Hybrid-Attack-Resistant Distributed State Estimation for Nonlinear Complex Networks with Random Coupling Strength and Sensor Delays

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Abstract

In this paper, a recursive distributed hybrid-attack-resistant state estimation (SE) scheme is proposed for a class of time-varying nonlinear complex networks (NCNs) subject to random coupling strength (RCS) and random sensor delays (RSDs) under hybrid attacks. A hybrid-attack model is considered to characterize the random occurrence of denial-of-service (DoS) attacks and deception attacks. The objective of the problem to be solved is to develop a recursive distributed estimation method such that, in the presence of RCS, RSDs and hybrid attacks, a locally optimized upper bound (UB) on the estimation error covariance (EEC) is ensured. By employing the mathematical induction method, a UB is firstly derived on the EEC. Subsequently, the obtained UB is minimized by appropriately designing the estimator gain (EG). Furthermore, a sufficient criterion guaranteeing the exponential boundedness (EB) of SE error is elaborated in the mean square sense (MSS). Finally, simulation experiments with localization applications of multiple mobile indoor robots are conducted to illustrate the applicability of the proposed SE scheme.

Index Terms

Time-varying nonlinear complex networks, Random hybrid attacks, Random sensor delays, Random coupling strength, Mean-square boundedness.

I. Introduction

In recent years, the dynamical behavior analysis of complex networks (CNs) has received particular research attention [1]–[4]. Because of the characteristics and advantages of describing the scale-free networks and small-world networks, it can characterize many real-world networks, such as social networks, information networks, traffic and transportation networks [5]–[9]. Generally, the complexity of CNs is mainly reflected in the huge number of nodes, the diversity of connections, and the complexity of dynamics. Consequently, multiple nodes in the CNs interact with each other and their dynamical behavior is more difficult to predict than a network with only one isolated node. Thus, for CNs, the state estimation (SE) issues have recently received increasing research attention,

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where the methods presented mainly contain the linear matrix inequality method for time-invariant CNs over infinite horizon and recursive matrix equation method for time-varying CNs over finite horizon [10]–[12]. For example, in [11], the distributed SE algorithm has been given for stochastic CNs with switching topology and the estimator gain (EG) has been determined by the recursive matrix equation method. In fact, the study of SE problems for CNs is useful to understand the process of network dynamics and plays an important role in practical engineering applications. As a consequence, it has great practical significance to study SE issues for CNs [13]–[16].

Nowadays, many applications of CNs assume that the network topology structure is known, with a deterministic coupling strength [17], [18]. However, such an assumption has certain limitations in practical applications, and the coupling strength might be uncertain due to the presence of noise disturbances and unknown inputs. Recently, the corresponding study on CNs problems with different types of coupling strength has attracted the attention of scholars. Furthermore, it has become a hot research topic and some encouraging results have been achieved in [19]–[21]. For example, in [20], a state estimator has been constructed to handle the random coupling strength (RCS) obeying a uniform distribution over a certain interval. In addition to the RCS, it is well known that random sensor delays (RSDs) and other network-induced phenomena could happen in the process of data transmission from sensor to estimator/controller [22]–[25]. Furthermore, the occurrence of RSDs is independent of each other in the CNs. In other words, some nodes may receive measurement signals normally, while others may suffer from sensor delays. So far, the SE problems affected by RSDs have been discussed accordingly [26]–[29]. Nevertheless, the distributed SE issue for CNs with RCS and RSDs has not yet received sufficient research efforts, which constitutes one of the main motivations of the present investigation.

Due to the complexity of networked environment, the data transmission in the sensor-to-estimator communication channels may be subject to cyber attacks and failures [30]–[33]. Generally, loopholes and security flaws have been utilized by malicious cyber attackers to attack systems and resources [34]–[37]. Therefore, the network security issue has become an essential topic in the networked system, which mainly discusses three types of cyber attacks, namely, deception attack, replay attack and denial-of-service (DoS) attack [38]–[40]. It should be noted that cyber attacks are not always successful due to the presence of firewall software, which might lead to the existence of the phenomenon of randomly occurring cyber attacks. Up to now, scholars have made great efforts to address the problem of distributed SE for stochastic CNs under cyber attacks [41]–[43]. In the existing literature, most research results have considered only a single type of cyber attack. For example, in [42], the state-saturated recursive state estimator has been constructed to deal with the influence caused by the deception attacks. However, in practical applications, hybrid attacks often occur frequently as they can generally increase the probability of successful attacks. In the CNs, the study on SE problems subject to hybrid attacks is scarce, let alone taking the RCS and RSDs into account.

Inspired by the aforementioned discussions, we aim to provide a distributed SE scheme for a class of nonlinear complex networks (NCNs) with RCS and RSDs subject to hybrid attacks. With this goal in mind, we are obliged to confront some challenging issues: 1) how to develop a distributed SE method of recursive application potential that simultaneously tackles the influence of RCS, RSDs and hybrid attacks; 2) how to analyze the exponential boundedness (EB) with respect to SE error in the mean square sense (MSS) from the theoretical analysis perspective; 3) how to reduce the complexity of high-dimensional matrix inversion as well as cross-covariance matrix calculation between coupled nodes. Consequently, the following three main contributions can be summarized: 1) the impacts of RSDs and hybrid attacks are addressed when dealing with the distributed SE problem with RCS for time-varying NCNs; 2) the boundedness analysis with respect to the SE error is discussed by providing a sufficient condition ensuring EB of the SE error in the MSS; 3) a recursive matrix equation method is utilized to avoid the inversion operation of high-dimensional matrix and calculation of cross-covariance matrix between coupled nodes, which can reduce the computational burden.

TABLE I
ACRONYMS AND NOTATIONS

SE	State estimation.	$\mathbb{R}^n$	n-dimensional Euclidean space.
CNs	Complex networks.	X^T	The transpose of matrix X.
NCNs	Nonlinear complex networks.	X^{-1}	The inverse of matrix X.
RCS	Random coupling strength.	$X \geq 0$	X is a nonnegative definite symmetric matrix.
RSDs	Random sensor delays.	$X > 0$	X is a positive definite symmetric matrix.
DoS	Denial-of-service.	$\mathrm{tr}\{X\}$	The trace of matrix X.
UB	Upper bound.	I	Identity matrix with appropriate dimensions.
PEC	Prediction error covariance.	0	Zero matrix with appropriate dimensions.
EEC	Estimation error covariance.	$\mathbb{E}\{y\}$	The expectation of y.
EG	Estimator gain.	$\mathrm{Prob}\{\cdot\}$	The probability of "$\cdot$".
EB	Exponential boundedness.	$\mathrm{diag}\{\cdots\}$	The diagonal block-matrix.
MSS	Mean square sense.	$\|\cdot\|$	The Euclidean norm of a vector.
MSE	Mean square error.		

II. Problem Formulation

In this paper, we consider a class of time-varying NCNs with N coupled nodes of the following form:

$$\vec{x}_{i,\varrho+1} = \vec{h}(\vec{x}_{i,\varrho}) + \sum_{j=1}^{N} \alpha_{ij,\varrho}\vec{\Gamma}\vec{x}_{j,\varrho} + \vec{B}_{i,\varrho}\varpi_{i,\varrho}, \tag{1}$$

$$\vec{z}_{i,\varrho} = \vec{C}_{i,\varrho}\vec{x}_{i,\varrho} + \vec{\nu}_{i,\varrho}, \tag{2}$$

$$z_{i,\varrho} = \theta_{i,\varrho}\vec{z}_{i,\varrho} + (1-\theta_{i,\varrho})\vec{z}_{i,\varrho-1}, \tag{3}$$

where $\vec{x}_{i,\varrho} \in \mathbb{R}^n$ represents the state vector of node i at time ϱ, with initial value $\vec{x}_{i,0}$ (the mean is $\bar{\vec{x}}_{i,0}$). $\vec{z}_{i,\varrho} \in \mathbb{R}^m$ stands for the measurement of the ith node. $z_{i,\varrho}$ is the measurement output with the RSDs. The RCS is described by random variables $\alpha_{ij,\varrho}$, which take values on the intervals $[c_{ij,\varrho}, d_{ij,\varrho}]$ with $0 < c_{ij,\varrho} \leq d_{ij,\varrho} \leq 1$ being known constants. Moreover, the random variables $\alpha_{ij,\varrho}$ satisfy $\mathbb{E}\{\alpha_{ij,\varrho}\} = \bar{\alpha}_{ij,\varrho}$, $\mathbb{E}\left\{(\alpha_{ij,\varrho} - \bar{\alpha}_{ij,\varrho})^2\right\} \triangleq \mathbb{E}\left\{\tilde{\alpha}^2_{ij,\varrho}\right\} = \hat{\alpha}_{ij,\varrho}$, where $\bar{\alpha}_{ij,\varrho}$ and $\hat{\alpha}_{ij,\varrho}$ are known positive scalars and $\alpha_{ij,\varrho}$ are mutually independent. $\vec{\Gamma} = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ stands for the inner coupling matrix, where there exists a connection with the jth state component if $\gamma_j \neq 0$. $\theta_{i,\varrho} \in \mathbb{R}$ $(i = 1, 2, \cdots, N)$ are a set of Bernoulli random variables. $\vec{B}_{i,\varrho}$ and $\vec{C}_{i,\varrho}$ are known time-varying matrices. $\varpi_{i,\varrho}$ and $\vec{\nu}_{i,\varrho}$ are zero-mean Gaussian white noises with covariances $Q_{i,\varrho}$ and $\vec{R}_{i,\varrho}$.

For $\forall\, \mathfrak{a}, \mathfrak{b} \in \mathbb{R}^n$, the nonlinear function $\vec{h}(\cdot)$: $\mathbb{R}^n \to \mathbb{R}^n$ satisfies

$$\|\vec{h}(\mathfrak{a}) - \vec{h}(\mathfrak{b}) - \vec{E}_\varrho(\mathfrak{a} - \mathfrak{b})\| \leq \ell_\varrho \|\mathfrak{a} - \mathfrak{b}\|, \tag{4}$$

where $\vec{E}_\varrho$ is a known time-varying matrix with proper dimensions and $\ell_\varrho > 0$ is a known scalar.

The phenomenon of RSDs is described by Bernoulli distributed random variables $\theta_{i,\varrho}$ satisfying

$$\mathbb{E}\{\theta_{i,\varrho}\} = \mathrm{Prob}\{\theta_{i,\varrho} = 1\} = \bar{\theta}_{i,\varrho},$$

where $\bar{\theta}_{i,\varrho} \in [0, 1]$ are known constants.

In addition to the RSDs, due to the opening-up and sharing characteristics of typical communication networks, the measurement data faces the risk of cyber attacks launched by adversaries in the signal transmission through the network channels. In order to more realistically describe the cyber attacks, the actual measurement received by the estimator is described by

$$y_{i,\varrho} = \beta_{1i,\varrho}(z_{i,\varrho} + \beta_{2i,\varrho}\tilde{\zeta}_{i,\varrho}), \tag{5}$$

where $\beta_{1i,\varrho}$ and $\beta_{2i,\varrho}$ are Bernoulli random variables, which satisfy the following conditions:

$$\mathbb{E}\{\beta_{1i,\varrho}\} = \text{Prob}\{\beta_{1i,\varrho}=1\} = \bar{\beta}_{1i,\varrho},$$
$$\mathbb{E}\{\beta_{2i,\varrho}\} = \text{Prob}\{\beta_{2i,\varrho}=1\} = \bar{\beta}_{2i,\varrho}$$

with $0 < \bar{\beta}_{1i,\varrho} \le 1$ and $0 < \bar{\beta}_{2i,\varrho} \le 1$ being known scalars. In (5), $\tilde{\zeta}_{i,\varrho} = -z_{i,\varrho} + \zeta_{i,\varrho}$ represents the deception signal, where $\zeta_{i,\varrho}$ is a bounded attack signal satisfying the condition $\zeta_{i,\varrho}^T \zeta_{i,\varrho} \le \bar{\zeta}_i$ and $\bar{\zeta}_i$ being a positive scalar. Generally, assume that all random variables $\varpi_{i,\varrho}$, $\vec{\nu}_{i,\varrho}$, $\alpha_{ij,\varrho}$, $\theta_{i,\varrho}$, $\beta_{1i,\varrho}$, $\beta_{2i,\varrho}$ and $\vec{x}_{i,0}$ are mutually independent.

Remark 1: It should be noticed that the hybrid attacks with model (5) are considered in this paper to cater the following real engineering scenarios. 1) With the rapid development of communication networks, a large number of nodes in the CNs realize the transmission of measurement data based on communication networks. Nevertheless, the potential attackers in the networks can exploit vulnerabilities and security flaws to attack the system, which constitutes a major threat to network security. 2) Most of the engineering systems possess powerful security measures, such as firewalls, intrusion detection systems, and intrusion defense systems, which can detect and prevent the occurrence of attacks. Thus, malicious attackers may fail to launch attacks against transmitted signal, which is the reason for introducing the Bernoulli random variables in (5) to portray the attack. 3) A single attack type may be easily detected and blocked by existing security system, but hybrid attacks can bypass these defenses more effectively by combining different attack types. Furthermore, different attack types can target different vulnerabilities or security flaws, which means that hybrid attacks can exploit multiple vulnerabilities at the same time, thereby increasing the probability of a successful attack.

