} \\ \in [(i-1)/g, (i+1)/g] & i = 1, 2, \ldots, g-1; \quad i \neq \frac{g}{2} \\ = 1 & i = g \end{cases} \tag{6}$$

Besides, NS^k must be ordered. We introduce a constraint value $\lambda \in (0, 1)$ to restrict the distance between $NS^k(s_i)$ and $NS^k(s_{i+1})$, i.e.,

$$NS^k(s_{i+1}) - NS^k(s_i) \geq \lambda \tag{7}$$

Based on Eqs. (5)–(7), the consistency-driven optimization model P to obtain personalized numerical scales for linguistic terms with HFLPR H^k is constructed as follows,

$$\begin{cases} \max \quad ACI(H^k) \\ s.t. \quad ACI(H^k) = \frac{1}{\#N_{H^k}} \sum_{h=1}^{\#N_{H^k}} \left(1 - \frac{2\sum_{i,j,z=1}^{n} \left| NS(l_{ij}^{h,k}) + NS(l_{jz}^{h,k}) - NS(l_{iz}^{h,k}) - 0.5 \right|}{3n(n-1)(n-2)}\right) \\ \quad l_{ij}^{h,k} \in H_{ij}^k \qquad i, j = 1, 2, \ldots, n \\ \quad l_{ij}^{h,k} = Neg(l_{ji}^{h,k}) \qquad i, j = 1, 2, \ldots, n \\ \quad NS^k(s_0) = 0 \\ \quad NS^k(s_{\frac{g}{2}}) = 0.5 \\ \quad NS^k(s_i) \in [(i-1)/g, (i+1)/g] \qquad i = 1, \ldots, g-1; \quad i \neq \frac{g}{2} \\ \quad NS^k(s_g) = 1 \\ \quad NS^k(s_{i+1}) - NS^k(s_i) \geq \lambda \qquad i = 0, 1, \ldots, g-1 \end{cases}$$

Solving model P uses the software packages Lingo or Matlab, we obtain the personalized numerical scales for each term in S associated with decision maker d_k, i.e., $NS^k(s_0)$, $NS^k(s_1), \ldots, NS^k(s_g)$, and the optimal ACI of H^k. The personalized numerical scales will provide a basis to personalize individual semantics of HFLTSs in Section 3.3.

3.3. Personalizing individual semantics with HFLTSs in GDM

Based on the personalized numerical scales, we propose an approach to personalize individual semantics of HFLTSs by computing the fuzzy envelope expressed by trapezoidal fuzzy membership functions in GDM. The personalized individual semantics of HFLTSs reflect the decision makers' different understanding for HFLTSs.

The process to represent the personalized individual semantics of HFLTSs can be implemented by a two-step procedure: (1) Representing the personalized individual semantics of linguistic terms; and (2) representing the personalized individual semantics of HFLTSs via fuzzy envelope. They are developed as follows:

(1) Representing the personalized individual semantics of linguistic terms
As stated above, in this paper we assume that the semantics of linguistic terms $s_i \in S$ are represented by the fuzzy membership functions $A(s_i) = (a_L^i, a_M^i, a_R^i)$. According to the fuzzy partitions [24], we have $a_R^{i-1} = a_M^i = a_L^{i+1}$, $i = 1, 2, \ldots, g-1$. Thus the set of points of all membership functions of the linguistic term set is given as

$$T = \{a_L^0, a_M^0, a_M^1, \ldots, a_M^g, a_R^g\} \tag{8}$$

From Section 3.2, the personalized numerical scales for the linguistic term set associated with decision maker e^k, $\{NS^k(s_0), NS^k(s_1), \ldots, NS^k(s_g)\}$, are obtained by the

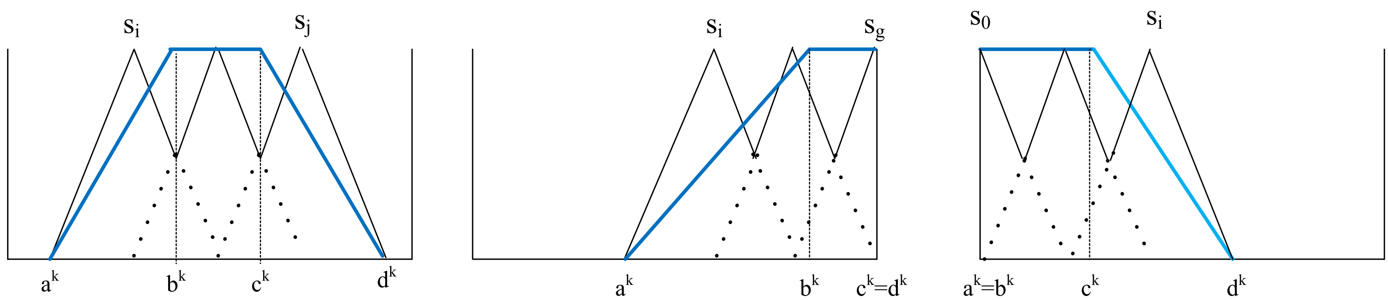

Fig. 2. Fuzzy envelopes for the HFLTSs $\{s_i, s_{i+1}, \ldots, s_j\}$, $\{s_i, s_{i+1}, \ldots, s_g\}$ and $\{s_0, s_1, \ldots, s_i\}$.

consistency-driven optimization model *P*. Based on the personalized numerical scale, each linguistic term s_i can be defined by triangular membership function as follows,

$$A^k(s_i) = \begin{cases} T(NS^k(s_0), NS^k(s_0), NS^k(s_1)) & i = 0 \\ T(NS^k(s_{i-1}), NS^k(s_i), NS^k(s_{i+1})) & i = 1, \ldots, g-1 \\ T(NS^k(s_{g-1}), NS^k(s_g), NS^k(s_g)) & i = g \end{cases} \tag{9}$$

Thus, Eq. (8) can be equivalently transformed into Eq. (10).

$$T = \{NS^k(s_0), NS^k(s_0), NS^k(s_1), \ldots, NS^k(s_{g-1}), NS^k(s_g), NS^k(s_g)\} \tag{10}$$

In this way, the personalized individual semantics of the linguistic terms s_i and the linguistic term set *S*, associated with each decision maker, can be represented by Eqs. (9) and (10).