Remark 2: Model (3) describes the phenomenon of RSDs and (5) reflects the phenomenon of measurement subject to hybrid attacks, which include the DoS attack and the deception attack within the same framework. From (3), it can be seen that the measurement signal is transmitted to the estimator without suffering from the RSDs when $\theta_{i,\varrho} = 1$. Meanwhile, the measurement output undergoes the DoS attack in a probabilistic way when $\beta_{1i,\varrho} = 0$, and the corresponding measurement is $y_{i,\varrho} = 0$. In addition, the measurement signal is affected by a deception attack signal launched by the malicious cyber attacker when $\beta_{1i,\varrho} = 1$ and $\beta_{2i,\varrho} = 1$, and the real measurement under this case is $y_{i,\varrho} = \zeta_{i,\varrho}$. In particular, the measurement signal is transmitted to the estimator without suffering from cyber attack when $\beta_{1i,\varrho} = 1$ and $\beta_{2i,\varrho} = 0$, i.e., the ideal measurement is $y_{i,\varrho} = \vec{C}_{i,\varrho}\vec{x}_{i,\varrho} + \vec{\nu}_{i,\varrho}$. On the contrary, the phenomenon of RSDs occurs when $\theta_{i,\varrho} = 0$, and the phenomenon of cyber attacks is same as mentioned above.

Summing up above discussions, when the phenomenon of RSDs does not occur (i.e., $\theta_{i,\varrho} = 1$), the following attack situations hold:

$$y_{i,\varrho} = \begin{cases} 0, & \text{if } \beta_{1i,\varrho} = 0, \text{ then the DoS attack occurs;} \\ \zeta_{i,\varrho}, & \text{if } \beta_{1i,\varrho} = 1 \text{ and } \beta_{2i,\varrho} = 1, \text{ then the deception attack occurs;} \\ \vec{C}_{i,\varrho}\vec{x}_{i,\varrho} + \vec{\nu}_{i,\varrho}, & \text{if } \beta_{1i,\varrho} = 1 \text{ and } \beta_{2i,\varrho} = 0, \text{ then no attack occurs.} \end{cases}$$

Before further analysis, we set

$$x_{i,\varrho} = \begin{bmatrix} \vec{x}_{i,\varrho} \\ \vec{x}_{i,\varrho-1} \end{bmatrix},\ h(x_{i,\varrho}) = \begin{bmatrix} \vec{h}(\vec{x}_{i,\varrho}) \\ 0 \end{bmatrix},\ \tilde{I} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix},\ B_{i,\varrho} = \begin{bmatrix} \vec{B}_{i,\varrho} \\ 0 \end{bmatrix},$$
$$C_{i,\varrho} = \begin{bmatrix} \vec{C}_{i,\varrho} & 0 \\ 0 & \vec{C}_{i,\varrho-1} \end{bmatrix},\ \nu_{i,\varrho} = \begin{bmatrix} \vec{\nu}_{i,\varrho} \\ \vec{\nu}_{i,\varrho-1} \end{bmatrix},\ \Gamma = \begin{bmatrix} \vec{\Gamma} & 0 \\ 0 & 0 \end{bmatrix},\ E_{\varrho} = \begin{bmatrix} \vec{E}_{\varrho} & 0 \\ 0 & 0 \end{bmatrix},$$
$$R_{i,\varrho} = \begin{bmatrix} \vec{R}_{i,\varrho} & 0 \\ 0 & \vec{R}_{i,\varrho-1} \end{bmatrix},\ \Theta_{i,\varrho} = \begin{bmatrix} \beta_{1i,\varrho}(1-\beta_{2i,\varrho})\theta_{i,\varrho} I & \beta_{1i,\varrho}(1-\beta_{2i,\varrho})(1-\theta_{i,\varrho}) I \end{bmatrix}.$$

Then, the system can be rewritten as

$$x_{i,\varrho+1} = h(x_{i,\varrho}) + \tilde{I}x_{i,\varrho} + \sum_{j=1}^{N} \alpha_{ij,\varrho}\Gamma x_{j,\varrho} + B_{i,\varrho}\varpi_{i,\varrho}, \tag{6}$$

$$y_{i,\varrho} = \Theta_{i,\varrho}(C_{i,\varrho}x_{i,\varrho} + \nu_{i,\varrho}) + \beta_{1i,\varrho}\beta_{2i,\varrho}\zeta_{i,\varrho}. \tag{7}$$

For the ith node, based on the received measurements $y_{i,\varrho+1}$, an estimator is designed as follows:

$$\hat{x}_{i,\varrho+1|\varrho} = h(\hat{x}_{i,\varrho|\varrho}) + \tilde{I}\hat{x}_{i,\varrho|\varrho} + \sum_{j=1}^{N} \bar{\alpha}_{ij,\varrho}\Gamma\hat{x}_{j,\varrho|\varrho}, \tag{8}$$

$$\hat{x}_{i,\varrho+1|\varrho+1} = \hat{x}_{i,\varrho+1|\varrho} + K_{i,\varrho+1}(y_{i,\varrho+1} - \bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1}\hat{x}_{i,\varrho+1|\varrho}), \tag{9}$$

where $\hat{x}_{i,\varrho+1|\varrho}$ and $\hat{x}_{i,\varrho+1|\varrho+1}$ are one-step prediction and estimation of $x_{i,\varrho}$, respectively; $K_{i,\varrho+1}$ is the EG to be designed and $\bar{\Theta}_{i,\varrho+1} = \begin{bmatrix} \bar{\beta}_{1i,\varrho+1}(1-\bar{\beta}_{2i,\varrho+1})\bar{\theta}_{i,\varrho+1}I & \bar{\beta}_{1i,\varrho+1}(1-\bar{\beta}_{2i,\varrho+1})(1-\bar{\theta}_{i,\varrho+1})I \end{bmatrix}$.

Remark 3: In this paper, the designed state estimator (8)-(9) is composed of two steps including prediction and updating steps, which is similar to the structure of traditional Kalman filter. Nevertheless, due to the consideration of the effect of RCS, RSDs and hybrid attacks, some additional terms with respect to the above phenomena are introduced in the state estimator design. To be specific, it is noted that the coupling strength $\alpha_{ij,\varrho}$ is random, that is to say, the exact value of $\alpha_{ij,\varrho}$ at each sampling moment is unknown. Thus, the expectation of $\alpha_{ij,\varrho}$ (i.e., $\bar{\alpha}_{ij,\varrho}$) is employed to reflect the coupling relationship between nodes in the prediction step. Meanwhile, the innovation sequence as a correction is used to construct the state estimator. It can be seen that $\bar{\Theta}_{i,\varrho+1}$ is adopted in the updating step, and $\bar{\theta}_{i,\varrho+1}$, $\bar{\beta}_{1i,\varrho+1}$ and $\bar{\beta}_{2i,\varrho+1}$ are included in $\bar{\Theta}_{i,\varrho+1}$. This is due to the fact that the measurement output affected by sensor delays and attacks can be available only in the remote estimator side. Similar to the coupling strength $\alpha_{ij,\varrho}$, we only know the occurrence probabilities of sensor delays and attacks (i.e., $\bar{\theta}_{i,\varrho+1}$, $\bar{\beta}_{1i,\varrho+1}$ and $\bar{\beta}_{2i,\varrho+1}$), which are employed to reflect the effect of sensor delays and attacks in the updating step.

III. Main Results

We are in a position to tackle the design problem of the SE scheme in this section. First, we will obtain upper bounds (UBs) on the prediction error covariance (PEC) and the estimation error covariance (EEC). Subsequently, the EG that minimizes the trace of the UB on EEC will be derived. The following lemmas are essential to deduce our results more directly.

Lemma 1: [43] For real vectors h, $p \in \mathbb{R}^n$, the following inequality holds

$$hp^T + ph^T \leq \epsilon hh^T + \epsilon^{-1}pp^T,$$

where $\epsilon > 0$ is a scalar.

Lemma 2: [44] Given matrices $\mathfrak{L}_1$, $\mathfrak{L}_2$ and $\mathfrak{L}_3$, where $0 < \mathfrak{L}_1 = \mathfrak{L}_1^T$ and $0 < \mathfrak{L}_2 = \mathfrak{L}_2^T$, then $\mathfrak{L}_1 - \mathfrak{L}_3^T\mathfrak{L}_2\mathfrak{L}_3 \geq 0$ if and only if

$$\begin{bmatrix} \mathfrak{L}_1 & \mathfrak{L}_3^T \\ \mathfrak{L}_3 & \mathfrak{L}_2^{-1} \end{bmatrix} \geq 0, \quad \text{or} \quad \begin{bmatrix} \mathfrak{L}_2^{-1} & \mathfrak{L}_3 \\ \mathfrak{L}_3^T & \mathfrak{L}_1 \end{bmatrix} \geq 0, \quad \text{or} \quad \mathfrak{L}_2^{-1} - \mathfrak{L}_3\mathfrak{L}_1^{-1}\mathfrak{L}_3^T \geq 0.$$

Denote the prediction error and SE error as $\tilde{x}_{i,\varrho+1|\varrho} = x_{i,\varrho+1} - \hat{x}_{i,\varrho+1|\varrho}$ and $\tilde{x}_{i,\varrho+1|\varrho+1} = x_{i,\varrho+1} - \hat{x}_{i,\varrho+1|\varrho+1}$, and their corresponding error covariances are characterized by $P_{i,\varrho+1|\varrho} = \mathbb{E}\{\tilde{x}_{i,\varrho+1|\varrho}\tilde{x}_{i,\varrho+1|\varrho}^T\}$ and $P_{i,\varrho+1|\varrho+1} = \mathbb{E}\{\tilde{x}_{i,\varrho+1|\varrho+1}\tilde{x}_{i,\varrho+1|\varrho+1}^T\}$, respectively.

Based on (6) and (8), $\tilde{x}_{i,\varrho+1|\varrho}$ is derived as

$$\tilde{x}_{i,\varrho+1|\varrho} = h(x_{i,\varrho}) - h(\hat{x}_{i,\varrho|\varrho}) + \tilde{I}\tilde{x}_{i,\varrho|\varrho} + \sum_{j=1}^{N} \tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho} + \sum_{j=1}^{N} \bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho} + B_{i,\varrho}\varpi_{i,\varrho}. \tag{10}$$

Similarly, the SE error of node i is given by

$$\begin{aligned}\tilde{x}_{i,\varrho+1|\varrho+1} =& (I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})\tilde{x}_{i,\varrho+1|\varrho}-K_{i,\varrho+1}\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}\\ &-K_{i,\varrho+1}\Theta_{i,\varrho+1}\nu_{i,\varrho+1}-\beta_{1i,\varrho+1}\beta_{2i,\varrho+1}K_{i,\varrho+1}\zeta_{i,\varrho+1},\end{aligned} \tag{11}$$

where $\tilde{\Theta}_{i,\varrho+1}=\Theta_{i,\varrho+1}-\bar{\Theta}_{i,\varrho+1}$. Then, it follows that $\mathbb{E}\{\tilde{\Theta}_{i,\varrho+1}\}=0$.

Lemma 3: The PEC can be described as

$$\begin{aligned}P_{i,\varrho+1|\varrho} =& \mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\mathfrak{Y}_{i,\varrho}^T\right\}+\mathbb{E}\left\{\mathfrak{C}_{i,\varrho}\mathfrak{C}_{i,\varrho}^T\right\}+\mathbb{E}\left\{\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]^T\right\}\\ &+\mathbb{E}\left\{\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\}+B_{i,\varrho}Q_{i,\varrho}B_{i,\varrho}^T+\sum_{s=1}^{3}\left(\mathcal{A}_{is,\varrho}+\mathcal{A}_{is,\varrho}^T\right),\end{aligned} \tag{12}$$

where

$$\begin{aligned}\mathfrak{Y}_{i,\varrho} =& h(x_{i,\varrho})-h(\hat{x}_{i,\varrho|\varrho})-E_\varrho\tilde{x}_{i,\varrho|\varrho},\\ \mathfrak{C}_{i,\varrho} =& (E_\varrho+\tilde{I})\tilde{x}_{i,\varrho|\varrho},\\ \mathcal{A}_{i1,\varrho} =& \mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\mathfrak{C}_{i,\varrho}^T\right\},\\ \mathcal{A}_{i2,\varrho} =& \mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\},\\ \mathcal{A}_{i3,\varrho} =& \mathbb{E}\left\{\mathfrak{C}_{i,\varrho}\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\}.\end{aligned}$$

Proof: According to the definition of PEC and (10), we have

$$\begin{aligned}P_{i,\varrho+1|\varrho} =& \mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\mathfrak{Y}_{i,\varrho}^T\right\}+\mathbb{E}\left\{\mathfrak{C}_{i,\varrho}\mathfrak{C}_{i,\varrho}^T\right\}+\mathbb{E}\left\{\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]^T\right\}\\ &+\mathbb{E}\left\{\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\}+B_{i,\varrho}Q_{i,\varrho}B_{i,\varrho}^T+\sum_{s=1}^{10}\left(\mathcal{A}_{is,\varrho}+\mathcal{A}_{is,\varrho}^T\right),\end{aligned}$$

where $\mathcal{A}_{is,\varrho}$ $(s=4,5,\cdots,10)$ are defined as follows:

$$\begin{aligned}\mathcal{A}_{i4,\varrho} =& \mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]^T\right\},\\ \mathcal{A}_{i5,\varrho} =& \mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\left[B_{i,\varrho}\varpi_{i,\varrho}\right]^T\right\},\\ \mathcal{A}_{i6,\varrho} =& \mathbb{E}\left\{\mathfrak{C}_{i,\varrho}\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]^T\right\},\\ \mathcal{A}_{i7,\varrho} =& \mathbb{E}\left\{\mathfrak{C}_{i,\varrho}\left[B_{i,\varrho}\varpi_{i,\varrho}\right]^T\right\},\\ \mathcal{A}_{i8,\varrho} =& \mathbb{E}\left\{\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\},\end{aligned}$$

$$\mathcal{A}_{i9,\varrho} = \mathbb{E}\left\{\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right][B_{i,\varrho}\varpi_{i,\varrho}]^T\right\},$$
$$\mathcal{A}_{i10,\varrho} = \mathbb{E}\left\{\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma \tilde{x}_{j,\varrho|\varrho}\right][B_{i,\varrho}\varpi_{i,\varrho}]^T\right\}.$$

Noting $\mathbb{E}\{\tilde{\alpha}_{ij,\varrho}\} = 0$ and $\mathbb{E}\{\varpi_{i,\varrho}\} = 0$, it is easy to find $\mathcal{A}_{is,\varrho} = 0$ ($s = 4, 5, \cdots, 10$). Then, (12) can be easily obtained. ■

Lemma 4: The EEC is calculated as follows:

$$\begin{aligned}P_{i,\varrho+1|\varrho+1} =& (I - K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})P_{i,\varrho+1|\varrho}(I - K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})^T \\ &+K_{i,\varrho+1}\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}x_{i,\varrho+1}^T C_{i,\varrho+1}^T\tilde{\Theta}_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T \\ &+K_{i,\varrho+1}\mathbb{E}\left\{\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T \\ &+\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}K_{i,\varrho+1}\mathbb{E}\left\{\zeta_{i,\varrho+1}\zeta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T + \mathcal{A}_{i11,\varrho+1} + \mathcal{A}_{i11,\varrho+1}^T,\end{aligned} \tag{13}$$

where $\mathcal{A}_{i11,\varrho+1} = -(I - K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})\mathbb{E}\{\beta_{1i,\varrho+1}\beta_{2i,\varrho+1}\tilde{x}_{i,\varrho+1|\varrho}\zeta_{i,\varrho+1}^T\}K_{i,\varrho+1}^T$.

Proof: Considering (11), it is obvious that

$$\begin{aligned}P_{i,\varrho+1|\varrho+1} =& (I - K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})P_{i,\varrho+1|\varrho}(I - K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})^T \\ &+K_{i,\varrho+1}\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}x_{i,\varrho+1}^T C_{i,\varrho+1}^T\tilde{\Theta}_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T \\ &+\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}K_{i,\varrho+1}\mathbb{E}\left\{\zeta_{i,\varrho+1}\zeta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T \\ &+K_{i,\varrho+1}\mathbb{E}\left\{\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T + \sum_{s=11}^{16}\left(\mathcal{A}_{is,\varrho+1} + \mathcal{A}_{is,\varrho+1}^T\right),\end{aligned}$$

where $\mathcal{A}_{is,\varrho+1}$ ($s = 12, 13, \cdots, 16$) are defined as follows:

$$\begin{aligned}\mathcal{A}_{i12,\varrho+1} =& -(I - K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})\mathbb{E}\left\{\tilde{x}_{i,\varrho+1|\varrho}x_{i,\varrho+1}^T C_{i,\varrho+1}^T\tilde{\Theta}_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T, \\ \mathcal{A}_{i13,\varrho+1} =& -(I - K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})\mathbb{E}\left\{\tilde{x}_{i,\varrho+1|\varrho}\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T, \\ \mathcal{A}_{i14,\varrho+1} =& K_{i,\varrho+1}\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T, \\ \mathcal{A}_{i15,\varrho+1} =& \bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}K_{i,\varrho+1}\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}\zeta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T, \\ \mathcal{A}_{i16,\varrho+1} =& \bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}K_{i,\varrho+1}\mathbb{E}\left\{\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\zeta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T.\end{aligned}$$

Noting $\mathbb{E}\{\tilde{\Theta}_{i,\varrho+1}\} = 0$ and $\mathbb{E}\{\nu_{i,\varrho+1}\} = 0$, it is easy to find $\mathcal{A}_{is,\varrho+1} = 0$ ($s = 12, 13, \cdots, 16$). Then, the proof of this lemma is complete. ■

Up to now, the expressions of PEC and EEC have been given in Lemmas 3 and 4, respectively. It is obvious to see from (12)-(13) that the exact values of covariance cannot be obtained due to the effect of nonlinearity, RCS, RSDs and hybrid attacks, which result in the existence of uncertain factors and unknown cross-terms. Therefore, the traditional Kalman filter is not longer applicable, and an alternative approach is to find the UBs on the PEC and EEC, which is a suboptimal method to conduct the operations. Subsequently, the following theorem gives the suboptimal UBs on the PEC and EEC based on the results of Lemmas 3 and 4, and designs the EG to minimize the obtained UB on the EEC.

Theorem 1: For the system (6)-(7) and the state estimator (8)-(9), let ϵ_h ($h = 1, 2, \cdots, 6$) be positive parameters, and let us denote $\bar{\alpha}_{i,\varrho} = \sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}$ and $\bar{I} = [I \quad I]$. If the following matrix equations:

$$\mathcal{P}_{i,\varrho+1|\varrho} = (1 + \epsilon_1 + \epsilon_2)\ell_\varrho^2 \mathrm{tr}\{\mathcal{P}_{i,\varrho|\varrho}\}I + (1 + \epsilon_1^{-1} + \epsilon_3)(E_\varrho + \tilde{I})\mathcal{P}_{i,\varrho|\varrho}(E_\varrho + \tilde{I})^T$$

$$+(1+\epsilon_2^{-1}+\epsilon_3^{-1})\bar{\alpha}_{i,\varrho}\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\mathcal{P}_{j,\varrho|\varrho}\Gamma^T+(1+\epsilon_4)\sum_{j=1}^{N}\hat{\alpha}_{ij,\varrho}\Gamma\mathcal{P}_{j,\varrho|\varrho}\Gamma^T$$

$$+(1+\epsilon_4^{-1})\sum_{j=1}^{N}\hat{\alpha}_{ij,\varrho}\Gamma\hat{x}_{j,\varrho|\varrho}\hat{x}_{j,\varrho|\varrho}^T\Gamma^T+B_{i,\varrho}Q_{i,\varrho}B_{i,\varrho}^T, \tag{14}$$

and

$$\begin{aligned}\mathcal{P}_{i,\varrho+1|\varrho+1} = {} & (1+\epsilon_5)(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})\mathcal{P}_{i,\varrho+1|\varrho}(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})^T \\ & +(1+\epsilon_5^{-1})\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}\bar{\zeta}_iK_{i,\varrho+1}K_{i,\varrho+1}^T+K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^T \\ & \times\mathrm{tr}\left\{C_{i,\varrho+1}\left[(1+\epsilon_6)\mathcal{P}_{i,\varrho+1|\varrho}+(1+\epsilon_6^{-1})\hat{x}_{i,\varrho+1|\varrho}\hat{x}_{i,\varrho+1|\varrho}^T\right]C_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T \\ & +\bar{\beta}_{1i,\varrho+1}(1-\bar{\beta}_{2i,\varrho+1})\mathrm{tr}\{R_{i,\varrho+1}\}K_{i,\varrho+1}K_{i,\varrho+1}^T,\end{aligned} \tag{15}$$

with initial values $\mathcal{P}_{i,0|0}\geq P_{i,0|0}>0$, have solutions $\mathcal{P}_{i,\varrho+1|\varrho}>0$ and $\mathcal{P}_{i,\varrho+1|\varrho+1}>0$, then it can be shown that $P_{i,\varrho+1|\varrho+1}\leq\mathcal{P}_{i,\varrho+1|\varrho+1}$. Moreover, if the EG matrix $K_{i,\varrho+1}$ is designed as

$$K_{i,\varrho+1} = (1+\epsilon_5)\mathcal{P}_{i,\varrho+1|\varrho}C_{i,\varrho+1}^T\bar{\Theta}_{i,\varrho+1}^T\Xi_{i,\varrho+1}^{-1}, \tag{16}$$

where

$$\begin{aligned}\Xi_{i,\varrho+1} = {} & (1+\epsilon_5)\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1}\mathcal{P}_{i,\varrho+1|\varrho}C_{i,\varrho+1}^T\bar{\Theta}_{i,\varrho+1}^T+\bar{\beta}_{1i,\varrho+1}(1-\bar{\beta}_{2i,\varrho+1})\mathrm{tr}\{R_{i,\varrho+1}\}I \\ & +(1+\epsilon_5^{-1})\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}\bar{\zeta}_iI+\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^T \\ & \times\mathrm{tr}\left\{C_{i,\varrho+1}\left[(1+\epsilon_6)\mathcal{P}_{i,\varrho+1|\varrho}+(1+\epsilon_6^{-1})\hat{x}_{i,\varrho+1|\varrho}\hat{x}_{i,\varrho+1|\varrho}^T\right]C_{i,\varrho+1}^T\right\}I,\end{aligned}$$

then it can be verified that $\mathrm{tr}\{\mathcal{P}_{i,\varrho+1|\varrho+1}\}$ is minimized at every time instant.

Proof: By means of the mathematical induction method and noting that the initial values satisfy $P_{i,0|0}\leq\mathcal{P}_{i,0|0}$, we assume that $P_{i,\varrho|\varrho}\leq\mathcal{P}_{i,\varrho|\varrho}$. Then, what we need to show is $P_{i,\varrho+1|\varrho+1}\leq\mathcal{P}_{i,\varrho+1|\varrho+1}$.

Firstly, according to Lemma 1, the cross-terms on (12) can be tackled as

$$\mathcal{A}_{i1,\varrho}+\mathcal{A}_{i1,\varrho}^T \leq \epsilon_1\mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\mathfrak{Y}_{i,\varrho}^T\right\}+\epsilon_1^{-1}\mathbb{E}\left\{\mathfrak{C}_{i,\varrho}\mathfrak{C}_{i,\varrho}^T\right\}, \tag{17}$$

$$\mathcal{A}_{i2,\varrho}+\mathcal{A}_{i2,\varrho}^T \leq \epsilon_2\mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\mathfrak{Y}_{i,\varrho}^T\right\}+\epsilon_2^{-1}\mathbb{E}\left\{\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\}, \tag{18}$$

$$\mathcal{A}_{i3,\varrho}+\mathcal{A}_{i3,\varrho}^T \leq \epsilon_3\mathbb{E}\left\{\mathfrak{C}_{i,\varrho}\mathfrak{C}_{i,\varrho}^T\right\}+\epsilon_3^{-1}\mathbb{E}\left\{\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\}, \tag{19}$$

where ϵ_1, ϵ_2 and ϵ_3 are given positive scalars.

Substituting (17)-(19) into (12) yields

$$\begin{aligned}P_{i,\varrho+1|\varrho} \leq {} & (1+\epsilon_1+\epsilon_2)\mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\mathfrak{Y}_{i,\varrho}^T\right\}+(1+\epsilon_1^{-1}+\epsilon_3)\mathbb{E}\left\{\mathfrak{C}_{i,\varrho}\mathfrak{C}_{i,\varrho}^T\right\} \\ & +(1+\epsilon_2^{-1}+\epsilon_3^{-1})\mathbb{E}\left\{\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]\left[\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\} \\ & +\mathbb{E}\left\{\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]\left[\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]^T\right\}+B_{i,\varrho}Q_{i,\varrho}B_{i,\varrho}^T.\end{aligned} \tag{20}$$

Considering the nonlinear constraint (4), it can be seen that

$$\mathbb{E}\left\{\mathfrak{Y}_{i,\varrho}\mathfrak{Y}_{i,\varrho}^T\right\} \leq \mathbb{E}\left\{\|\mathfrak{Y}_{i,\varrho}\|^2\right\}I$$

$$\begin{aligned} &\leq \ell_\varrho^2 \mathbb{E}\left\{\left\|\tilde{x}_{i,\varrho|\varrho}\right\|^2\right\} I \\ &= \ell_\varrho^2 \mathrm{tr}\left\{P_{i,\varrho|\varrho}\right\} I. \end{aligned} \tag{21}$$

By using Lemma 1 again, the unknown term of (20) can be upper bounded as

$$\begin{aligned} &\mathbb{E}\left\{\left[\sum_{j=1}^N \bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]\left[\sum_{j=1}^N \bar{\alpha}_{ij,\varrho}\Gamma\tilde{x}_{j,\varrho|\varrho}\right]^T\right\} \\ &\leq \frac{1}{2}\sum_{j=1}^N\sum_{l=1}^N \bar{\alpha}_{ij,\varrho}\bar{\alpha}_{il,\varrho}\Gamma\mathbb{E}\left\{\tilde{x}_{j,\varrho|\varrho}\tilde{x}_{j,\varrho|\varrho}^T + \tilde{x}_{l,\varrho|\varrho}\tilde{x}_{l,\varrho|\varrho}^T\right\}\Gamma^T \\ &= \frac{1}{2}\sum_{l=1}^N \bar{\alpha}_{il,\varrho}\sum_{j=1}^N \bar{\alpha}_{ij,\varrho}\Gamma P_{j,\varrho|\varrho}\Gamma^T + \frac{1}{2}\sum_{j=1}^N \bar{\alpha}_{ij,\varrho}\sum_{l=1}^N \bar{\alpha}_{il,\varrho}\Gamma P_{l,\varrho|\varrho}\Gamma^T \\ &= \bar{\alpha}_{i,\varrho}\sum_{j=1}^N \bar{\alpha}_{ij,\varrho}\Gamma P_{j,\varrho|\varrho}\Gamma^T. \end{aligned} \tag{22}$$