(2) Representing the personalized individual semantics of HFLTSs via fuzzy envelope
Liu and Rodríguez [11] proposed a method to represent the semantics of the HFLTS via fuzzy envelope, using a trapezoidal fuzzy membership function *T*(*a*, *b*, *c*, *d*) obtained by aggregating the fuzzy membership functions of the linguistic terms of the HFLTS. This method provides a basis for personalizing individual semantics of HFLTSs.

Based on the computation method proposed in [11], we propose the fuzzy envelopes for HFLTSs provided by decision makers in GDM to personalize individual semantics by means of trapezoidal membership functions $T(a^k, b^k, c^k, d^k)$. From the context-free grammar in Definition 3, the comparative linguistic expressions can be divided into three types: between s_i and s_j ($i \neq 0$, $j \neq g$), at least s_i and at most s_i. Based on the transformations between comparative linguistic expressions and HFLTSs in Definition 6, we consider the following three cases to compute the fuzzy envelope for HFLTSs:

Case A: Fuzzy envelope for the HFLTS $\{s_i, s_{i+1}, \ldots, s_j\}$ ($i \neq 0$, $j \neq g$)
In order to represent the personalized individual semantics of the HFLTS $\{s_i, s_{i+1}, \ldots, s_j\}$ by using the fuzzy envelope defined by the trapezoidal membership function $T(a^k, b^k, c^k, d^k)$, the min and the max operators to compute a^k and d^k are used, i.e.,

$$a^k = \min\left\{NS^k(s_{i-1}), NS^k(s_i), \ldots, NS^k\left(s_j\right), NS^k\left(s_{j+1}\right)\right\} = NS^k(s_{i-1})$$

$$d^k = \max\left\{NS^k(s_{i-1}), NS^k(s_i), \ldots, NS^k\left(s_j\right), NS^k\left(s_{j+1}\right)\right\} = NS^k\left(s_{j+1}\right)$$

The way to obtain the parameters b^k and c^k is as follows,

(i) If $i+j$ is odd and $i+1 = j$, then $b^k = NS^k(s_i)$ and $c^k = NS^k(s_{i+1})$.

(ii) If $i+j$ is odd and $i+1 < j$, then

$$b^k = OWA_{W^2}\left\{NS^k(s_i), NS^k(s_{i+1}), \ldots, NS^k(s_{\frac{i+j-1}{2}})\right\};$$

$$c^k = OWA_{W^1}\left\{NS^k(s_j), NS^k(s_{j-1}), \ldots, NS^k(s_{\frac{i+j+1}{2}})\right\}.$$

(iii) If $i+j$ is even, then

$$b^k = OWA_{W^2}\left\{NS^k(s_i), NS^k(s_{i+1}), \ldots, NS^k(s_{\frac{i+j}{2}})\right\};$$

$$c^k = OWA_{W^1}\left\{NS^k(s_j), NS^k(s_{j-1}), \ldots, NS^k(s_{\frac{i+j}{2}})\right\},$$

where $W^1 = (w_1^1, w_2^1, \ldots, w_n^1)^T$ with $w_1^1 = \alpha_1$, $w_2^1 = \alpha_1(1-\alpha_1)$, $w_3^1 = \alpha_1(1-\alpha_1)^2, \ldots$, $w_{n-1}^1 = \alpha_1(1-\alpha_1)^{n-2}$, $w_n^1 = (1-\alpha_1)^{n-1}$ and $\alpha_1 = \frac{j-i-1}{g-1}$.
$W^2 = (w_1^2, w_2^2, \ldots, w_n^2)^T$ with $w_1^2 = {\alpha_2}^{n-1}$, $w_2^2 = (1-\alpha_2){\alpha_2}^{n-2}$, $w_3^2 = (1-\alpha_2){\alpha_2}^{n-3}, \ldots$, $w_{n-1}^2 = (1-\alpha_2)\alpha_2$, $w_n^2 = 1-\alpha_2$ and $\alpha_2 = \frac{g-(j-i)}{g-1}$.

Case B: Fuzzy envelope for HFLTS $\{s_i, s_{i+1}, \ldots, s_g\}$
The fuzzy envelope for $\{s_i, s_{i+1}, \ldots, s_g\}$ defined by $T(a^k, b^k, c^k, d^k)$ is computed as follows,

$$a^k = \min\left\{NS^k(s_{i-1}), NS^k(s_i), \ldots, NS^k(s_g)\right\} = NS^k(s_{i-1});$$

$$b^k = OWA_{W^2}\left\{NS^k(s_i), NS^k(s_{i+1}), \ldots, NS^k(s_g)\right\};$$

$$c^k = NS^k(s_g);$$

$$d^k = \max\left\{NS^k(s_{i-1}), NS^k(s_i), \ldots, NS^k(s_g)\right\} = NS^k(s_g),$$

where $W^2 = ({\alpha_2}^{g-i}, (1-\alpha_2){\alpha_2}^{g-i-1}, (1-\alpha_2){\alpha_2}^{g-i-2}, \ldots, (1-\alpha_2)\alpha_2, 1-\alpha_2)^T$ with $\alpha_2 = \frac{i}{g}$.