Noting that $\mathbb{E}\left\{\tilde{\alpha}_{ij,\varrho}\tilde{\alpha}_{il,\varrho}\right\} = \mathbb{E}\left\{(\alpha_{ij,\varrho} - \bar{\alpha}_{ij,\varrho})(\alpha_{il,\varrho} - \bar{\alpha}_{il,\varrho})\right\} = 0$ $(j \neq l)$ and $\mathbb{E}\big\{\tilde{\alpha}_{ij,\varrho}^2\big\} = \hat{\alpha}_{ij,\varrho}$, so we can obtain

$$\begin{aligned} &\mathbb{E}\left\{\left[\sum_{j=1}^N \tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]\left[\sum_{j=1}^N \tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]^T\right\} \\ &= \mathbb{E}\left\{\sum_{j=1}^N \tilde{\alpha}_{ij,\varrho}^2\Gamma x_{j,\varrho}x_{j,\varrho}^T\Gamma^T\right\} \\ &= \sum_{j=1}^N \hat{\alpha}_{ij,\varrho}\Gamma\mathbb{E}\left\{x_{j,\varrho}x_{j,\varrho}^T\right\}\Gamma^T. \end{aligned} \tag{23}$$

Furthermore, with the help of Lemma 1, we have

$$\begin{aligned} \mathbb{E}\left\{x_{j,\varrho}x_{j,\varrho}^T\right\} &= \mathbb{E}\left\{(\tilde{x}_{j,\varrho|\varrho} + \hat{x}_{j,\varrho|\varrho})(\tilde{x}_{j,\varrho|\varrho} + \hat{x}_{j,\varrho|\varrho})^T\right\} \\ &\leq (1+\epsilon_4)P_{j,\varrho|\varrho} + (1+\epsilon_4^{-1})\hat{x}_{j,\varrho|\varrho}\hat{x}_{j,\varrho|\varrho}^T, \end{aligned} \tag{24}$$

where ϵ_4 is a positive scalar. Then, it is observed from (23) and (24) that

$$\begin{aligned} &\mathbb{E}\left\{\left[\sum_{j=1}^N \tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]\left[\sum_{j=1}^N \tilde{\alpha}_{ij,\varrho}\Gamma x_{j,\varrho}\right]^T\right\} \\ &\leq (1+\epsilon_4)\sum_{j=1}^N \hat{\alpha}_{ij,\varrho}\Gamma P_{j,\varrho|\varrho}\Gamma^T + (1+\epsilon_4^{-1})\sum_{j=1}^N \hat{\alpha}_{ij,\varrho}\Gamma\hat{x}_{j,\varrho|\varrho}\hat{x}_{j,\varrho|\varrho}^T\Gamma^T. \end{aligned} \tag{25}$$

Substituting (21), (22), (25) into (20) results in

$$\begin{aligned} P_{i,\varrho+1|\varrho} \leq\ & (1+\epsilon_1+\epsilon_2)\ell_\varrho^2\mathrm{tr}\{P_{i,\varrho|\varrho}\}I + (1+\epsilon_1^{-1}+\epsilon_3)(E_\varrho+\tilde{I})P_{i,\varrho|\varrho}(E_\varrho+\tilde{I})^T \\ & +(1+\epsilon_2^{-1}+\epsilon_3^{-1})\bar{\alpha}_{i,\varrho}\sum_{j=1}^N \bar{\alpha}_{ij,\varrho}\Gamma P_{j,\varrho|\varrho}\Gamma^T + (1+\epsilon_4)\sum_{j=1}^N \hat{\alpha}_{ij,\varrho}\Gamma P_{j,\varrho|\varrho}\Gamma^T \\ & +(1+\epsilon_4^{-1})\sum_{j=1}^N \hat{\alpha}_{ij,\varrho}\Gamma\hat{x}_{j,\varrho|\varrho}\hat{x}_{j,\varrho|\varrho}^T\Gamma^T + B_{i,\varrho}Q_{i,\varrho}B_{i,\varrho}^T, \end{aligned} \tag{26}$$

which implies $P_{i,\varrho+1|\varrho} \leq \mathcal{P}_{i,\varrho+1|\varrho}$.

Next, we are ready to show that $P_{i,\varrho+1|\varrho+1} \le \mathcal{P}_{i,\varrho+1|\varrho+1}$ is true. Again, from Lemma 1, we obtain

$$\begin{aligned}\mathcal{A}_{i11,\varrho+1}+\mathcal{A}_{i11,\varrho+1}^T \le\ & \epsilon_5(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})\mathbb{E}\left\{\tilde{x}_{i,\varrho+1|\varrho}\tilde{x}_{i,\varrho+1|\varrho}^T\right\}(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})^T\\ &+\epsilon_5^{-1}K_{i,\varrho+1}\mathbb{E}\left\{\beta_{1i,\varrho+1}^2\beta_{2i,\varrho+1}^2\zeta_{i,\varrho+1}\zeta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T\\ =\ & \epsilon_5(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})P_{i,\varrho+1|\varrho}(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})^T\\ &+\epsilon_5^{-1}\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}K_{i,\varrho+1}\mathbb{E}\left\{\zeta_{i,\varrho+1}\zeta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T,\end{aligned}\tag{27}$$

where ϵ_5 is a positive scalar. Then, taking (13) and (27) into consideration, one has

$$\begin{aligned}P_{i,\varrho+1|\varrho+1} \le\ & (1+\epsilon_5)(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})P_{i,\varrho+1|\varrho}(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})^T\\ &+K_{i,\varrho+1}\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}x_{i,\varrho+1}^TC_{i,\varrho+1}^T\tilde{\Theta}_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T\\ &+K_{i,\varrho+1}\mathbb{E}\left\{\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T\\ &+(1+\epsilon_5^{-1})\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}K_{i,\varrho+1}\mathbb{E}\left\{\zeta_{i,\varrho+1}\zeta_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T.\end{aligned}\tag{28}$$

The unknown term of (28) can be upper bounded as follows:

$$\begin{aligned}&\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}x_{i,\varrho+1}^TC_{i,\varrho+1}^T\tilde{\Theta}_{i,\varrho+1}^T\right\}\\ \le\ & \mathbb{E}\left\{\mathrm{tr}\{C_{i,\varrho+1}x_{i,\varrho+1}x_{i,\varrho+1}^TC_{i,\varrho+1}^T\}\tilde{\Theta}_{i,\varrho+1}\tilde{\Theta}_{i,\varrho+1}^T\right\}\\ =\ & \mathbb{E}\left\{\mathrm{tr}\{C_{i,\varrho+1}x_{i,\varrho+1}x_{i,\varrho+1}^TC_{i,\varrho+1}^T\}\right\}\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}\tilde{\Theta}_{i,\varrho+1}^T\right\}\\ =\ & \mathrm{tr}\left\{C_{i,\varrho+1}\mathbb{E}\{x_{i,\varrho+1}x_{i,\varrho+1}^T\}C_{i,\varrho+1}^T\right\}\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^T,\end{aligned}\tag{29}$$

where $\mathbb{E}\{\tilde{\Theta}_{i,\varrho+1}\tilde{\Theta}_{i,\varrho+1}^T\}=\mathbb{E}\{(\Theta_{i,\varrho+1}-\bar{\Theta}_{i,\varrho+1})(\Theta_{i,\varrho+1}-\bar{\Theta}_{i,\varrho+1})^T\}=\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^T$ and $\bar{I}=[I\quad I]$.

Similarly, we have the following inequality:

$$\begin{aligned}&\mathbb{E}\left\{\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T\right\}\\ \le\ & \mathbb{E}\left\{\mathrm{tr}\left\{\nu_{i,\varrho+1}\nu_{i,\varrho+1}^T\right\}\Theta_{i,\varrho+1}\Theta_{i,\varrho+1}^T\right\}\\ =\ & \mathbb{E}\left\{\mathrm{tr}\{\nu_{i,\varrho+1}\nu_{i,\varrho+1}^T\}\right\}\mathbb{E}\left\{\Theta_{i,\varrho+1}\Theta_{i,\varrho+1}^T\right\}\\ =\ & \mathrm{tr}\left\{R_{i,\varrho+1}\right\}\mathbb{E}\left\{\Theta_{i,\varrho+1}\Theta_{i,\varrho+1}^T\right\}\\ =\ & \bar{\beta}_{1i,\varrho+1}(1-\bar{\beta}_{2i,\varrho+1})\mathrm{tr}\left\{R_{i,\varrho+1}\right\}I,\end{aligned}\tag{30}$$

where $\mathbb{E}\{\Theta_{i,\varrho+1}\Theta_{i,\varrho+1}^T\}=\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^T+\bar{\Theta}_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}^T=\bar{\Theta}_{i,\varrho+1}\bar{I}^T$.

Using Lemma 1 again, we have

$$\begin{aligned}\mathbb{E}\left\{x_{i,\varrho+1}x_{i,\varrho+1}^T\right\} =\ & \mathbb{E}\left\{(\tilde{x}_{i,\varrho+1|\varrho}+\hat{x}_{i,\varrho+1|\varrho})(\tilde{x}_{i,\varrho+1|\varrho}+\hat{x}_{i,\varrho+1|\varrho})^T\right\}\\ \le\ & (1+\epsilon_6)P_{i,\varrho+1|\varrho}+(1+\epsilon_6^{-1})\hat{x}_{i,\varrho+1|\varrho}\hat{x}_{i,\varrho+1|\varrho}^T,\end{aligned}\tag{31}$$

where ϵ_6 is a positive scalar. Applying (31) to (29) yields

$$\begin{aligned}&\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}x_{i,\varrho+1}^TC_{i,\varrho+1}^T\tilde{\Theta}_{i,\varrho+1}^T\right\}\\ \le\ & \mathrm{tr}\left\{C_{i,\varrho+1}\left[(1+\epsilon_6)\mathcal{P}_{i,\varrho+1|\varrho}+(1+\epsilon_6^{-1})\hat{x}_{i,\varrho+1|\varrho}\hat{x}_{i,\varrho+1|\varrho}^T\right]C_{i,\varrho+1}^T\right\}\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^T.\end{aligned}\tag{32}$$

Moreover, noting the fact that $\zeta_{i,\varrho}^T\zeta_{i,\varrho}\le\bar{\zeta}_i$, one obtains

$$\mathbb{E}\left\{\zeta_{i,\varrho+1}\zeta_{i,\varrho+1}^T\right\} \le \mathbb{E}\left\{\zeta_{i,\varrho+1}^T\zeta_{i,\varrho+1}\right\}I\le\bar{\zeta}_iI.\tag{33}$$

Subsequently, substituting (30), (32) and (33) into (28) leads to

$$P_{i,\varrho+1|\varrho+1} \le (1+\epsilon_5)(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})P_{i,\varrho+1|\varrho}(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})^T$$

$$+(1+\epsilon_5^{-1})\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}\bar{\zeta}_i K_{i,\varrho+1}K_{i,\varrho+1}^T + K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^T$$
$$\times \mathrm{tr}\left\{C_{i,\varrho+1}\left[(1+\epsilon_6)P_{i,\varrho+1|\varrho} + (1+\epsilon_6^{-1})\hat{x}_{i,\varrho+1|\varrho}\hat{x}_{i,\varrho+1|\varrho}^T\right]C_{i,\varrho+1}^T\right\}K_{i,\varrho+1}^T$$
$$+\bar{\beta}_{1i,\varrho+1}(1-\bar{\beta}_{2i,\varrho+1})\mathrm{tr}\{R_{i,\varrho+1}\}K_{i,\varrho+1}K_{i,\varrho+1}^T. \tag{34}$$

According to the initial values $\mathcal{P}_{i,0|0} \geq P_{i,0|0} > 0$, $P_{i,\varrho+1|\varrho} \leq \mathcal{P}_{i,\varrho+1|\varrho}$ and (34), we conclude that $P_{i,\varrho+1|\varrho+1} \leq \mathcal{P}_{i,\varrho+1|\varrho+1}$ by the mathematical induction method.