Case C: Fuzzy envelope for HFLTS $\{s_0, s_1, \ldots, s_i\}$
The way to compute the fuzzy envelope for $\{s_0, s_1, \ldots, s_i\}$ defined by $T(a^k, b^k, c^k, d^k)$ is as follows,

$$a^k = \min\left\{NS^k(s_0), NS^k(s_0), NS^k(s_1), \ldots, NS^k(s_i), NS^k(s_{i+1})\right\} = NS^k(s_0);$$

$$b^k = NS^k(s_0);$$

$$c^k = OWA_{W^1}\left\{NS^k(s_0), NS^k(s_1), NS^k(s_2), \ldots, NS^k(s_i)\right\};$$

$$d^k = \max\left\{NS^k(s_0), NS^k(s_0), NS^k(s_1), \ldots, NS^k(s_i), NS^k(s_{i+1})\right\} = NS^k(s_{i+1}),$$

where $W^1 = (\alpha_1, \alpha_1(1-\alpha_1), \alpha_1(1-\alpha_1)^2, \ldots, \alpha_1(1-\alpha_1)^{g-i-1}, (1-\alpha_1)^{g-i})^T$ with $\alpha_1 = \frac{i}{g}$.
Fig. 2 shows the fuzzy envelopes for HFLTSs $\{s_i, s_{i+1}, \ldots, s_j\}$ ($i \neq 0$; $j \neq g$), $\{s_i, s_{i+1}, \ldots, s_g\}$ and $\{s_0, s_1, \ldots, s_i\}$ in Cases A-C, respectively.
In Cases A–C, the OWA weights W^1 and W^2 presented in [4] are used for computing the fuzzy envelope for HFLTS. Besides, more details about the computation method to obtain the values α_1 and α_2 can be found in [11].

In this way, we generalize the use of fuzzy envelope to represent the personalized individual semantics of the HFLTS. Because semantics play a key role in CW, our proposal can provide a potential tool to help decision makers obtain the optimal solution in hesitant linguistic GDM when dealing with the idea that words mean different things to different people.

4. Numerical examples and analysis

In this section, numerical examples and a comparative study are provided to justify the feasibility of the proposed approach to personalize individual semantics in GDM with comparative linguistic expressions.

4.1. Numerical examples

In this example, there are five alternatives $X = \{x_1, x_2, x_3, x_4, x_5\}$ and four decision makers $E = \{e^1, e^2, e^3, e^4\}$. Each decision maker provides his/her preference relation with comparative linguistic expressions over X using the following linguistic term set,

$$\begin{aligned} S = \{&s_0 = extremely\ poor, s_1 = very\ poor, s_2 = poor,\\ s_3 &= slightly\ poor, s_4 = Fair,\\ s_5 &= slightly\ good, s_6 = good, s_7 = very\ good, s_8 = extremely\ good\} \end{aligned}$$

The four HFLPRs transformed from preference relations with comparative linguistic expressions are provided as follows,

$$H^1 = \begin{pmatrix} \{s_4\} & \{s_5\} & \{s_0\} & \{s_2, s_3, s_4\} & \{s_5, s_6\} \\ \{s_3\} & \{s_4\} & \{s_4, s_5\} & \{s_1, s_2\} & \{s_2, s_3\} \\ \{s_8\} & \{s_3, s_4\} & \{s_4\} & \{s_6, s_7\} & \{s_1\} \\ \{s_4, s_5, s_6\} & \{s_6, s_7\} & \{s_1, s_2\} & \{s_4\} & \{s_0, s_1\} \\ \{s_2, s_3\} & \{s_5, s_6\} & \{s_7\} & \{s_7, s_8\} & \{s_4\} \end{pmatrix}$$

$$H^2 = \begin{pmatrix} \{s_4\} & \{s_3, s_4\} & \{s_5, s_6\} & \{s_1, s_2\} & \{s_0, s_1\} \\ \{s_4, s_5\} & \{s_4\} & \{s_6, s_7, s_8\} & \{s_2, s_3\} & \{s_0, s_1, s_2\} \\ \{s_2, s_3\} & \{s_0, s_1, s_2\} & \{s_4\} & \{s_5, s_6\} & \{s_4, s_5, s_6\} \\ \{s_6, s_7\} & \{s_5, s_6\} & \{s_2, s_3\} & \{s_4\} & \{s_4, s_5, s_6\} \\ \{s_7, s_8\} & \{s_6, s_7, s_8\} & \{s_2, s_3, s_4\} & \{s_2, s_3, s_4\} & \{s_4\} \end{pmatrix}$$

$$H^3 = \begin{pmatrix} \{s_4\} & \{s_1, s_2, s_3\} & \{s_4, s_5\} & \{s_6\} & \{s_7\} \\ \{s_5, s_6, s_7\} & \{s_4\} & \{s_2, s_3\} & \{s_0, s_1\} & \{s_7, s_8\} \\ \{s_3, s_4\} & \{s_5, s_6\} & \{s_4\} & \{s_6, s_7\} & \{s_5\} \\ \{s_2\} & \{s_7, s_8\} & \{s_1, s_2\} & \{s_4\} & \{s_5\} \\ \{s_1\} & \{s_0, s_1\} & \{s_3\} & \{s_3\} & \{s_4\} \end{pmatrix}$$