Finally, $\mathrm{tr}\{\mathcal{P}_{i,\varrho+1|\varrho+1}\}$ can be minimized by the following process. We calculate $\frac{\partial \mathrm{tr}\{\mathcal{P}_{i,\varrho+1|\varrho+1}\}}{\partial K_{i,\varrho+1}}$ and set it to be zero resulting in

$$\begin{aligned}\frac{\partial \mathrm{tr}\left\{\mathcal{P}_{i,\varrho+1|\varrho+1}\right\}}{\partial K_{i,\varrho+1}} =\ & 2(1+\epsilon_5)K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1}\mathcal{P}_{i,\varrho+1|\varrho}C_{i,\varrho+1}^T\bar{\Theta}_{i,\varrho+1}^T\\ &+2\bar{\beta}_{1i,\varrho+1}(1-\bar{\beta}_{2i,\varrho+1})\mathrm{tr}\{R_{i,\varrho+1}\}K_{i,\varrho+1} + 2(1+\epsilon_5^{-1})\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}\bar{\zeta}_i K_{i,\varrho+1}\\ &-2(1+\epsilon_5)\mathcal{P}_{i,\varrho+1|\varrho}C_{i,\varrho+1}^T\bar{\Theta}_{i,\varrho+1}^T + 2K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^T\\ &\times\mathrm{tr}\left\{C_{i,\varrho+1}\left[(1+\epsilon_6)\mathcal{P}_{i,\varrho+1|\varrho} + (1+\epsilon_6^{-1})\hat{x}_{i,\varrho+1|\varrho}\hat{x}_{i,\varrho+1|\varrho}^T\right]C_{i,\varrho+1}^T\right\}\\ =\ & 0,\end{aligned}$$

from which expression (16) for the EG matrix $K_{i,\varrho+1}$ is derived after some manipulations. The proof of this theorem is now complete. ■

Remark 4: In general, the existing research methods for the SE issues of CNs mainly include linear matrix inequality method for time-invariant CNs and recursive matrix equation method for time-varying CNs. In this paper, based on the recursive matrix equation method, a novel hybrid-attack-resistant recursive distributed SE scheme has been proposed for time-varying NCNs with RCS, RSDs and hybrid attacks. Compared with the linear matrix inequality method, the newly proposed recursive scheme is more applicable for online computations. Moreover, the recursive SE issues can be divided into centralized and distributed structures. For the distributed structure considered in this paper, its distinguishing features are low computation burden, high reliability and strong robustness. On the other hand, the hybrid attacks model that includes DoS attack and deception attack in a unified framework has been, for the first time, considered for time-varying NCNs with RCS and RSDs to reflect the complexity of the networked environment.

Till now, the hybrid-attack-resistant recursive distributed SE problem has been solved. Accordingly, the developed hybrid-attack-resistant recursive distributed SE algorithm can be summarized in the following Algorithm 1.

Algorithm 1 Hybrid-Attack-Resistant Recursive Distributed SE Algorithm.

Step 1: Set $\varrho = 0$ and select the initial values and related parameters.
Step 2: Update the one-step prediction $\hat{x}_{i,\varrho+1|\varrho}$ by (8).
Step 3: Compute the UB on the PEC $\mathcal{P}_{i,\varrho+1|\varrho}$ by (14).
Step 4: Calculate the EG matrix $K_{i,\varrho+1}$ for the ith node by (16).
Step 5: Compute the SE $\hat{x}_{i,\varrho+1|\varrho+1}$ by (9).
Step 6: Compute the UB on the EEC $\mathcal{P}_{i,\varrho+1|\varrho+1}$ by (15).
Step 7: Let $\varrho = \varrho + 1$, and go back to *Step* 2.

IV. Boundedness Analysis

A sufficient condition is given to guarantee that the SE error is EB in the MSS in this section. Before further analysis, the following definition, lemma and assumption are given.

Definition 1: [45] The random process is said to be EB in the MSS, if there are real constants σ, $\iota > 0$ and $0 < \wp < 1$ such that

$$\mathbb{E}\{\ \|\psi_\varrho\|^2\ \} \leq \sigma\|\psi_0\|^2\wp^\varrho + \iota$$

holds for every $\varrho > 0$.

Lemma 5: [45] Suppose that there are a random process $V_\varrho(\psi_\varrho)$, real constants $\mu_{\min}$, $\mu_{\max}$, $\upsilon > 0$ and $0 < \rho \leq 1$ such that

$$\mu_{\min} \|\psi_\varrho\|^2 \leq V_\varrho(\psi_\varrho) \leq \mu_{\max} \|\psi_\varrho\|^2 ,$$

and

$$\mathbb{E}\left\{V_\varrho(\psi_\varrho)|\psi_{\varrho-1}\right\} \leq (1-\rho)V_{\varrho-1}(\psi_{\varrho-1}) + \upsilon.$$

Then, ψ_ϱ satisfies

$$\mathbb{E}\{ \|\psi_\varrho\|^2 \} \leq \frac{\mu_{\max}}{\mu_{\min}}\mathbb{E}\{ \|\psi_0\|^2 \}(1-\rho)^\varrho + \frac{\upsilon}{\mu_{\min}}\sum_{i=1}^{\varrho-1}(1-\rho)^i,$$

and, consequently, it is EB in the MSS.

Assumption 1: There are positive constants $\underline{b}_i$, $\bar{b}_i$, $\underline{c}_i$, $\bar{c}_i$, $\underline{q}_i$, $\bar{q}_i$, $\bar{r}_i$, $\bar{\chi}$, $\bar{\sigma}_i$, τ, $\underline{\beta}_{1i}$, $\bar{\beta}_{1i}$, $\underline{\beta}_{2i}$, $\bar{\beta}_{2i}$, $\underline{\theta}_i$, $\bar{\theta}_i$, $\underline{\ell}$ and $\bar{\ell}$ such that

$$\begin{aligned}
&\underline{b}_i I \leq B_{i,\varrho}B_{i,\varrho}^T \leq \bar{b}_i I,\ \underline{c}_i I \leq C_{i,\varrho}C_{i,\varrho}^T \leq \bar{c}_i I,\ \underline{q}_i I \leq Q_{i,\varrho} \leq \bar{q}_i I,\\
&R_{i,\varrho} \leq \bar{r}_i I,\ \hat{x}_{i,\varrho+1|\varrho}\hat{x}_{i,\varrho+1|\varrho}^T \leq \bar{\chi} I,\ \mathcal{P}_{i,\varrho+1|\varrho} \leq \bar{\sigma}_i I,\ \Gamma\Gamma^T \leq \tau I,\\
&\underline{\beta}_{1i} \leq \bar{\beta}_{1i,\varrho} \leq \bar{\beta}_{1i},\ \underline{\beta}_{2i} \leq \bar{\beta}_{2i,\varrho} \leq \bar{\beta}_{2i},\ \underline{\theta}_i \leq \bar{\theta}_{i,\varrho} \leq \bar{\theta}_i,\ \underline{\ell} \leq \ell_\varrho \leq \bar{\ell}
\end{aligned}$$

for any i and ϱ.

Theorem 2: Consider the NCNs (6)-(7) subject to RSDs and hybrid attacks. Under Assumption 1, if there are positive scalars ϵ_1, ϵ_2, ϵ_3 and ϵ_5 such that

$$N < (1+\epsilon_2^{-1}+\epsilon_3^{-1})\left[\frac{1}{4}(1+\epsilon_5) - (1+\epsilon_1^{-1}+\epsilon_3)^{-1}\right] \tag{35}$$

for any i and ϱ, then the SE error $\tilde{x}_{i,\varrho|\varrho}$ is EB in the MSS.

Proof: For the sake of simplicity of expressions, let us define $\tilde{x}_{\varrho|\varrho} = [\tilde{x}_{1,\varrho|\varrho}^T\ \tilde{x}_{2,\varrho|\varrho}^T\ \cdots\ \tilde{x}_{N,\varrho|\varrho}^T]^T$ and construct the Lyapunov function as

$$V_\varrho(\tilde{x}_{\varrho|\varrho}) = \sum_{i=1}^{N} \tilde{x}_{i,\varrho|\varrho}^T \mathcal{P}_{i,\varrho|\varrho}^{-1}\tilde{x}_{i,\varrho|\varrho}. \tag{36}$$

Based on Lemma 5, we need to do the following processing. According to (10), (11) and (36), we get

$$\begin{aligned}
&\mathbb{E}\left\{V_{\varrho+1}(\tilde{x}_{\varrho+1|\varrho+1})|\tilde{x}_{\varrho|\varrho}\right\}\\
=\ &\mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{Y}_{i,\varrho}^T\Phi_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{Y}_{i,\varrho}\right\}\\
&+\mathbb{E}\left\{\sum_{i=1}^{N}x_{i,\varrho+1}^T C_{i,\varrho+1}^T\tilde{\Theta}_{i,\varrho+1}^T K_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}\right\}\\
&+\mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{C}_{i,\varrho}^T\Phi_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{C}_{i,\varrho}\right\}\\
&+\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\tilde{\alpha}_{ij,\varrho}\tilde{\alpha}_{id,\varrho}x_{j,\varrho}^T\Gamma^T\Phi_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma x_{d,\varrho}\right\}
\end{aligned}$$

$$
\begin{aligned}
&+\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\bar{\alpha}_{ij,\varrho}\bar{\alpha}_{id,\varrho}\tilde{x}_{j,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{d,\varrho|\varrho}\right\}\\
&+\mathbb{E}\left\{\sum_{i=1}^{N}\nu_{i,\varrho+1}^{T}\Theta_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\right\}\\
&+\mathbb{E}\left\{\sum_{i=1}^{N}\beta_{1i,\varrho+1}^{2}\beta_{2i,\varrho+1}^{2}\zeta_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\}\\
&+\mathbb{E}\left\{\sum_{i=1}^{N}\varpi_{i,\varrho}^{T}B_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}B_{i,\varrho}\varpi_{i,\varrho}\right\}\\
&+\sum_{s=1}^{6}(\mathcal{B}_{is,\varrho}+\mathcal{B}_{is,\varrho}^{T}),
\end{aligned}
\tag{37}
$$

where

$$
\begin{aligned}
\Phi_{i,\varrho+1} &= I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1},\\
\mathcal{B}_{i1,\varrho} &= \mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{Y}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{C}_{i,\varrho}\right\},\\
\mathcal{B}_{i2,\varrho} &= \mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\mathfrak{Y}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{j,\varrho|\varrho}\right\},\\
\mathcal{B}_{i3,\varrho} &= -\mathbb{E}\left\{\sum_{i=1}^{N}\beta_{1i,\varrho+1}\beta_{2i,\varrho+1}\mathfrak{Y}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\},\\
\mathcal{B}_{i4,\varrho} &= \mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\mathfrak{C}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{j,\varrho|\varrho}^{T}\right\},\\
\mathcal{B}_{i5,\varrho} &= -\mathbb{E}\left\{\sum_{i=1}^{N}\beta_{1i,\varrho+1}\beta_{2i,\varrho+1}\mathfrak{C}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\},\\
\mathcal{B}_{i6,\varrho} &= -\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{\alpha}_{ij,\varrho}\beta_{1i,\varrho+1}\beta_{2i,\varrho+1}\tilde{x}_{j,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\}.
\end{aligned}
$$

According to Lemma 1, the cross terms in (37) are upper bounded as follows:

$$
\begin{aligned}
\mathcal{B}_{i1,\varrho}+\mathcal{B}_{i1,\varrho}^{T} \leq\ &\mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{Y}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{Y}_{i,\varrho}\right\}\\
&+\mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{C}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{C}_{i,\varrho}\right\},
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
\mathcal{B}_{i2,\varrho}+\mathcal{B}_{i2,\varrho}^{T} \leq\ &\mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{Y}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{Y}_{i,\varrho}\right\}\\
&+\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\bar{\alpha}_{ij,\varrho}\bar{\alpha}_{id,\varrho}\tilde{x}_{j,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{d,\varrho|\varrho}\right\},
\end{aligned}
\tag{39}
$$

$$
\mathcal{B}_{i3,\varrho}+\mathcal{B}_{i3,\varrho}^{T} \leq \mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{Y}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{Y}_{i,\varrho}\right\}
$$

$$+\mathbb{E}\left\{\sum_{i=1}^{N}\beta_{1i,\varrho+1}^{2}\beta_{2i,\varrho+1}^{2}\zeta_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\}, \tag{40}$$

$$\begin{aligned}\mathcal{B}_{i4,\varrho}+\mathcal{B}_{i4,\varrho}^{T} \le\ & \mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{C}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{C}_{i,\varrho}\right\}\\ &+\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\bar{\alpha}_{ij,\varrho}\bar{\alpha}_{id,\varrho}\tilde{x}_{j,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{d,\varrho|\varrho}\right\},\end{aligned} \tag{41}$$

$$\begin{aligned}\mathcal{B}_{i5,\varrho}+\mathcal{B}_{i5,\varrho}^{T} \le\ & \mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{C}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{C}_{i,\varrho}\right\}\\ &+\mathbb{E}\left\{\sum_{i=1}^{N}\beta_{1i,\varrho+1}^{2}\beta_{2i,\varrho+1}^{2}\zeta_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\},\end{aligned} \tag{42}$$

$$\begin{aligned}\mathcal{B}_{i6,\varrho}+\mathcal{B}_{i6,\varrho}^{T} \le\ & \mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\bar{\alpha}_{ij,\varrho}\bar{\alpha}_{id,\varrho}\tilde{x}_{j,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{d,\varrho|\varrho}\right\}\\ &+\mathbb{E}\left\{\sum_{i=1}^{N}\beta_{1i,\varrho+1}^{2}\beta_{2i,\varrho+1}^{2}\zeta_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\}.\end{aligned} \tag{43}$$