$$H^4 = \begin{pmatrix} \{s_4\} & \{s_1, s_2\} & \{s_2, s_3\} & \{s_3\} & \{s_3, s_4\} \\ \{s_6, s_7\} & \{s_4\} & \{s_4, s_5\} & \{s_6\} & \{s_7, s_8\} \\ \{s_5, s_6\} & \{s_3, s_4\} & \{s_4\} & \{s_8\} & \{s_0\} \\ \{s_5\} & \{s_2\} & \{s_0\} & \{s_4\} & \{s_1, s_2\} \\ \{s_4, s_5\} & \{s_0, s_1\} & \{s_8\} & \{s_6, s_7\} & \{s_4\} \end{pmatrix}$$

4.1.1. Illustration for setting personalized numerical scales

Based on the data H^1, H^2, H^3 and H^4, we illustrate the use of the consistency-driven optimization model to set the personalized numerical scale for linguistic term set.

Based on Eq. (6), the range of $NS^k(s_i) (k = 1, 2, 3, 4)$ is set as follows,

$$NS^k(s_i) \begin{cases} = 0 & i = 0 \\ = 0.5 & i = 4 \\ \in [(i-1)/8, (i+1)/8] & i = 1, 2, \ldots, 7; i \neq 4 \\ = 1 & i = 8 \end{cases}$$

Following, we show how to obtain the set of linguistic preference relations, N_{H^k}, associated to H^k. Here, we take the HFLPR H^1 as an example.

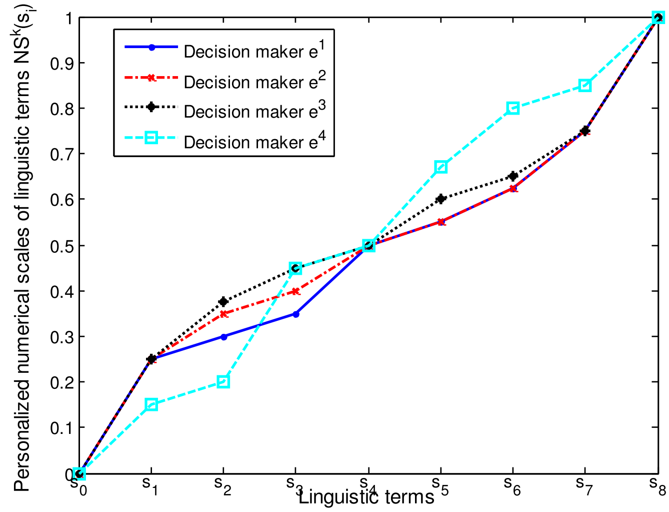

Fig. 3. Personalized numerical scales $NS^k(s_i)$.

The linguistic preference relation set associated to H^1 is $N_{H^1} = \{L^{h,1} | h = 1, 2, \ldots, \#N_{H^1}\}$, where $\#N_{H^1} = \prod_{i=1}^5 \prod_{j=2}^5 \#H_{ij}^1 = 192$. For $L^{h,1} = (l_{ij}^{h,1})_{n\times n}$, where $l_{ij}^{h,1} \in H_{ij}^1$ and $l_{ij}^{h,1} = Neg(l_{ji}^{h,1})$, it is obtained by the permutation and combination of the elements in H^1, such as

$$L^{1,1} = \begin{pmatrix} \{s_4\} & \{s_5\} & \{s_0\} & \{s_2\} & \{s_5\} \\ \{s_3\} & \{s_4\} & \{s_4\} & \{s_1\} & \{s_2\} \\ \{s_8\} & \{s_4\} & \{s_4\} & \{s_6\} & \{s_1\} \\ \{s_6\} & \{s_7\} & \{s_1\} & \{s_4\} & \{s_1\} \\ \{s_3\} & \{s_6\} & \{s_7\} & \{s_7\} & \{s_4\} \end{pmatrix},$$

$$L^{2,1} = \begin{pmatrix} \{s_4\} & \{s_5\} & \{s_0\} & \{s_3\} & \{s_5\} \\ \{s_3\} & \{s_4\} & \{s_5\} & \{s_1\} & \{s_2\} \\ \{s_8\} & \{s_3\} & \{s_4\} & \{s_7\} & \{s_1\} \\ \{s_5\} & \{s_7\} & \{s_1\} & \{s_4\} & \{s_0\} \\ \{s_3\} & \{s_6\} & \{s_7\} & \{s_8\} & \{s_4\} \end{pmatrix} \ldots$$

According to Eq. (7), without loss of generality, we set the constraint value $\lambda = 0.05$. Then, the optimization model to obtain the personalized numerical scale NS^k is as follows.

$$\begin{cases} \max \quad ACI(H^k) \\ s.t.\ ACI(H^k) = \frac{1}{\#N_{H^k}} \times \sum_{h=1}^{\#N_{H^k}} \left(1 - \frac{1}{90} \sum_{i,j,z=1}^{n} \left| NS^k(l_{ij}^{h,k}) + NS^k(l_{jz}^{h,k}) \right.\right. \\ \qquad \left.\left. - NS^k(l_{iz}^{h,k}) - 0.5 \right| \right) \\ \quad l_{ij}^{h,k} \in H_{ij}^k \qquad i, j = 1, 2, \ldots, 5 \\ \quad l_{ij}^{h,k} = Neg(l_{ji}^{h,k}) \qquad i, j = 1, 2, \ldots, 5 \\ \quad NS^k(s_0) = 0 \\ \quad NS^k(s_4) = 0.5 \\ \quad NS^k(s_8) = 1 \\ \quad NS^k(s_i) \in [(i-1)/8, (i+1)/8] \qquad i = 1, \ldots, 7; i \neq 4 \\ \quad NS^k(s_{i+1}) - NS^k(s_i) \geq 0.05 \qquad i = 0, 1, \ldots, 7 \end{cases} \tag{11}$$

where $NS^k(s_i) \quad (k = 0, 1, \ldots, 8)$ are decision variables.

We solve the above model to obtain the personalized numerical scales $NS^k(s_i)$ $(k = 1, 2, 3, 4;\ i = 0, 1, \ldots, 8)$ (see Table 1). Besides, we provide Fig. 3 to show the difference of $NS^k(s_i)$ among different decision makers more clearly.

Moreover, we use four HFLPRs from [26] to compute the personalized individual semantics of linguistic terms by solving model (11). The obtained semantics are shown in Table 2 and Fig. 4.

According to Tables 1 and 2 and Figs. 3 and 4, the personalized numerical scales of linguistic term set associated with each

Table 1
Obtained values of $NS^k(s_i)$.

	$NS^k(s_0)$	$NS^k(s_1)$	$NS^k(s_2)$	$NS^k(s_3)$	$NS^k(s_4)$	$NS^k(s_5)$	$NS^k(s_6)$	$NS^k(s_7)$	$NS^k(s_8)$
e_1	0	0.25	0.3	0.35	0.5	0.55	0.625	0.75	1
e_2	0	0.25	0.35	0.4	0.5	0.55	0.625	0.75	1
e_3	0	0.25	0.375	0.45	0.5	0.6	0.65	0.75	1
e_4	0	0.15	0.2	0.45	0.5	0.67	0.8	0.85	1

Table 2
Obtained values of $NS^k(s_i)$ using the HFLPRs in [26].

	$NS^k(s_0)$	$NS^k(s_1)$	$NS^k(s_2)$	$NS^k(s_3)$	$NS^k(s_4)$	$NS^k(s_5)$	$NS^k(s_6)$	$NS^k(s_7)$	$NS^k(s_8)$
e_1	0	0.25	0.366	0.407	0.5	0.593	0.634	0.75	1
e_2	0	0.2375	0.375	0.49	0.5	0.5625	0.625	0.75	1
e_3	0	0.25	0.333	0.417	0.5	0.583	0.667	0.75	1
e_4	0	0.232	0.262	0.323	0.5	0.597	0.738	0.768	1

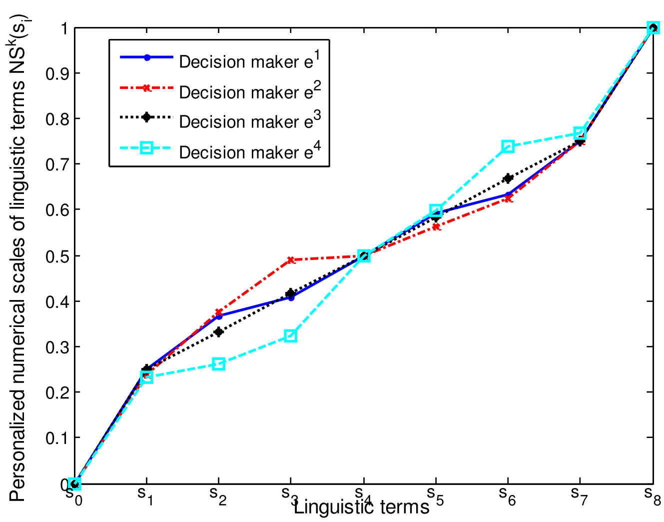

Fig. 4. Personalized numerical scales $NS^k(s_i)$ using the HFLPRs in [26].

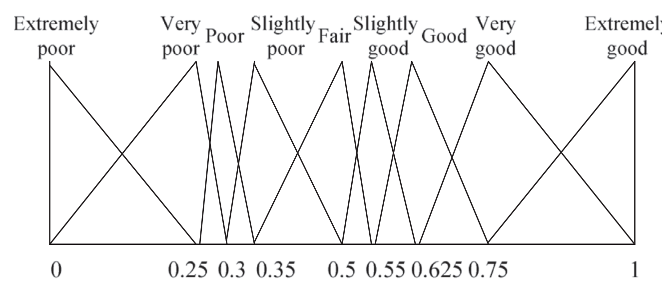

Fig. 5. Personalized individual semantics for linguistic terms associated with e^1.

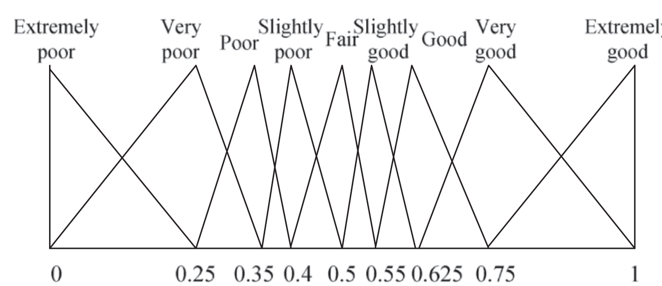

Fig. 6. Personalized individual semantics for linguistic terms associated with e^2.

decision maker are different, which provides a basis to personalize individual semantics to reflect the different meanings of linguistic terms for different decision makers.