Substituting (38)-(43) into (37) yields

$$\begin{aligned}&\mathbb{E}\left\{V_{\varrho+1}(\tilde{x}_{\varrho+1|\varrho+1})|\tilde{x}_{\varrho|\varrho}\right\}\\ \le\ & 4\mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{Y}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{Y}_{i,\varrho}\right\}\\ &+\mathbb{E}\left\{\sum_{i=1}^{N}x_{i,\varrho+1}^{T}C_{i,\varrho+1}^{T}\tilde{\Theta}_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}\right\}\\ &+4\mathbb{E}\left\{\sum_{i=1}^{N}\mathfrak{C}_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{C}_{i,\varrho}\right\}\\ &+\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\tilde{\alpha}_{ij,\varrho}\tilde{\alpha}_{id,\varrho}x_{j,\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma x_{d,\varrho}\right\}\\ &+4\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\bar{\alpha}_{ij,\varrho}\bar{\alpha}_{id,\varrho}\tilde{x}_{j,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{d,\varrho|\varrho}\right\}\\ &+\mathbb{E}\left\{\sum_{i=1}^{N}\nu_{i,\varrho+1}^{T}\Theta_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\right\}\\ &+4\mathbb{E}\left\{\sum_{i=1}^{N}\beta_{1i,\varrho+1}^{2}\beta_{2i,\varrho+1}^{2}\zeta_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\}\\ &+\mathbb{E}\left\{\sum_{i=1}^{N}\varpi_{i,\varrho}^{T}B_{i,\varrho}^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}B_{i,\varrho}\varpi_{i,\varrho}\right\}.\end{aligned} \tag{44}$$

From (14) and (15), we can easily obtain the following inequalities:

$$\begin{aligned}\mathcal{P}_{i,\varrho+1|\varrho+1} \ge\ & (1+\epsilon_5)(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})\mathcal{P}_{i,\varrho+1|\varrho}(I-K_{i,\varrho+1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1})^{T}\\ =\ & (1+\epsilon_5)\Phi_{i,\varrho+1}\mathcal{P}_{i,\varrho+1|\varrho}\Phi_{i,\varrho+1}^{T},\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{i,\varrho+1|\varrho+1} &\geq (1+\epsilon_5^{-1})\bar{\beta}_{1i,\varrho+1}\bar{\beta}_{2i,\varrho+1}\bar{\zeta}_i K_{i,\varrho+1}K_{i,\varrho+1}^T, \\
\mathcal{P}_{i,\varrho+1|\varrho} &\geq (1+\epsilon_1^{-1}+\epsilon_3)(E_\varrho+\tilde{I})\mathcal{P}_{i,\varrho|\varrho}(E_\varrho+\tilde{I})^T, \\
\mathcal{P}_{i,\varrho+1|\varrho} &\geq (1+\epsilon_2^{-1}+\epsilon_3^{-1})\bar{\alpha}_{i,\varrho}\bar{\alpha}_{ij,\varrho}\Gamma\mathcal{P}_{j,\varrho|\varrho}\Gamma^T, \ (j=1,2,\cdots,N), \\
\mathcal{P}_{i,\varrho+1|\varrho} &\geq (1+\epsilon_1+\epsilon_2)\ell_\varrho^2 \mathrm{tr}\{\mathcal{P}_{i,\varrho|\varrho}\}I, \\
\mathcal{P}_{i,\varrho+1|\varrho} &\geq B_{i,\varrho}Q_{i,\varrho}B_{i,\varrho}^T.
\end{aligned} \tag{45}$$

Then, according to Lemma 2 and Assumption 1, it follows from (45) that

$$\begin{aligned}
\Phi_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1} &\leq (1+\epsilon_5)^{-1}\mathcal{P}_{i,\varrho+1|\varrho}^{-1}, \\
(E_\varrho+\tilde{I})^T\mathcal{P}_{i,\varrho+1|\varrho}^{-1}(E_\varrho+\tilde{I}) &\leq (1+\epsilon_1^{-1}+\epsilon_3)^{-1}\mathcal{P}_{i,\varrho|\varrho}^{-1}, \\
\Gamma^T\mathcal{P}_{i,\varrho+1|\varrho}^{-1}\Gamma &\leq (1+\epsilon_2^{-1}+\epsilon_3^{-1})^{-1}\bar{\alpha}_{i,\varrho}^{-1}\bar{\alpha}_{ij,\varrho}^{-1}\mathcal{P}_{j,\varrho|\varrho}^{-1}, \ (j=1,2,\cdots,N), \\
\mathrm{tr}\{\mathcal{P}_{i,\varrho|\varrho}\}I &\leq \frac{\bar{\sigma}_i}{(1+\epsilon_1+\epsilon_2)\underline{\ell}^2}I, \\
\mathcal{P}_{i,\varrho+1|\varrho}^{-1} &\leq \frac{1}{\underline{q}_i\underline{b}_i}I.
\end{aligned} \tag{46}$$

From (46), we get

$$\begin{aligned}
&\mathbb{E}\left\{\sum_{i=1}^N \mathfrak{C}_{i,\varrho}^T\Phi_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{C}_{i,\varrho}\right\} \\
\leq\ &\mathbb{E}\left\{\sum_{i=1}^N (1+\epsilon_5)^{-1}\tilde{x}_{i,\varrho|\varrho}^T(E_\varrho+\tilde{I})^T\mathcal{P}_{i,\varrho+1|\varrho}^{-1}(E_\varrho+\tilde{I})\tilde{x}_{i,\varrho|\varrho}\right\} \\
\leq\ &\mathbb{E}\left\{\sum_{i=1}^N (1+\epsilon_5)^{-1}(1+\epsilon_1^{-1}+\epsilon_3)^{-1}\tilde{x}_{i,\varrho|\varrho}^T\mathcal{P}_{i,\varrho|\varrho}^{-1}\tilde{x}_{i,\varrho|\varrho}\right\} \\
=\ &(1+\epsilon_5)^{-1}(1+\epsilon_1^{-1}+\epsilon_3)^{-1}\mathbb{E}\left\{V_\varrho(\tilde{x}_{\varrho|\varrho})\right\}.
\end{aligned}$$

According to (46) and Assumption 1, one has

$$\begin{aligned}
&\mathbb{E}\left\{\sum_{i=1}^N \mathfrak{Y}_{i,\varrho}^T\Phi_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\mathfrak{Y}_{i,\varrho}\right\} \\
\leq\ &\mathbb{E}\left\{\sum_{i=1}^N (1+\epsilon_5)^{-1}\frac{1}{\underline{q}_i\underline{b}_i}\mathfrak{Y}_{i,\varrho}^T\mathfrak{Y}_{i,\varrho}\right\} \\
\leq\ &(1+\epsilon_5)^{-1}\bar{\ell}^2\mathrm{tr}\left\{\sum_{i=1}^N \frac{1}{\underline{q}_i\underline{b}_i}\mathcal{P}_{i,\varrho|\varrho}\right\} \\
\leq\ &\frac{2n\bar{\ell}^2}{(1+\epsilon_1+\epsilon_2)(1+\epsilon_5)\underline{\ell}^2}\sum_{i=1}^N \frac{\bar{\sigma}_i}{\underline{q}_i\underline{b}_i} \\
\triangleq\ &\xi_1.
\end{aligned}$$

Similarly, in light of Assumption 1, from (46), we have

$$\begin{aligned}
&\mathbb{E}\left\{\sum_{i=1}^N \varpi_{i,\varrho}^T B_{i,\varrho}^T\Phi_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}B_{i,\varrho}\varpi_{i,\varrho}\right\} \\
\leq\ &\mathbb{E}\left\{\sum_{i=1}^N (1+\epsilon_5)^{-1}\frac{1}{\underline{q}_i\underline{b}_i}\varpi_{i,\varrho}^T B_{i,\varrho}^T B_{i,\varrho}\varpi_{i,\varrho}\right\}
\end{aligned}$$

$$
\begin{aligned}
&= \operatorname{tr}\left\{\mathbb{E}\left\{\sum_{i=1}^{N}(1+\epsilon_5)^{-1}\frac{1}{\underline{q}_i\underline{b}_i}B_{i,\varrho}\varpi_{i,\varrho}\varpi_{i,\varrho}^T B_{i,\varrho}^T\right\}\right\} \\
&= \operatorname{tr}\left\{\sum_{i=1}^{N}(1+\epsilon_5)^{-1}\frac{1}{\underline{q}_i\underline{b}_i}B_{i,\varrho}Q_{i,\varrho}B_{i,\varrho}^T\right\} \\
&\le 2n(1+\epsilon_5)^{-1}\sum_{i=1}^{N}\frac{1}{\underline{q}_i\underline{b}_i}\bar{q}_i\bar{b}_i \\
&\triangleq \xi_2.
\end{aligned}
$$

Also, it follows from (16) and (46) that

$$
\begin{aligned}
K_{i,\varrho+1}^T K_{i,\varrho+1} &= (1+\epsilon_5)^2\Xi_{i,\varrho+1}^{-1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1}\mathcal{P}_{i,\varrho+1|\varrho}\mathcal{P}_{i,\varrho+1|\varrho}C_{i,\varrho+1}^T\bar{\Theta}_{i,\varrho+1}^T\Xi_{i,\varrho+1}^{-1} \\
&\le (1+\epsilon_5)^2\bar{\sigma}_i^2\bar{c}_i\bar{\beta}_{1i}^2(1-\underline{\beta}_{2i})^2\left[\bar{\theta}_i^2+(1-\underline{\theta}_i)^2\right]\left[(1+\epsilon_5^{-1})\underline{\beta}_{1i}\underline{\beta}_{2i}\bar{\zeta}_i\right]^{-2}I \\
&\triangleq \bar{\kappa}_i I,
\end{aligned} \tag{47}
$$

and

$$
\begin{aligned}
K_{i,\varrho+1}K_{i,\varrho+1}^T &= (1+\epsilon_5)^2\mathcal{P}_{i,\varrho+1|\varrho}C_{i,\varrho+1}^T\bar{\Theta}_{i,\varrho+1}^T\Xi_{i,\varrho+1}^{-1}\Xi_{i,\varrho+1}^{-1}\bar{\Theta}_{i,\varrho+1}C_{i,\varrho+1}\mathcal{P}_{i,\varrho+1|\varrho} \\
&\ge (1+\epsilon_5)^2\underline{\beta}_{1i}^2(1-\bar{\beta}_{2i})^2\left[\underline{\theta}_i^2+(1-\bar{\theta}_i)^2\right]\underline{c}_i\underline{q}_i^2\underline{b}_i^2\Big[(1+\epsilon_5)\bar{\sigma}_i\bar{c}_i\bar{\beta}_{1i}^2(1-\underline{\beta}_{2i})^2\left[\bar{\theta}_i^2+(1-\underline{\theta}_i)^2\right] \\
&\quad +2m\bar{\beta}_{1i}(1-\underline{\beta}_{2i})\bar{r}_i+(1+\epsilon_5^{-1})\bar{\beta}_{1i}\bar{\beta}_{2i}\bar{\zeta}_i+2m\bar{c}_i\left[\bar{\beta}_{1i}(1-\underline{\beta}_{2i})-\underline{\beta}_{1i}^2(1-\bar{\beta}_{2i})^2[\underline{\theta}_i^2+(1-\bar{\theta}_i)^2]\right] \\
&\quad \times\left[(1+\epsilon_6)\bar{\sigma}_i+(1+\epsilon_6^{-1})\bar{\chi}\right]\Big]^{-2}I \\
&\triangleq \underline{\kappa}_i I.
\end{aligned} \tag{48}
$$

According to (45) and (48), we can derive

$$
\mathcal{P}_{i,\varrho+1|\varrho+1} \ge (1+\epsilon_5^{-1})\underline{\beta}_{1i}\underline{\beta}_{2i}\bar{\zeta}_i\underline{\kappa}_i I \triangleq \frac{1}{\underline{\hbar}_i}I.
$$

Thus, it is obtained that

$$
\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1} \le \underline{\hbar}_i I. \tag{49}
$$

Based on (47), (49) and Assumption 1, we have

$$
\begin{aligned}
&\mathbb{E}\left\{\sum_{i=1}^{N}\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T K_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\right\} \\
&\le \mathbb{E}\left\{\sum_{i=1}^{N}\underline{\hbar}_i\bar{\kappa}_i\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\right\} \\
&= \operatorname{tr}\left\{\mathbb{E}\left\{\sum_{i=1}^{N}\underline{\hbar}_i\bar{\kappa}_i\Theta_{i,\varrho+1}\nu_{i,\varrho+1}\nu_{i,\varrho+1}^T\Theta_{i,\varrho+1}^T\right\}\right\} \\
&\le \sum_{i=1}^{N}\underline{\hbar}_i\bar{\kappa}_i m\bar{\beta}_{1i}^2(1-\underline{\beta}_{2i})^2\left[\bar{\theta}_i^2+(1-\underline{\theta}_i)^2\right]\bar{r}_i \\
&\triangleq \xi_3.
\end{aligned}
$$

Similarly, by utilizing Lemma 1, in view of (46), we get

$$
\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\bar{\alpha}_{ij,\varrho}\bar{\alpha}_{id,\varrho}\tilde{x}_{j,\varrho|\varrho}^T\Gamma^T\Phi_{i,\varrho+1}^T\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{d,\varrho|\varrho}\right\}
$$

$$
\begin{aligned}
&\le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\bar{\alpha}_{ij,\varrho}\bar{\alpha}_{id,\varrho}\mathbb{E}\left\{\tilde{x}_{j,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{j,\varrho|\varrho}+\tilde{x}_{d,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{d,\varrho|\varrho}\right\}\\
&=\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{\alpha}_{i,\varrho}\bar{\alpha}_{ij,\varrho}\mathbb{E}\left\{\tilde{x}_{j,\varrho|\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma\tilde{x}_{j,\varrho|\varrho}\right\}\\
&\le (1+\epsilon_5)^{-1}(1+\epsilon_2^{-1}+\epsilon_3^{-1})^{-1}\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{x}_{j,\varrho|\varrho}^{T}\mathcal{P}_{j,\varrho|\varrho}^{-1}\tilde{x}_{j,\varrho|\varrho}\right\}\\
&=(1+\epsilon_5)^{-1}(1+\epsilon_2^{-1}+\epsilon_3^{-1})^{-1}N\mathbb{E}\left\{V_{\varrho}(\tilde{x}_{\varrho|\varrho})\right\}.
\end{aligned}
$$

Furthermore, based on (31), (46) and Assumption 1, we have

$$
\begin{aligned}
&\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{d=1}^{N}\tilde{\alpha}_{ij,\varrho}\tilde{\alpha}_{id,\varrho}x_{j,\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma x_{d,\varrho}\right\}\\
&=\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}^{2}x_{j,\varrho}^{T}\Gamma^{T}\Phi_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}\Phi_{i,\varrho+1}\Gamma x_{j,\varrho}\right\}\\
&\le (1+\epsilon_5)^{-1}\mathbb{E}\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\alpha}_{ij,\varrho}^{2}\frac{\tau}{\underline{q}_i\underline{b}_i}x_{j,\varrho|\varrho}^{T}x_{j,\varrho|\varrho}\right\}\\
&\le (1+\epsilon_5)^{-1}\sum_{i=1}^{N}\sum_{j=1}^{N}\hat{\alpha}_{ij,\varrho}^{2}\frac{\tau}{\underline{q}_i\underline{b}_i}\mathrm{tr}\left\{(1+\epsilon_6)\mathcal{P}_{j,\varrho+1|\varrho}+(1+\epsilon_6^{-1})\hat{x}_{j,\varrho+1|\varrho}\hat{x}_{j,\varrho+1|\varrho}^{T}\right\}\\
&\le (1+\epsilon_5)^{-1}\sum_{i=1}^{N}\sum_{j=1}^{N}\hat{\alpha}_{ij,\varrho}^{2}\frac{\tau}{\underline{q}_i\underline{b}_i}\mathrm{tr}\left\{(1+\epsilon_6)\bar{\sigma}_jI+(1+\epsilon_6^{-1})\bar{\chi}I\right\}\\
&\triangleq \xi_4.
\end{aligned}
$$

Similarly, by (47), (49) and Assumption 1, it is derived that

$$
\mathbb{E}\left\{\sum_{i=1}^{N}\beta_{1i,\varrho+1}^{2}\beta_{2i,\varrho+1}^{2}\zeta_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\zeta_{i,\varrho+1}\right\}\le\sum_{i=1}^{N}\bar{\beta}_{1i}\bar{\beta}_{2i}\bar{\kappa}_i\bar{\zeta}_i\underline{\hbar}_i\triangleq\xi_5.
$$

Subsequently, it is directly obtained that

$$
\begin{aligned}
&\mathbb{E}\left\{\sum_{i=1}^{N}x_{i,\varrho+1}^{T}C_{i,\varrho+1}^{T}\tilde{\Theta}_{i,\varrho+1}^{T}K_{i,\varrho+1}^{T}\mathcal{P}_{i,\varrho+1|\varrho+1}^{-1}K_{i,\varrho+1}\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}\right\}\\
&\le\sum_{i=1}^{N}\bar{\kappa}_i\underline{\hbar}_i\mathbb{E}\left\{x_{i,\varrho+1}^{T}C_{i,\varrho+1}^{T}\tilde{\Theta}_{i,\varrho+1}^{T}\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}\right\}\\
&=\sum_{i=1}^{N}\bar{\kappa}_i\underline{\hbar}_i\mathrm{tr}\left\{\mathbb{E}\left\{\tilde{\Theta}_{i,\varrho+1}C_{i,\varrho+1}x_{i,\varrho+1}x_{i,\varrho+1}^{T}C_{i,\varrho+1}^{T}\tilde{\Theta}_{i,\varrho+1}^{T}\right\}\right\}\\
&\le\sum_{i=1}^{N}\bar{\kappa}_i\underline{\hbar}_i\mathrm{tr}\left\{\mathrm{tr}\left\{C_{i,\varrho+1}\left[(1+\epsilon_6)\mathcal{P}_{i,\varrho+1|\varrho}+(1+\epsilon_6^{-1})\hat{x}_{i,\varrho+1|\varrho}\hat{x}_{i,\varrho+1|\varrho}^{T}\right]C_{i,\varrho+1}^{T}\right\}\bar{\Theta}_{i,\varrho+1}(\bar{I}-\bar{\Theta}_{i,\varrho+1})^{T}\right\}\\
&\le\sum_{i=1}^{N}\bar{\kappa}_i\underline{\hbar}_i m\big[\bar{\beta}_{1i}(1-\underline{\beta}_{2i})-\underline{\beta}_{1i}^{2}(1-\bar{\beta}_{2i})^{2}[\underline{\theta}_i^{2}+(1-\bar{\theta}_i)^{2}]\big]\bar{c}_i\mathrm{tr}\left\{\left[(1+\epsilon_6)\bar{\sigma}_iI+(1+\epsilon_6^{-1})\bar{\chi}I\right]\right\}\\
&\triangleq\xi_6.
\end{aligned}
$$

Next, according to (46), it is obvious that

$$\mathcal{P}_{i,\varrho|\varrho}^{-1} \geq \frac{(1+\epsilon_1+\epsilon_2)\underline{\ell}^2}{\bar{\sigma_i}} I \triangleq \bar{\hbar}_i I. \tag{50}$$

Hence, combining (36), (49) with (50) yields

$$\bar{\hbar}_i \mathbb{E}\left\{\|\tilde{x}_{\varrho|\varrho}\|^2\right\} \leq \mathbb{E}\left\{V_\varrho(\tilde{x}_{\varrho|\varrho})\right\} \leq \underline{\hbar}_i \mathbb{E}\left\{\|\tilde{x}_{\varrho|\varrho}\|^2\right\}. \tag{51}$$

Based on the above analysis, it can be deduced that

$$\begin{aligned}
&\mathbb{E}\left\{V_{\varrho+1}(\tilde{x}_{\varrho+1|\varrho+1})|\tilde{x}_{\varrho|\varrho}\right\}\\
\leq\ &\left[4(1+\epsilon_5)^{-1}(1+\epsilon_1^{-1}+\epsilon_3)^{-1}+4(1+\epsilon_5)^{-1}(1+\epsilon_2^{-1}+\epsilon_3^{-1})^{-1}N\right]V_\varrho(\tilde{x}_{\varrho|\varrho})\\
&+4\xi_1+\xi_2+\xi_3+\xi_4+4\xi_5+\xi_6\\
\triangleq\ &\phi V_\varrho(\tilde{x}_{\varrho|\varrho})+\upsilon,
\end{aligned} \tag{52}$$

where $\phi = 4(1+\epsilon_5)^{-1}(1+\epsilon_1^{-1}+\epsilon_3)^{-1}+4(1+\epsilon_5)^{-1}(1+\epsilon_2^{-1}+\epsilon_3^{-1})^{-1}N$ and $\upsilon = 4\xi_1+\xi_2+\xi_3+\xi_4+4\xi_5+\xi_6$. It follows from (35) that $0 \leq \phi < 1$.

Finally, in light of (51) and (52), we have

$$\begin{aligned}
\mathbb{E}\left\{\|\tilde{x}_{\varrho|\varrho}\|^2\right\} &\leq \frac{\mathbb{E}\left\{V_\varrho(\tilde{x}_{\varrho|\varrho})\right\}}{\bar{\hbar}_i}\\
&\leq \frac{\phi\mathbb{E}\left\{V_{\varrho-1}(\tilde{x}_{\varrho-1|\varrho-1})\right\}+\upsilon}{\bar{\hbar}_i}\\
&\leq \frac{\phi^\varrho\mathbb{E}\left\{V_0(\tilde{x}_{0|0})\right\}+\upsilon\sum_{i=1}^{\varrho-1}\phi^i}{\bar{\hbar}_i}\\
&\leq \frac{\underline{\hbar}_i}{\bar{\hbar}_i}\phi^\varrho\mathbb{E}\left\{\|\tilde{x}_{0|0}\|^2\right\}+\frac{\upsilon}{\bar{\hbar}_i}\sum_{i=1}^{\varrho-1}\phi^i.
\end{aligned}$$

Then, we can conclude from Lemma 5 that the SE error is EB in the MSS. ■

Remark 5: It is noted that the sufficient condition given in Theorem 2 is dependent on the information of the system matrices, covariance matrices of different noises and coupling parameters. From the derivation process of the theorem, it is clear to see that the Assumption 1 is a crucial prerequisite for analyzing the EB of SE error, which reflects the realistic constraints imposed by energy-limited physical processes in practical application. The involved constraints in Assumption 1 are fairly reasonable based on the implementation of the proposed SE method in the energy-limited physical processes. Moreover, it remains open to further establish more broader sufficient condition with less conservatism.

Remark 6: Note that the main results of this paper in (14)-(16) of Theorem 1 are dependent on some parameters, i.e., ϵ_h ($h = 1, 2, \cdots, 6$) generated by Lemma 1. Obviously, the selection of these parameters can affect the UB of the EEC and their selection principle lies in the minimization of the obtained UB as much as possible. Nevertheless, as stated in [46], the UB of the EEC is a non-convex function with respect to these parameters, which leads to the fact that finding the optimal parameters is extremely difficult in the minimum variance sense. Meanwhile, in Theorem 2, a sufficient condition for the EB of the SE error has been established, which requires that the parameters (i.e., ϵ_1, ϵ_2, ϵ_3 and ϵ_5) satisfy (35). Hence, this adds constraint to the determination of the optimal parameters. In this case, it would be necessary to select or design algorithms that can handle non-convex optimization problems, such as genetic algorithm, particle swarm optimization and Bayesian optimization, and this is one of the future research topics.

Remark 7: So far, we have developed an optimized recursive distributed SE method for time-varying NCNs with RCS, RSDs and hybrid attacks. In particular, an alternative optimal technique has been adopted to obtain the

UB on the EEC for each node by properly determining the EG. In addition, an easy-to-testify sufficient condition has been given guaranteeing the EB with respect to SE error in the MSS. Accordingly, the related information of RCS, RSDs and hybrid attacks has been explicitly reflected in the main results. To be specific, the RCS has been reflected by $\bar{\alpha}_{ij,\varrho}$ and $\hat{\alpha}_{ij,\varrho}$. The case of RSDs can be found in $\bar{\Theta}_{i,\varrho+1}$ as $\bar{\theta}_{i,\varrho+1}$ is included in $\bar{\Theta}_{i,\varrho+1}$. Similarly, the occurrence probabilities of hybrid attacks have been reflected by $\bar{\beta}_{1i,\varrho+1}$, $\bar{\beta}_{2i,\varrho+1}$ and $\bar{\Theta}_{i,\varrho+1}$, and the deception attack signal has been reflected by $\bar{\zeta}_i$.

V. An Illustrative Example

In this section, an illustrative example is utilized to illustrate the feasibility and applicability of the developed distributed SE scheme with applications on the localization of multiple indoor mobile robots.

The localization problem of four indoor mobile robots is solved by the proposed distributed SE approach in this example. On the basis of [21], the kinematic for the robot can be modelled by

$$\begin{aligned} u_{i,\varrho+1} &= u_{i,\varrho} + \delta_{i,\varrho}\cos(\lambda_{i,\varrho}) + \sum_{j=1}^{4}\alpha_{ij,\varrho}\vec{\Gamma}^{u}u_{j,\varrho} + \vec{B}^{u}_{i,\varrho}\varpi^{u}_{i,\varrho}, \\ \eta_{i,\varrho+1} &= \eta_{i,\varrho} + \delta_{i,\varrho}\sin(\lambda_{i,\varrho}) + \sum_{j=1}^{4}\alpha_{ij,\varrho}\vec{\Gamma}^{\eta}\eta_{j,\varrho} + \vec{B}^{\eta}_{i,\varrho}\varpi^{\eta}_{i,\varrho}, \\ \lambda_{i,\varrho+1} &= \lambda_{i,\varrho} + \varsigma_{i,\varrho} + \sum_{j=1}^{4}\alpha_{ij,\varrho}\vec{\Gamma}^{\lambda}\lambda_{j,\varrho} + \vec{B}^{\lambda}_{i,\varrho}\varpi^{\lambda}_{i,\varrho}, \end{aligned}$$

where $(u_{i,\varrho}, \eta_{i,\varrho})$ represents the position and $\lambda_{i,\varrho}$ stands for the orientation for the ith mobile robot. $\delta_{i,\varrho}$ and $\varsigma_{i,\varrho}$ are displacement velocity and angular velocity. $\vec{\Gamma} = \mathrm{diag}\{\vec{\Gamma}^{u}, \vec{\Gamma}^{\eta}, \vec{\Gamma}^{\lambda}\}$ and $\vec{B}_{i,\varrho} = \mathrm{diag}\{\vec{B}^{u}_{i,\varrho}, \vec{B}^{\eta}_{i,\varrho}, \vec{B}^{\lambda}_{i,\varrho}\}$ characterize the inner coupling matrix and system matrix, respectively. $\varpi_{i,\varrho} = [\varpi^{u}_{i,\varrho}\ \varpi^{\eta}_{i,\varrho}\ \varpi^{\lambda}_{i,\varrho}]^T$ is a zero-mean white noise with covariance $Q_{i,\varrho}$.