4.1.2. Illustration for representing the personalized individual semantics

Next, we illustrate the process to represent the personalized individual semantics in GDM with HFLTSs based on the results shown in Table 1.

(1) Personalized individual semantics of linguistic terms
Following Eq. (9), we construct the personalized individual semantics for the linguistic terms $s_i \in S$ associated with decision makers $e^k (k = 1, 2, 3, 4)$ defined by the triangular membership functions as follows,

$$A^k(s_i) = \begin{cases} T(NS^k(s_0), NS^k(s_0), NS^k(s_1)) & i = 0 \\ T(NS^k(s_{i-1}), NS^k(s_i), NS^k(s_{i+1})) & i = 1, \ldots, 7 \\ T(NS^k(s_7), NS^k(s_8), NS^k(s_8)) & i = 8 \end{cases}$$

(i) Personalized individual semantics for linguistic terms associated with e^1 (see Fig. 5)

$A^1(s_0) = T(0, 0, 0.25)$; $A^1(s_1) = T(0, 0.25, 0.3)$;
$A^1(s_2) = T(0.25, 0.3, 0.35)$;
$A^1(s_3) = T(0.3, 0.35, 0.5)$;
$A^1(s_4) = T(0.35, 0.5, 0.55)$;
$A^1(s_5) = T(0.5, 0.55, 0.625)$;
$A^1(s_6) = T(0.55, 0.625, 0.75)$;
$A^1(s_7) = T(0.625, 0.75, 1)$ and
$A^1(s_8) = T(0.75, 1, 1)$.

(ii) Personalized individual semantics for linguistic terms associated with e^2 (see Fig. 6)

$A^2(s_0) = T(0, 0, 0.25)$; $A^2(s_1) = T(0, 0.25, 0.35)$;
$A^2(s_2) = T(0.25, 0.35, 0.4)$;
$A^2(s_3) = T(0.35, 0.4, 0.5)$; $A^2(s_4) = T(0.4, 0.5, 0.55)$;
$A^2(s_5) = T(0.5, 0.55, 0.625)$;
$A^2(s_6) = T(0.55, 0.625, 0.75)$;
$A^2(s_7) = T(0.625, 0.75, 1)$ and
$A^2(s_8) = T(0.75, 1, 1)$.

(iii) Personalized individual semantics for linguistic terms associated with e^3 (see Fig. 7)

$A^3(s_0) = T(0, 0, 0.25)$; $A^3(s_1) = T(0, 0.25, 0.375)$;
$A^3(s_2) = T(0.25, 0.375, 0.45)$;
$A^3(s_3) = T(0.375, 0.45, 0.5)$; $A^3(s_4) = T(0.45, 0.5, 0.6)$;
$A^3(s_5) = T(0.5, 0.6, 0.65)$;
$A^3(s_6) = T(0.6, 0.65, 0.75)$;

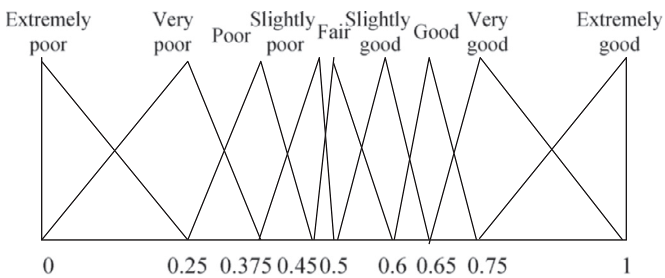

Fig. 7. Personalized individual semantics for linguistic terms associated with e^3.

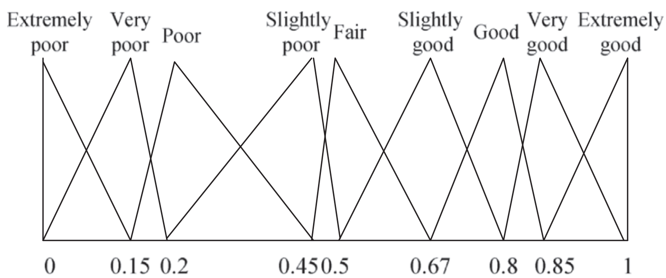

Fig. 8. Personalized individual semantics for linguistic terms associated with e^4.

$A^3(s_7) = T(0.65, 0.75, 1)$ and
$A^3(s_8) = T(0.75, 1, 1)$.

(iv) Personalized individual semantics for linguistic terms associated with e^4 (see Fig. 8)

$A^4(s_0) = T(0, 0, 0.15)$; $A^4(s_1) = T(0, 0.15, 0.2)$;
$A^4(s_2) = T(0.15, 0.2, 0.45)$;
$A^4(s_3) = T(0.2, 0.45, 0.5)$; $A^4(s_4) = T(0.45, 0.5, 0.67)$;
$A^4(s_5) = T(0.5, 0.67, 0.8)$;
$A^4(s_6) = T(0.67, 0.8, 0.85)$;
$A^4(s_7) = T(0.8, 0.85, 1)$ and
$A^4(s_8) = T(0.85, 1, 1)$.

(2) The personalized individual semantics for HFLTSs via fuzzy envelope

We take the HFLTSs $H^2_{12} = \{s_3, s_4\}$, $H^2_{23} = \{s_6, s_7, s_8\}$ and $H^2_{25} = \{s_0, s_1, s_2\}$ in the HFLPR H^2 as an example to illustrate the way to compute the personalized individual semantics of HFLTSs via fuzzy envelope defined by trapezoidal fuzzy membership functions.

(i) For the HFLTS $H^2_{12} = \{s_3, s_4\}$, according to Case A in Section 3.3, the set of elements to aggregate is
$T = \{NS^2(s_2), NS^2(s_3), NS^2(s_4), NS^2(s_5)\}$.
The parameters of the fuzzy envelope $env_F(H^2_{12}) = T\{a_{12}, b_{12}, c_{12}, d_{12}\}$ for H^2_{12} is computed as follows,

$$a_{12} = \min\{NS^2(s_2), NS^2(s_3), NS^2(s_4), NS^2(s_5)\} = NS^2(s_2) = 0.35$$
$$d_{12} = \max\{NS^2(s_2), NS^2(s_3), NS^2(s_4), NS^2(s_5)\} = NS^2(s_5) = 0.55$$
$$b_{12} = NS^2(s_3) = 0.4$$
$$c_{12} = NS^2(s_4) = 0.5$$

Thus, the personalized individual semantics for H^2_{12} is $T\{0.35, 0.4, 0.5, 0.55\}$.

(ii) For the HFLTS $H^2_{23} = \{s_6, s_7, s_8\}$, according to Case B in Section 3.3, the set of elements to aggregate is
$T = \{NS^2(s_5), NS^2(s_6), NS^2(s_7), NS^2(s_8), NS^2(s_8)\}$.

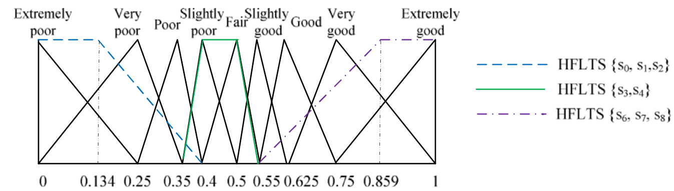

Fig. 9. Personalized individual semantics for H^2_{12}, H^2_{23} and H^2_{25}.

The parameters of the fuzzy envelope $env_F(H^2_{23}) = T\{a_{23}, b_{23}, c_{23}, d_{23}\}$ for H^2_{23} is computed as follows,

$$a_{23} = \min\{NS^2(s_5), NS^2(s_6), NS^2(s_7), NS^2(s_8), NS^2(s_8)\} = NS^2(s_5) = 0.55$$
$$d_{23} = \max\{NS^2(s_5), NS^2(s_6), NS^2(s_7), NS^2(s_8), NS^2(s_8)\} = NS^2(s_8) = 1$$

and the parameter $c_{23} = NS^2(s_8) = 1$.
The point b_{23} is computed by the OWA operator with $\alpha_2 = \frac{3}{4}$ and the weighting vector $W^2 = ((\frac{3}{4})^2, (1-\frac{3}{4})\cdot\frac{3}{4}, (1-\frac{3}{4}))^T$. Thus,

$$b_{23} = \left(\frac{3}{4}\right)^2 \cdot NS^2(s_8) + \left(1-\frac{3}{4}\right)\cdot\frac{3}{4}\cdot NS^2(s_7) + \left(1-\frac{3}{4}\right)\cdot NS^2(s_6) = \frac{9}{16}\cdot 1 + \frac{3}{16}\cdot 0.75 + \frac{1}{4}\cdot 0.625 = 0.859$$

Thus, the personalized individual semantics for H^2_{23} is $T\{0.55, 0.859, 1, 1\}$.

(iii) For the HFLTS $H^2_{25} = \{s_0, s_1, s_2\}$, according to Case C in Section 3.3, the set of elements to aggregate is
$T = \{NS^2(s_0), NS^2(s_0), NS^2(s_1), NS^2(s_2), NS^2(s_3)\}$.
The parameters of the fuzzy envelope $env_F(H^2_{25}) = T\{a_{25}, b_{25}, c_{25}, d_{25}\}$ for H^2_{25} is computed as follows,

$$a_{25} = \min\{NS^2(s_0), NS^2(s_0), NS^2(s_1), NS^2(s_2), NS^2(s_3)\} = NS^2(s_0) = 0$$
$$d_{25} = \max\{NS^2(s_0), NS^2(s_0), NS^2(s_1), NS^2(s_2), NS^2(s_3)\} = NS^2(s_3) = 0.4$$

and the parameter $b_{25} = NS^2(s_0) = 0$.
The point c_{25} is computed by the OWA operator with $\alpha_1 = \frac{1}{4}$ and the weighting vector $W^1 = (\frac{1}{4}, \frac{1}{4}\cdot(1-\frac{1}{4}), (1-\frac{1}{4})^2)^T$. Thus,

$$c_{25} = \frac{1}{4}\cdot NS^2(s_2) + \frac{1}{4}\cdot\left(1-\frac{1}{4}\right)\cdot NS^2(s_1) + \left(1-\frac{1}{4}\right)^2\cdot NS^2(s_0) = \frac{1}{4}\cdot 0.35 + \frac{1}{4}\cdot\left(1-\frac{1}{4}\right)\cdot 0.25 + \left(1-\frac{1}{4}\right)^2\cdot 0 = 0.134$$

Thus, the personalized individual semantics for H^2_{25} is $T\{0, 0, 0.134, 0.4\}$.Fig. 9 shows the obtained personalized individual semantics for HFLTSs $H^2_{12} = \{s_3, s_4\}$, $H^2_{23} = \{s_6, s_7, s_8\}$ and $H^2_{25} = \{s_0, s_1, s_2\}$.
Based on our proposal to personalize individual semantics of HFLTSs, it is easy to obtain the personalized individual semantics of all HFLTSs in the HFLPRs H^1, H^2, H^3 and H^4. However, for saving space, we do not present them here.