The position measurement of the ith robot is described by

$$\vec{z}_{i,\varrho} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vec{x}_{i,\varrho} + \vec{\nu}_{i,\varrho},$$

where $\vec{x}_{i,\varrho} = [u_{i,\varrho}\ \eta_{i,\varrho}\ \lambda_{i,\varrho}]^T$.

The initial positions of the four robots are set as (0.5, 3.3), (2.2, 2.2), (3.3, 4.4) and (5.5, 4.4). The initial estimation satisfies $\hat{\vec{x}}_{i,0|0} = \bar{\vec{x}}_{i,0}$ and initial UB of EEC satisfies $\mathcal{P}_{i,0|0} = 2I_6$. The system matrices are chosen as $\vec{B}_{1,\varrho} = \mathrm{diag}\{0.12, 0.12, 0.12\}$, $\vec{B}_{2,\varrho} = \mathrm{diag}\{0.24, 0.24, 0.06\}$, $\vec{B}_{3,\varrho} = \mathrm{diag}\{0.1, 0.1, 0.08\}$ and $\vec{B}_{4,\varrho} = \mathrm{diag}\{0.4, 0.4, 0.1\}$. Assume that $\alpha_{ij,\varrho}$ obeys the uniform distribution on the interval [0.2, 0.4]. The inner coupling matrix is given as $\vec{\Gamma} = 0.01I_3$. Moreover, the nonlinear function is $\vec{h}(\vec{x}_{i,\varrho}) = [u_{i,\varrho}\ \eta_{i,\varrho}\ \lambda_{i,\varrho}]^T + [\delta_{i,\varrho}\cos(\lambda_{i,\varrho})\ \delta_{i,\varrho}\sin(\lambda_{i,\varrho})\ \varsigma_{i,\varrho}]^T$, where $\delta_{i,\varrho} = 5$ and $\varsigma_{i,\varrho} = 0.1$. Some parameters are set as $Q_{i,\varrho} = 0.1I_3$, $\vec{R}_{i,\varrho} = 0.1I_2$, $\bar{\beta}_{1i,\varrho} = 0.99$, $\bar{\beta}_{2i,\varrho} = 0.01$, $\bar{\theta}_{i,\varrho} = 0.75$, $\bar{\zeta}_i = 0.03$, $\epsilon_1 = 0.65$, $\epsilon_2 = 0.01$, $\epsilon_3 = 0.1$, $\epsilon_4 = 0.7$, $\epsilon_5 = 1$ and $\epsilon_6 = 0.5$.

Based on the above given conditions, the experimental results are shown in Figs. 1-7. The actual and estimated trajectories of the four robots are given in Figs. 1-4, respectively. Fig. 5 plots the logarithm of $\mathrm{tr}\{\mathcal{P}_{i,\varrho|\varrho}\}$ and $\mathrm{MSE}_{i,\varrho}$ for the position of the four robots. Here, $\mathrm{MSE}_{i,\varrho}$ means the mean square error of the ith robot at the ϱth instant. It is not difficult to see that the developed SE scheme has a good performance in the estimation of the trajectories of mobile robots.

In addition, in order to further clarify how the cyber attacks influence the SE performance, the impact of the attack probabilities on the estimation performance is analyzed through some comparative experiments. Figs. 6 and 7 plot the average of the $\mathrm{MSE}_{i,\varrho}$ of the four robots under different DoS and deception attack probabilities, i.e., $\bar{\beta}_{1i,\varrho} = 0.9$,

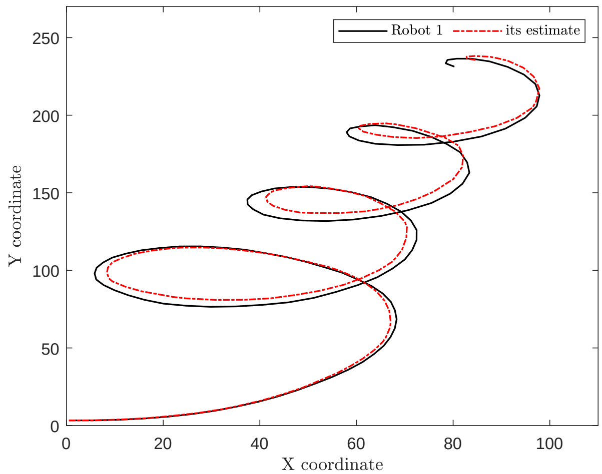

Fig. 1. State and its estimate of Robot 1.

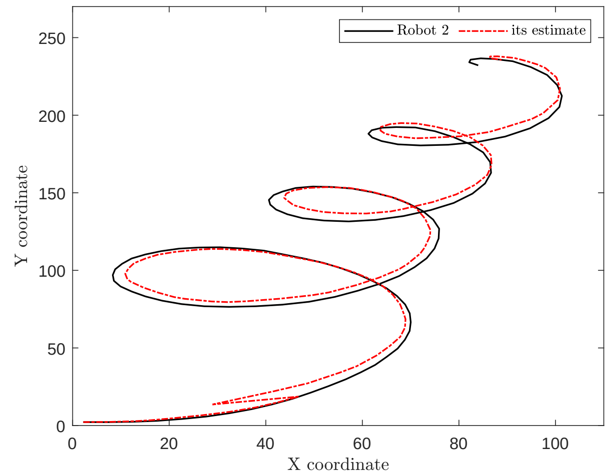

Fig. 2. State and its estimate of Robot 2.

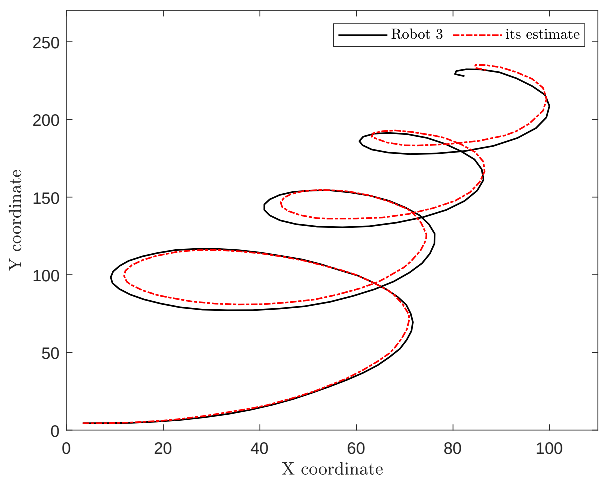

Fig. 3. State and its estimate of Robot 3.

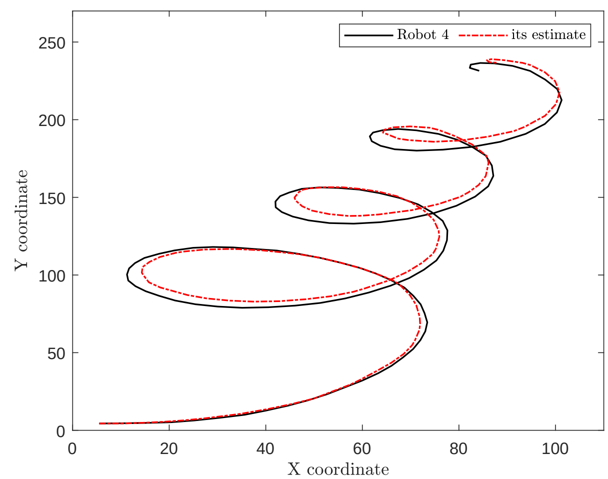

Fig. 4. State and its estimate of Robot 4.

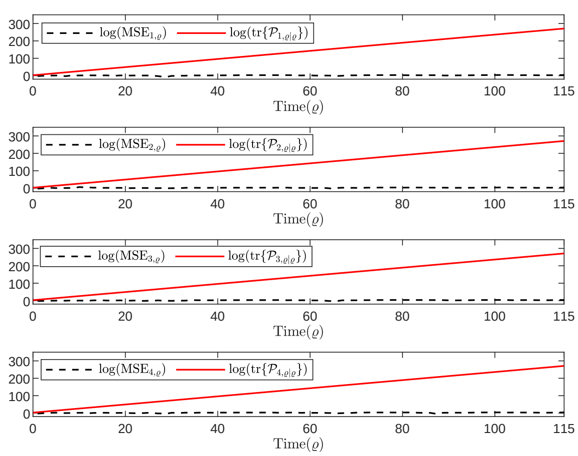

Fig. 5. $\log(\mathrm{MSE}_{i,\varrho})$ and $\log(\mathrm{tr}\{\mathcal{P}_{i,\varrho|\varrho}\})$ for four mobile robots (i=1, 2, 3, 4).

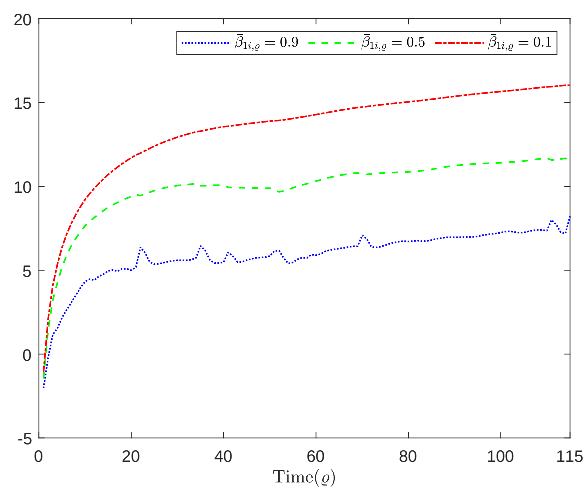

Fig. 6. log(MSE) with different DoS attack probabilities.

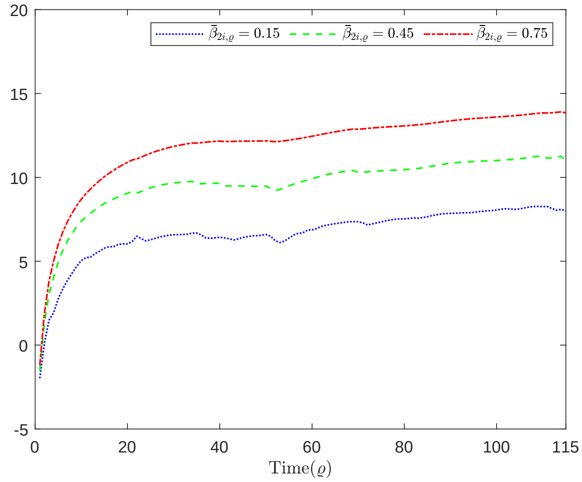

Fig. 7. log(MSE) with different deception attack probabilities.

TABLE II
log(MSE) WITH DIFFERENT DOS ATTACK PROBABILITIES

Time(ϱ)	$\cdots$	40	41	42	43	$\cdots$	100	101	102	103	$\cdots$
$\bar{\beta}_{1i,\varrho}=0.9$	$\cdots$	5.478	6.066	5.836	5.492	$\cdots$	7.237	7.301	7.320	7.277	$\cdots$
$\bar{\beta}_{1i,\varrho}=0.5$	$\cdots$	10.060	9.939	9.926	9.923	$\cdots$	11.405	11.424	11.444	11.461	$\cdots$
$\bar{\beta}_{1i,\varrho}=0.1$	$\cdots$	13.555	13.581	13.619	13.656	$\cdots$	15.651	15.678	15.704	15.730	$\cdots$

TABLE III
log(MSE) WITH DIFFERENT DECEPTION ATTACK PROBABILITIES

Time(ϱ)	$\cdots$	40	41	42	43	$\cdots$	100	101	102	103	$\cdots$
$\bar{\beta}_{2i,\varrho}=0.15$	$\cdots$	6.432	6.393	6.343	6.271	$\cdots$	8.057	8.105	8.127	8.111	$\cdots$
$\bar{\beta}_{2i,\varrho}=0.45$	$\cdots$	9.643	9.496	9.486	9.487	$\cdots$	11.006	11.026	11.046	11.062	$\cdots$
$\bar{\beta}_{2i,\varrho}=0.75$	$\cdots$	12.161	12.136	12.142	12.149	$\cdots$	13.598	13.618	13.638	13.658	$\cdots$

$\bar{\beta}_{1i,\varrho}=0.5$, $\bar{\beta}_{1i,\varrho}=0.1$, and $\bar{\beta}_{2i,\varrho}=0.15$, $\bar{\beta}_{2i,\varrho}=0.45$, $\bar{\beta}_{2i,\varrho}=0.75$, respectively. Subsequently, Tables II and III present the values of log(MSE) under different DoS and deception attack probabilities, respectively. It can be seen that the log(MSE) increases as the attack probability increases, that is to say, a higher attack probability can lead to the degraded accuracy of the proposed algorithm. In other words, more incomplete and inaccurate measurement information received by the estimator will lead to lower estimation accuracy.

VI. CONCLUSIONS

In this paper, we have tackled the recursive distributed SE issue for time-varying NCNs with RCS, RSDs and hybrid attacks. Accordingly, a UB on the EEC for each node has been deduced by the mathematical induction method, whose trace has been minimized by designing the EG in a proper way. Moreover, a sufficient criterion has been provided to ensure that the SE error is EB in the MSS. Finally, in order to illustrate the efficiency of the developed recursive distributed SE method, simulation experiments with comparisons and discussions have been provided. Furthermore, the possible topics for the future research can be the extension of the main results in this paper by considering the impact of communication resource constraints.

Declaration of competing interest

The authors claim that there are no potential conflicts of interest. This submission has been approved by all co-authors.

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