Table 3
The semantics for several HFLTSs from Section 4.1 using the approach in [11] and our proposed approach.

	In [11]	Our proposal
$H_{25}^1 = \{s_2, s_3\}$	[0.125,0.25,0.375,0.5]	[0.25,0.3,0.35,0.5]
$H_{24}^2 = \{s_2, s_3\}$	[0.125,0.25,0.375,0.5]	[0.25,0.35,0.4,0.5]
$H_{23}^3 = \{s_2, s_3\}$	[0.125,0.25,0.375,0.5]	[0.25,0.375,0.45,0.5]
$H_{13}^4 = \{s_2, s_3\}$	[0.125,0.25,0.375,0.5]	[0.15,0.2,0.45,0.5]
$H_{23}^1 = \{s_4, s_5\}$	[0.375,0.5,0.625,0.75]	[0.35,0.5,0.55,0.625]
$H_{13}^3 = \{s_4, s_5\}$	[0.375,0.5,0.625,0.75]	[0.45,0.5,0.6,0.65]
$H_{23}^4 = \{s_4, s_5\}$	[0.375,0.5,0.625,0.75]	[0.45,0.5,0.67,0.8]
$H_{24}^1 = \{s_1, s_2\}$	[0,0.167,0.333,0.5]	[0,0.25,0.3,0.35]
$H_{14}^2 = \{s_1, s_2\}$	[0,0.167,0.333,0.5]	[0,0.25,0.35,0.4]
$H_{12}^4 = \{s_1, s_2\}$	[0,0.167,0.333,0.5]	[0,0.15,0.2,0.45]

4.2. Comparative study

In [11] and our proposal, the semantics of HFLTS are both expressed as trapezoidal membership functions. Next, we take some HFLTSs from HFLPRs provided in Section 4.1 to show the difference for the representations of HFLTSs in the following methods:

(1) The semantics of HFLTSs proposed in [11];
(2) The personalized individual semantics of HFLTSs in Section 3.3.

Table 3 shows the semantics of HFLTSs for different decision makers using the approach in [11] and our proposed approach.

From Table 3, the following observations are highlighted:

- By applying the approach in Liu and Rodríguez [11], the semantics obtained for $\{s_2, s_3\}$, $\{s_4, s_5\}$ and $\{s_1, s_2\}$ of different decision makers are all expressed by the trapezoidal membership functions [0.125, 0.25, 0.375, 0.5], [0.375, 0.5, 0.625, 0.75] and [0, 0.167, 0.333, 0.5], respectively.
- Using our proposed approach, the semantics obtained for $\{s_2, s_3\}$, $\{s_4, s_5\}$ and $\{s_1, s_2\}$ of different decision makers are different, such as, the semantics of $\{s_2, s_3\}$ for decision makers e^1, e^2, e^3 and e^4 are [0.25, 0.3, 0.35, 0.5], [0.25, 0.35, 0.4, 0.5], [0.25, 0.375, 0.45, 0.5] and [0.15, 0.2, 0.45, 0.5], respectively.

The above observations show that the approach in [11] provided the semantics rules, but it reflects the same semantics of HFLTSs for different decision makers. While our proposed approach reflects the different understanding of HFLTSs for different decision makers, it reflects the personalized individual semantics.

5. Discussion: Advantages and weakness

In this section, we present some improvements and limitations of the proposed approach to personalize individual semantics in hesitant linguistic GDM.

1) Advantages. We find the following improvements of our proposal:
   a) In Mendel and Wu [15], type-2 fuzzy set is used to deal with the multiple meanings of words, but it cannot represent the specific meaning of words. Comparing with the method in [15], our proposal is based on a different assumption to personalize individual semantics via numerical scale and consistency-driven methodology.
   b) In recent years, HFLTSs are widely used in linguistic decision making (e.g., [2,26]) based on the operation rules of 2-tuple linguistic model, but the semantics for HFLTSs are not discussed in most of these studies. In Liu and Rodríguez [11] the fuzzy envelope for HFLTSs has been proposed to describe the semantics of HFLTSs. Comparing with the approach in [11], the proposed approach provides a way to show the individual difference in understanding the meaning of HFLTSs.
   c) Li et al. [10] proposed a model to personalize individual semantics of simple terms of a linguistic term set. Our proposal is a continuation of Li et al., and generalizes the work in [10] to personalize individual semantics of HFLTSs.
2) Weakness. We find the following limitations:
   a) To our knowledge, there is not any framework to compare different CW methodologies in decision making. However, it is necessary to propose some criteria to compare our proposal with other CW methodologies (e.g., Mendel and Wu [15]).
   b) Semantics should play an important role in linguistic GDM problems, but this paper mainly discusses how to personalize individual semantics in a hesitant linguistic and group context, and it is necessary to study how to use personalized individual semantics to improve the quality of hesitant linguistic GDM.

These limitations will be talked in the future research, to design CW comparison methodologies from different criteria, and to discuss the GDM improvements based on the personalized individual semantics.

6. Conclusion

In this paper, we propose the framework to personalize individual semantics in the hesitant linguistic GDM with comparative linguistic expressions to improve the management of different meanings of words for different people. An average consistency-driven approach to personalize numerical scales of the linguistic term set is first provided, then based on the personalized numerical scales, the fuzzy envelope for HFLTSs described by trapezoidal fuzzy membership function is proposed to personalize individual semantics with comparative linguistic expressions.

In the future, we plan to study the use of the personalized individual semantics in large scale GDM problems [12,13,18,30].

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